

Transition Rule Elicitation Methods for Urban Cellular Automata Models

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Abstract: In this chapter, transition rules used in urban CA models are reviewed and classified into two categories: transition potential rules and conflict resolution rules. Then, four widely used rule elicitation methods: Regression analysis, Artificial Neural network (ANN), Visual calibration, and Analytical Hierarchy Processing – Multi Criteria Evaluation (AHP-MCE) are discussed. Most of these methods are data driven methods and can be used to elicit the transition potential rules in the urban CA models. In the following, three possible rule elicitation methods: Interview, Document analysis, and Card sorting are explained and demonstrated. These three methods are driven by knowledge and can be used to elicit conflict resolution rules as well as transition potential rules in urban CA models.

1. INTRODUCTION

1.1 Cellular Automata Model

Cellular Automata (CA) were firstly devised by John von Neumann and Stanislaw Ulam in the 1940s as a framework to investigate the logical underpinnings of life. One can say that the “cellular” comes from Ulam and the “automata” comes from von Neumann (Rucker, 1999). CA can generate complex behaviors based on a relatively simple set of rules. It makes them suitable to be applied in complex system simulation, such as urban development, fire, disease spreading, traffic simulation etc. Basic CA, as

defined by Ulam, von Neumann, and Wolfram (Wolfram, 1984; Rucker, 1999) have five components: Lattice, Neighborhood, Cell State, Transition rules, and Time. The lattice is the space where CA exist and evolve over time. The neighborhood is the place, where the cells are exactly located. A neighborhood consists of an examined cell itself and any number of cells in a given configuration around the examined cell (Torrens, 2000). Cell state is the status or value, which a cell can take. Basic CA models often have a Boolean cell state, 0 or 1. The transition rule is the control component in the CA model that determines the future cell state as a function of the current state of a cell and the states of its surrounding cells. Time is the temporal scale in the CA model and is represented in discrete time steps.

1.2 Urban Cellular Automata Model

Since CA models are good at generating different scenarios based on pre-defined criteria or constraints, many researchers have employed CA models to simulate urban development and try to answer different “what-if” questions. These questions include land use dynamics (White and Engelen, 1993, White, Engelen, et al., 1997), regional scale urbanization (Semboloni, 1997; White and Engelen, 1997), poly centricity (Wu 1998), urban spatial development (Wu and Webster, 1998), and urban growth and sprawl (Batty, Xie, et al., 1999; Clarke, Hoppen, et al., 1997).

CA models have demonstrated their ability in urban research, especially for academic study. However, the use of CA in urban simulations often entails substantial departures from the original formal structure of CA described by von Neumann, Ulam, and Wolfram. Although the application of CA to urban systems seems natural and intuitive, this is not in itself sufficient justification for their use (Coculelis, 1985). Basic CA, as defined by Ulam, von Neumann, and Wolfram (Wolfram, 1984; Rucker, 1999), is not well suited to urban applications; since the framework is too simplified and constrained to represent real cities (Torrens, 2000).

In order to simulate an urban system successfully, it is necessary to make some modifications to the basic CA model. In the five components of the basic CA model, transition rules are the most important part. They serve as the algorithms that code real-world behavior into the artificial CA world. In fact, in the context of urban CA, transition rules are responsible for explaining *how cities work* (Torrens, 2000). Different transition rules will generate different simulation results and the simulation precision is mainly determined by transition rules. Therefore accurate elicitation and understanding of transition rules is at the heart of CA modeling.

To that end, we see the need to explicitly differentiate transition rules and consider conflict resolution rules, which we discuss in section 2. Then, in

section 3, we review a number of methods to establish transition rules, most of which are data driven. In section 4 we review methods to fire knowledge driven rule elicitation, which are of particular interest to establish conflict resolution rules. Conclusions are drawn in section 5.

2. TRANSITION RULES IN URBAN CA MODELS

2.1 Transition Rules in Basic CA Models

Transition rules specify the behavior of cells between time-steps, deciding the future states of cells. In urban models, states mostly are land-uses. In a strict CA, transition rules are applied uniformly across cells in a synchronous fashion (Torrens, 2000). As argued by Batty (1997) the basic CA rules are formulated with IF, THEN, ELSE sentences and rely on the input from a neighborhood template to evaluate cell state changes. For example, one typical CA transition rule can take the following form:

$$TP_{T+1} = f(S_T, NB) \quad (\text{Equation 1})$$

TP_{T+1} — Transition Potential of tested cell in time T + 1

S_T — Tested cell state in time T.

NB — Neighborhood states

In this formula, each cell's Transition Potential in time T + 1 is determined by neighborhood states and its own state in time T.

2.2 Transition Rules in Urban CA Models

To make CA models applicable to urban environments we need to subject transition rules to conflict resolution rules. When applying the CA model to urban systems, many influential factors need to be considered as well as many states a cell can take. The state of a tested cell will not only be affected by the neighborhood effect (e.g. neighboring land uses), but also be affected by other influential factors in the urban system. For example, in most urban CA models, factors such as accessibility and suitability will also be included in transition rules. Thus, the cell's transition potential for instance from rural to urban or from one land use type to another can be calculated as follows.

$$TP_{T+1} = f(S_T, NB, AC, SU...) \quad (\text{Equation 2})$$

TP_{T+1} , S_T and NB have the same meaning as Equation 1.

AC — Accessibility effect

SU — Suitability effect

From the above model, people can calculate the transition potential for different states in each of the tested cells. In this paper, the above formula

will be called the *transition potential rule*. In most available urban CA models, the above rule will be regarded as the transition rule. In these models, the cell's state will only be determined by its transition potential, where the state for which the highest potential was calculated, is the state that the cell will take in the next time step.

In some urban CA models, which aim to simulate the change from non-urban to urban area, such simplicity is accepted, since we only have one objective cell state: urban area. However, if we want to model more precise urban development process, such as the change between different land-use types, the above simplicity is not sufficient because in such a model, a cell's transition potential to different land use types will be different and because the final cell state will not only be determined by its transition potential, but also be affected by other factors.

For example, we have four cells: A, B, C and D. Each of them has two possible cell states, namely residential or industrial land use. Suppose each land use type will require two cells. In this case, the cells' transition potential can be calculated from above equation resulting in for example values found in Table 1.

Table 1. Changing Potentiality of four tested cells.

A	C	Changing Potential to	0.9	0.8	Changing Potential to	0.8	0.9
B	D	Residential Area	0.7	0.5	Industrial Area	0.6	0.6

However, to which state the cell A, B, C and D will change is still unknown. If the modeler changes the cell states according to the ranking of **its transition potentiality**, then the above cells will evolve into the following states. See the following table 2.

Table 2. One possible changing results.

A	C	Changing states according to their Potential Ranking	R	I
B	D		R	I

If the modeler uses a different rule, the result of these four cells will be different. For example, if the modeler decides that the residential land use type has changing priority, then the final cell states will be the following.

Table 3. Another possible changing results.

A	C	Changing states according to the priority of land use types.	R	R
B	D		I	I

In this case, although cell C has a higher potential to change into industrial area, yet since the residential land use type has the priority to select the cells and cell C has the second highest changing potential for residential land use type, thus its final state will be residential area. And if two land use types have different land demands, the case will be more complicated.

With the two different approaches the modeler implies that each cell changes to the state with the highest potential, or that each cell changes according to an outside power e.g. in the form of a policy, or financial power, etc.

From this discussion, we can see a cell's final state is not only determined by a cell transition potential value, but also affected by other transition rules. In this paper, these rules which will deal with possible land use conflict will be named *conflict resolution rules*. The whole structure of transition rules of urban CA models can be represented as in figure 1. In most urban CA models, the conflict resolution rule has been neglected. The cell's status is only be determined by its transition potential.

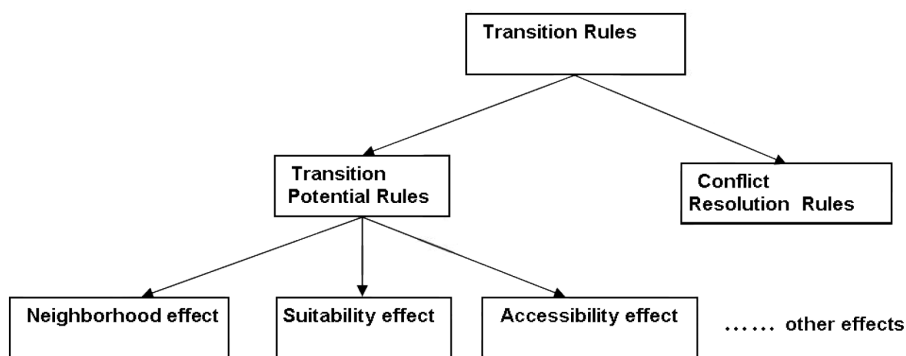


Figure 1. Transition rule structure in urban CA models.

Although in many cases transition potential and conflict resolution rules can be clearly differentiated, situations do exist where they cannot, for instance, if a cell with current state A can possibly change to state B or C, dependent on the concentration of state A cells or concentration of state B or C cells. To illustrate, the potential of a cell of poor quality residential state, to be converted to institutional state or to be converted to higher quality residential state, could be a function of the current concentration of poor quality residential cells, in relation to the current concentrations of institutional and higher quality neighbouring cells. Is that an expression of a neighbourhood effect or is it a conflict resolution rule? Without the

distinction it would have been considered a neighbourhood effect, but if powers of lobbies of developers are behind these transitions, conflict rules would be more meaningful. Obviously, with such explicit definition of conflict resolution rules, we seem to treat cells as agents, although we are certainly not talking about agent-based systems.

3. TRANSITION POTENTIAL RULE ELICITATION METHODS, MOSTLY DATA DRIVE, SOME KNOWLEDGE DRIVEN

In this part, four available rule elicitation methods: regression analysis, artificial neural network, visual observation (trial-error) and Analytical Hierarchy Process and Multi-Criteria Evaluation (AHP-MCE) method will be reviewed.

3.1 Regression Analysis

Regression analysis has been used to elicit transition potential rules in research by Wu (2000) and Sui and Zeng's (2001). In these urban CA models, the rules were elicited in the following way. First, the modeller identifies the possible influence factors, which affect urban cell's state changes. Probably these factors include neighbourhood effect, suitability effect, and accessibility effect. Wu's (2000) model included the neighbourhood effect and accessibility effect. Sui and Zeng's (2001) model included all these three effects.

Then the modeller uses some methods to measure these different effects. For example, the neighbourhood effect can be measured by the ratio of developed cells to all neighbourhood cells. The suitability of cells can be calculated by traditional land suitability analysis. And the accessibility of cells can be measured by the distance to different areas (urban centre, main road...) with GIS software.

In the third step, the modeller overlays different land use maps and identified changing areas. Then random samples will be selected from these changing areas. Next, modellers use multiple regression analysis to find explanatory coefficients to the different influence factors. These coefficients will be input in the transition potential rule to calculate the changing potentiality of different cells.

Finally the modeller uses regression analysis to calculate future land demand based on past urban development. The predicted land demand will be used as the threshold to determine how many cells will change states in

the simulation process. After the above analysis, the transition potential rule can be represented by the following formula.

$$TP = \sum_{i=1}^n CiRi \quad (\text{Equation 3})$$

Here, TP is the cell's transition potentiality. Ri is the influence factor which has been identified and regressed in the above process. Ci is its coefficient. This rule elicitation method is a data-driven method without consideration and understanding of people's decision making process, which will affect urban development directly.

3.2 Artificial Neural Network: ANN

In some research, Artificial Neural Networks have been used to elicit the transition rules in urban CA models. Li and Yeh (2001, 2002) have developed their models based on ANN. In their research, a Back-Propagation (BP) neural network was employed, which is good at capturing non-linear characteristics and strong in prediction. Firstly, the authors identified the possible factors which will affect land use change. In Li and Yeh's (2001, 2002) research neighbourhood effects, accessibility effects and suitability effects were identified. Then, some methods were used to measure the above three effects. In the third step, a neural network was formed. Generally, the modeller will select a three-layer network (input layer, hidden layer, and output layer). In such network, each layer will include some neurons. For the input layer, the amount of neurons will be equal to the selected variables. To the output layer, the number of neurons will be same as the predicted land use types. Suppose we want to predict the land use change from rural to urban area, we will have two neurons in the output layer. Neurons in hidden layer are selected based on experience.

After forming the network structure, the modeller still needs to select some functions to link the neurons. In Li and Yeh's case, since the authors used a Back-Propagation (BP) neural network to elicit the transition rules, they selected the functions according to BP network regulation. In the fourth step, a land use change map will be formed by overlaying historical data. Generally, the network will require at least two land use change maps, one will be used to select some random samples to train the network; the other change map will be used to test the formed network. After that, a mature neural network will be formed and can act as the transition potential rule in urban CA models. Finally, the total land consumption in a given period will be calculated from the historical data and be used to control the whole iteration times of the formed network.

However, the above rule elicitation method also inherits some shortcomings, e.g. the black box operation. People don't know what the

exact transition potential rules are. Only some data go in and some results come out. This drawback restricts its application in rule elicitation, in which a high interpretative ability is required. Furthermore, how to determine the neural network structure is still under discussion. Finally, such rule elicitation method is also a data-driven method, although good at generating urban configurations, yet lacking understanding of the real urban development process.

3.3 Visual Observation (Trial and Error) Method

Although many researchers have used visual observation to calibrate their models and elicit the transition rules, yet such a method is best demonstrated in the RIKS CA models, which are developed by the Research Institute of Knowledge System (RIKS), in the Netherlands. Generally, RIKS models include two parts: the micro model and the macro model. The former part is developed from CA model and will calculate the transition potential of different cells. The latter part will control the detailed cell state change based on the outside land demand. Here, RIKS models used the visual observation method to elicit the interactions of the different land use types within the neighbourhood, which can be represented by distance curves like the following figure 2. The horizontal value: 1...7 denotes the distance between tested cell and central cell. The vertical value: $-100 \sim +100$ reflects the interaction between tested cell and central cell. Based on these interactions, the modeller can describe the neighbourhood effect easily and then develop out the transition potential rules for the models.

In order to get these curves, firstly, the modeller will use some coarse curves to reflect the interaction of different land use types in the case study area. Then, some simulation results will be generated based on these curves. The modeller will compare the simulation results with the real urban land use map and try to adjust the parameter values. After rough adjustment, a new simulation result will be generated, which is then compared with the real land use map. If the accuracy was increased then the parameter will be modified in a subtler step and in the same direction as the first modification. If the simulation accuracy was decreased, it may suggest the first adjustment is in a wrong direction. Clearly, this process is a time consuming process and full of uncertainty, which limits its application greatly.

3.4 Analytical Hierarchy Process and Multi-Criteria Evaluation (AHP-MCE) Method

From the above introductions, we can see these rule elicitation methods are driven by data and lack of the consideration about people's decision making

process, which in the end is the determining force in urban development process. Comparing to these methods, Wu and Webster's (1998) method as it tries to elicit behaviour-driven transition rules. In their model, analytical hierarchy process (AHP) and multi-criteria evaluation (MCE) have been used to capture the characteristics in people's decision making process and the elicited rule can be quantified by pairwise comparison and weighting process.

In detail, the transition rule can be elicited in the following way. Firstly, the modeller identifies some factors which will affect the land development in a case study area. Then he will form a criteria hierarchy to represent the relationship between these factors and the simulation objective. Thirdly, the factors' importance will be determined by pairwise comparison, a weighting process to establish importance of the different factors. This step is very important in the whole rule elicitation process, since in this step, the decision makers' opinions will be represented by their different weight sets. Wu and Webster (1998) simplified this step and determined the factors' weights according to possible planning policies and their own understanding of the urban development.

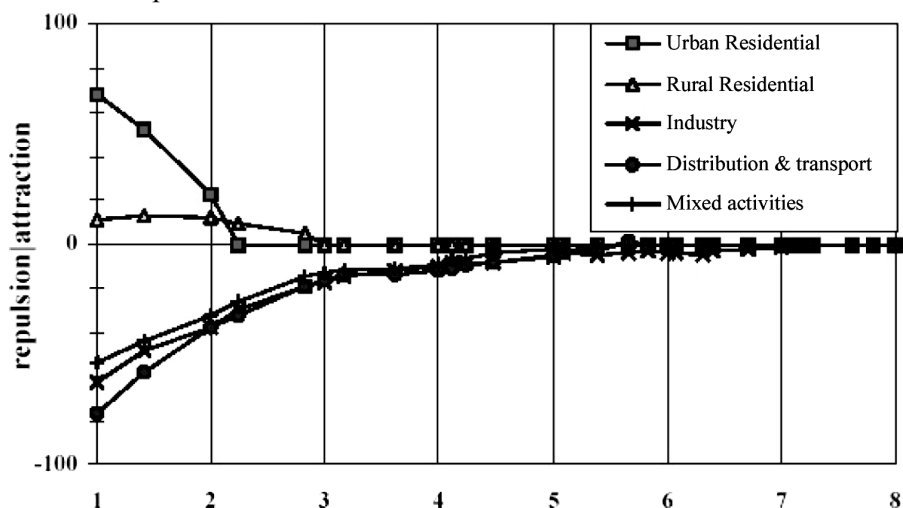


Figure 2. Distance Curves in RIKS models (Engelen, Geertman, et al., 1999).

Comparing with the above data-driven methods, this method is easier to reflect peoples' decision making priorities. And it is possible to generate different urban development scenarios based on different decision making processes. However, similar to the other methods, this method also only focuses on the transition potential rule in the CA models. Some questions such as how to select suitable decision makers and deal with their conflicts

about urban developments should be well considered in the future modelling process.

4. POSSIBLE KNOWLEDGE DRIVEN RULE ELICITATION METHODS

Generally speaking, rule elicitation is one kind of knowledge acquisition form, which aims to develop some transition rules based on available knowledge. Burge discussed (1998) many knowledge acquisition methods such as: *Interviewing, Case Study, Protocols, Critiquing, Role Playing, Simulation, Prototyping, Teach back, Observation, List Related, Construct Elicitation, Sorting, Laddering, and Document Analysis*. In the following the authors will use some of the above knowledge acquisition methods to elicit and represent conflict-resolution rules, which are derived from stakeholders' different opinions.

4.1 Interviewing, Document Analysis and Sorting

4.1.1 Interviewing

Interviewing is the most widely used knowledge elicitation techniques for knowledge based systems, especially in some special fields such as interface design, rule finding etc (McGraw, 1992). Here interviewing will be used to find out suitable stakeholders according to different simulation objectives. In detail, it might work in the following way.

1. Formulate the simulation objective to interviewees (experts, decision makers...). Since different simulation objectives might need different stakeholders, thus we should select stakeholders based on models' simulation objectives. For example, if one model wants to simulate urban sprawl process and another model wants to simulate the residential-commercial land use changing process, the selected stakeholders might be quite different.
2. Encourage the interviewees to list some possible stakeholders which are related to simulation objectives.

By this method, possible stakeholders to a given simulation objective might be identified.

4.1.2 Document Analysis

In this step, document analysis will be used to rank and validate the different stakeholders which have been identified in the above step. In detail it might work in the following way.

1. Select corresponding documents, which recorded the recent land use changes in the case study area. The source may come from official or unofficial records.
2. Analyze the land use changes process and try to find out the roles of different stakeholders in such process. The focus might be on those documents, which recorded the happened land use conflict in case study area. For example, in some period, a commercial investor and an industrial investor requested a same land parcel. If finally, the commercial investor won the conflict, it may suggest that the commercial investor had priority in the land conflict process. Based on these historical records, modeler can assign them with different weights or rank them in a sequence order to reflect such difference.

Following the above steps, possible stakeholders related to simulation objective and their relative importance can be identified.

4.1.3 Sorting

In sorting method, domain entities are sorted to determine how the expert classifies their knowledge. The domain expert is presented with a list of entities to be sorted. They are then asked to sort them either using pre-defined dimensions or along any dimension they feel is important. Subjects may be asked to perform multiple sorts, each using a different dimension (Burge 1998). Here, sorting will be used to elicit transition potential rule. The detailed working processes will be the following.

1. Write down the possible measurement factors such as, soil types, terrain, slope, and distance to urban center...on the cards and present them to the selected stakeholders. Make them clear and understandable. Also suggest the stakeholders list some new measurement factors.
2. Formulate some sub-effects which will be included in the transition potential rule, such as the neighborhood effect, accessibility effect, suitability effect, planning influence, and social-economic influence ...
3. Let the selected stakeholders sort the measurement factors according to these sub-effects. For example, soil type might be sorted to reflect the suitability effect. Distance factors will be sorted to reflect the accessibility effect...In this step the stakeholders can also form new

sub-effect. Require these stakeholders to sort the cards as possible as they can until they cannot find some new categorizing methods. Finally require the stakeholders to explain why they sort the cards in this way.

By card sorting, modeler can identify suitable factors to measure the transition potential rule in urban CA models.

4.2 Demonstrating the Rule Elicitation Process

In the following part, based on different simulation objectives, the above rule elicitation methods will be demonstrated.

4.2.1 Simulate the Change from Non-Urban to Urban Area

If the modelers want to simulate urban expansion in case study area, the transition rule might be elicited in the following way.

1. By interviewing, a modeler can identify the important stakeholders in the past urban expansion process. Suppose the modeler found a residential investor and a commercial investor are the main driving forces in the urban expansion process. Then some document analysis will be done, which aims to find out the investors' rankings in the urban development process. Here, the focus will be put on those documents, which recorded the past land use conflict in the case study area. By analysis of these conflicts, the investors' relative importance can be identified. Ideally this importance should be quantified in the form of a weight: *WR* and *WC*. If more than 2 important stakeholders have been identified in such process, their weights can also be attained through this method.
2. Let the identified stakeholder select suitable factors to represent the transition potential rules. Here card sorting will be used. And finally different stakeholder probably will form different transition potential rules.
3. Use the Analytical of Hierarchy Process (AHP) method to form two different hierarchy structures of transition potential rules: *TPr*, *TPc*. *TPr* comes from residential investor and *TPc* comes from commercial investor.
4. Use the pairwise comparison and weighting method to quantify the different factors in these two hierarchical transition rules; and use the following formula to calculate the transition potential rules: *TPr and TPc*. Here take *TPr* as an example.

$$TPr = w1*neighborhood\ effect + w2*suitability\ effect + w3*accessibility\ effect... \quad (\text{Equation 4})$$

$$= w1*deve-density + w2*(wa*slope + wb*terrain + wc*soil) + w3*(wd*Dist-Road + we*Dist-Center)... \quad (\text{Equation 5})$$

5. Use the firstly assigned weights of residential investor and commercial investor WR and WC to assemble these two hierarchical transition rules (TPr and TPc) as $TPwhole$. Its structure can be found in the following figure 3.

$$TPwhole = WR*TPr + WC*TPc \quad (\text{Equation 6})$$

(Here Deve-density means the development density, which can be measured by the ratio of developed cells to the total cells in the neighbourhood. We can use this method to reflect neighbourhood effect.)

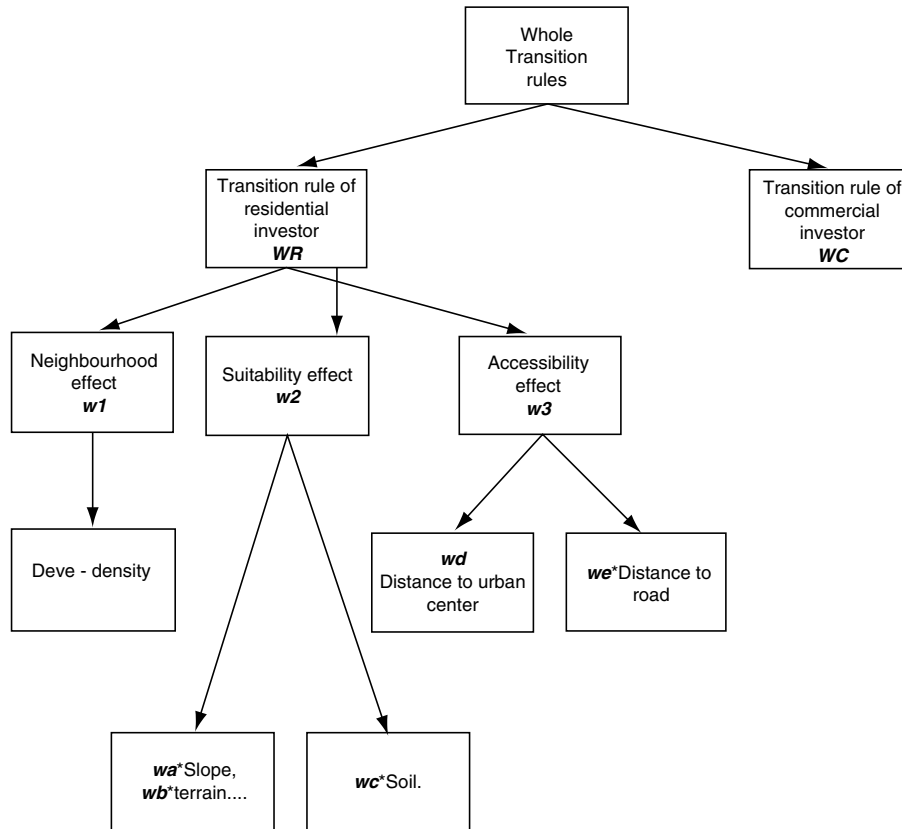


Figure 3. Combined Transition Rule elicited from residential and commercial investors.

In this model, the possible land use conflict between residential investor and commercial investor has been modeled in the above formula by their different weights WR and WC . The cell's changing potential from non-urban to urban area can be calculated from the above formula. The total amount of cells, which developed in urban area, will be controlled by the predicted land consumption.

4.2.2 Simulate the Inter-Changing Among Different Land Use Types

In the above example, the modeler only has one final cell state: urban area, thus it's possible to combine the transition potential rule and conflict resolution rule together. If modelers want to simulate the change among different land use types, the conflict resolution rule should be considered separately. The rule elicitation process might be the following.

1. Use the above step 2 and step 3 to get the transition potential rules from residential investor and commercial investor separately.
2. Run the above transition potential rules separately. Then, each cell will attain two values, which represent the transition potential to residential and commercial land use type respectively. Clearly in this situation, some land use conflict will happen, since some land parcels are suitable for both (residential area and commercial area). To solve this conflict, some resolution rules should be applied.
3. Use document analysis to attain suitable ranks about residential and commercial investors. After ranking their positions, the modeler still needs to attain their possible land demand per year. This information also can be elicited from document analysis. In some cases, these demand can be predicted from past land consumption data.
4. After identifying their ranking positions and land demands, the modeler can change the cell states as follows. Suppose, the commercial investor has the priority in land development process and its land demand will be 200 cells. Then, the modeler should satisfy its land demand first, even some potential to residential area is higher. When its land demand is satisfied, the residential investor will get the choice to allocate the cells. And if we have more than two land use types, the possible land use conflict can also be resolved in this way, which is based on their rankings and possible land use demands.

5. CONCLUSION AND DISCUSSION

In this paper, we have argued that a distinction should be made between transition potential rules and conflict resolution rules. The transition potential rules can calculate the transition potential of different cells. The conflict-resolution rules will determine the cell's final state in CA models, which is of particular interest for those cases where a cell has high potential for multiple purposes. In most CA models, conflict resolution rules have been ignored and the cell's state is determined by its transition potential only.

After defining the concept of transition rule in urban CA models, four rule elicitation methods were discussed. They are Regression analysis, ANN, Visual comparison (trial and error) and AHP-MCE method. The former two methods are data driven methods, which try to elicit the transition rules from historical data. The visual comparison method is very time consuming and full of uncertainty. Comparing with these three methods, AHP-MCE is more hopeful, and aims to develop the transition potential rule based on people's decision making process. However, all these 4 rule elicitation methods focus on the transition potent rules.

Therefore, we also discussed three possible rule elicitation methods: interview, document analysis and sorting. These three methods are knowledge driven methods and intended to elicit the conflict resolution rule as well as the transition potential rule. Finally, these rule elicitation methods have been demonstrated according to different simulation objectives.

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6. REFERENCES

- Batty, M., 1997, "CA and Urban Form: A primer", *Journal of the American Planning Association* **63**(2): 266-274.
- Batty, M., Y. Xie, and Z. Sun, 1999. "Modelling urban dynamics through GIS-based cellular automata", *Computers, Environment and Urban Systems* **23**(3): 205-233.
- Burge, J. E., 1998, *Knowledge Elicitation for Design Task Sequencing Knowledge*, Master thesis, Worcester Polytechnic Institute, Worcester, Mass, USA.
- Clarke, K. C., S. Hoppen, and Gaydos, L., 1997, "A self-modifying cellular automaton model of historical urbanization in the San Francisco Bay area." *Environment and Planning B* **24**(2): 247-261.
- Couclelis, H., 1985, "Cellular worlds: a framework for modelling micro-macro dynamics", *Environment and Planning A* **17**(5): 585-596.
- Engelen, G., S. Geertman, P. Smits, and C. Wessels, 1999, "Dynamic GIS and Strategic Physical Planning Support: a practical application to the IJmond/Zuid-Kennemerland region", in: Stillwell, Geertman, and Openshaw (eds.) *Geographical Information and Planning*, Springer, p. 87-111.

- Li, X. and A. G.-O. Yeh, 2001, "Calibration of cellular automata by using neural networks for the simulation of complex urban systems." *Environment and Planning A* **33**(8): 1445-1462.
- Li, X. and A. G.-O. Yeh, 2002. "Neural-network-based cellular automata for simulating multiple land use changes using GIS." *International Journal of Geographical Information Science*, **16**(4): 323-343.
- McGraw, K. L., 1992, "Review: Knowledge Elicitation for User Interface Design", *Designing and Evaluating User Interfaces for Knowledge Based Systems-* Chapter 4: Ellis Horwood Limited, p. 45-62.
- Rucker, R. 1999, *Seek! Selected Nonfiction*, New York. Four Walls Eight Windows, New York.
- Semboloni, F., 1997, "An urban and regional model based on cellular automata", *Environment and Planning B*, **24**(4): 589-612.
- Sui, D. Z. and H. Zeng, 2001, "Modelling the dynamics of landscape structure in Asia's emerging desakota regions: a case study in Shenzhen." *Landscape and Urban Planning* **53**(1-4): 37-52.
- Torrens, P. M., 2000, "How cellular automata models of urban system work. (1.theory)" *Centre for Advanced Spatial Analysis Working Paper Series 28*, University of College London, London.
- White, R. and G. Engelen, 1993, "Cellular automata and fractal urban form: a cellular modelling approach to the evolution of urban land use patterns", *Environment and Planning A* **25**(8): 1175-1199.
- White, R. and G. Engelen, 1997, "Cellular automata as the basis of integrated dynamic regional modelling," *Environment and Planning B* **24**(2): 235-246.
- White, R., G. Engelen, and I. Ujje, 1997, "The use of constrained cellular automata for high-resolution modelling of urban land use dynamics", *Environment and Planning B* **24**(3): 323-343.
- Wolfram, S. (1984) "Cellular automata as models of complexity," *Nature*, 31(4), p. 419-424.
- Wu, F., 1998, "Simulating urban encroachment on rural land with fuzzy-logic-controlled cellular automata in a geographical information system." *Journal of Environmental Management* **53**(4): 293-308.
- Wu, F., 2000, "A Parameterised Urban Cellular Model Combining Spontaneous and Self-Organising Growth", in: P. Atkinson and D. Martin (eds.) *Geocomputation: Innovation in GIS 7*, Taylor & Francis, London, p. 73-85.
- Wu, F. and C. J. Webster, 1998, "Simulation of land development through the integration of cellular automata and multi-criteria evaluation." *Environment and Planning B* **25**(1): 103-126.