A catalogue of built forms, using a binary representation

Philip Steadman and Linda Waddoups

University College London, and The Open University
Bartlett School, and Department of Design and Innovation
London and Milton Keynes
United Kingdom

ABSTRACT

A technique is described for the representation of a class of rectangular built forms. Each individual form is produced by applying a series of transformations to a single generic or ‘archetypal’ form, which is designed to take care of the broad constraints, on built space, of close-packing and the requirements for natural light and views. Parts of the archetype which are selected for inclusion in any particular built form are then designated by 1s, and parts which are suppressed by 0s. This makes it possible to assign a unique binary code to each different (undimensioned) built form produced from the archetype. Binary codes corresponding to all legitimate forms may then be arranged in ascending order, to create a comprehensive catalogue. The paper describes such a catalogue comprising forms with up to four courtyards, described by 22-digit binary strings. Metric values may be assigned to the various dimensions of each form, making it a matter of simple arithmetic to compute such attributes as volume, surface area, minimum site area or floor space index. From logical operations on the binary strings it is possible to identify a series of configurational characteristics of the corresponding forms, such as their overall plan shapes, the number of courtyards or the potential for symmetry. The catalogue may thus be searched for built forms fulfilling some set of specifications, for example total floor area, site size and certain desired shape attributes. Worked examples are illustrated from the design of multi-storey office buildings. Possible applications are suggested for this approach, in architectural science and the early strategic stages of architectural design.

1 AN ARCHETYPAL BUILT FORM

In this paper we present a technique for representing and cataloguing built forms. The term was introduced by March who defined built forms as “mathematical or quasi-mathematical models...which are used to represent buildings to any required degree of complexity in theoretical studies” (March 1972 p.56). A built form for the present purpose is a diagrammatic abstraction from the geometrical form of a real building, in which much of the fine detail is ignored. All of the articulation and decoration of facades is left out of account; and much of the internal arrangement of individual rooms is also ignored. What remains is a general description of the shape of the building envelope, plus a rather approximate representation of the subdivision of the
interior into zones of different kinds. We will confine discussion here to built forms whose geometry is orthogonal or close to orthogonal.

Imagine then a notional built form as in figure 1(a). It has a large number of storeys. In plan it consists of a regular pattern of nine courts, three in each direction. This number of courts is arbitrary – there could be more. The courtyards do not penetrate however through the full height of the form.

Below them (figure 1(b)) there is an (unspecified) number of storeys in which built space extends across the entire form. All space within the built form is then divided into three types of zone, depending on the lighting conditions. Space on the topmost level of the courtyard floors, and on the topmost level of the deep-plan floors below the bases of the courtyards, can potentially be top-lit by roof lights (dark tone). Space adjacent to external walls around the perimeter of the form, and facing into the courtyards, can potentially be side-lit by windows (medium tone). All remaining space in the interior of the form must necessarily be lit by artificial light (light tone). Notice the artificially-lit strips that run between side-lit zones in the courtyard floors. Although the number of courts shown in the figure is notional, the form should not be imagined as continuing indefinitely in either direction. On the contrary, it is bounded on all four sides by outward-facing side-lit zones. In previous work we have referred to this as an ‘archetypal’ built form (Steadman 1998, Steadman in press). All dimensions of the archetypal form are to be imagined as parameterised. The form may be described within a co-ordinate system whose origin is at the bottom right corner of the form. This system of co-ordinates in effect divides the form into a

Figure 1: (a) The archetypal built form. Light tone shows artificially-lit space, medium tone shows sidelit space, dark tone shows potentially toplit space. (b) The lower, deep-plan floors of the archetypal built form, all but the topmost of which are artificially lit at the centre and sidelit around the edges, while the topmost can have top-lighting from the bases of the courtyards above.
matrix of cuboids. Dimensions may be assigned in $z$ to specify storey heights, and in $x$ and $y$ to specify the depths in plan of ‘strips’ of accommodation across the form. Thus depth dimensions may be assigned to the side-lit zones around the perimeter and facing the courtyards, widths may be assigned to the courts themselves, and so on. Should any parameter values in $z$ be set to zero, then of course the complete floor in question will be removed. In this way it is possible to select just some of the lower deep-plan floors; just some of the courtyard floors; or some combination of the two

Figure 2: Binary encoding of the form of Mies van der Rohe’s Seagram Building, as proposed by March (1976)
types of floor. Similarly, by setting selected parameter values in \( x \) and \( y \) to zero, complete strips of accommodation across the form are removed. Thus for example the outward-facing sidelit zones may be suppressed, leaving blind facades without openings. The strips of artificially-lit space may be removed from the courtyard floors, leaving them wholly daylit. Entire courts or rows of courts may be removed, leaving perhaps only one court, or even no complete courts at all, just simple I, L or U shapes in plan.

It is more usual in architectural composition, or in the design of computer tools to support that compositional process, to conceive of elementary forms being combined in an \textit{additive} way to create more complex forms. The archetypal form is imagined, instead, as a complex theoretical arrangement from which many simpler individual built forms may be produced by the \textit{subtraction} of elements. To draw an analogy from sculpture, this is a method of \textit{carving}, rather than a method of \textit{modelling}. An earlier paper demonstrated how, by giving suitable parameter values to the dimensions of the archetype, it was possible to approximate the forms of a very diverse selection of actual buildings: a factory, a theatre, a town hall and a large hotel (Steadman 1998).

2 BINARY ENCODING

In the 1970s March proposed a method for describing the geometry of rectangular built forms by means of a binary encoding (March 1976). In essence, March’s technique involved enclosing the form of a given building within a bounding box. This box is divided up into cuboids by a series of orthogonal planes, coinciding with all external surfaces of the building. Any cuboid that corresponds to a part of the built form is then coded with a 1. Any cuboid that corresponds to empty space outside the form is encoded with a 0. March proposes a convention for ‘unpacking’ the cuboids, by first separating out vertical slices in \( y \), and then separating stacks of cuboids in \( x \). The 0s and 1s corresponding to the respective cuboids can then be listed in a single binary string. The binary encoding thus serves to represent the configuration of the built form, independent of its metric dimensions. Dimensional values can be associated with the 1s in the binary string, to generate any particular dimensioned instance of the configuration. March illustrates the example of Mies van der Rohe’s Seagram Building (figure 2).

We have adapted the principle of March’s binary encoding to the archetypal built form, but with certain key differences. The approach can be illustrated, without loss of generality, by considering just one floor level and one courtyard within the archetype. (We can imagine that all other floors and courtyards have been suppressed.) This leaves a 7 x 7 array of cuboids as seen in plan view in figure 3. Binary digits are assigned to each of the rows of cuboids in both \( x \) and \( y \) in the order shown in the figure. We can now describe any configuration by a 14-digit binary string, in which by convention the \( x \) values are given first, and the \( y \) values second. If a row of cuboids is to be included in the configuration, the respective binary digit is a 1. If a row is to be suppressed the digit is a 0.
3 19TH CENTURY OFFICE BUILDINGS IN CHICAGO

We can illustrate this method of encoding with an application to some real buildings, to indicate its potential in analysis and design. The sample is drawn from late 19th century and early 20th century office skyscrapers in Chicago. The plans of these buildings were constrained by typical block sizes in the central business district of the city, the ‘Loop’. Most blocks here are near-squares, measuring typically 98 x 115 metres. Some are subdivided by alleys into two, three or four rectangular parts. In the southern part of the Loop there are a few much longer, thinner blocks.

The blocks themselves are further subdivided into sites, whose shapes are again for the most part simple rectangles. If the specific dimensions are ignored, then there is only a small number of possible configurations for sites, where distinctions are made on the basis of the numbers and arrangements of their frontages onto streets. There are sites with: one frontage, two adjacent frontages (on the corner of a block), two opposite frontages (running right across a block), three frontages (on the end of a block) or four frontages (filling the whole block). (Sites with no frontages, in the interior of a block, are also theoretically possible, although rare in practice because of the lack of street access.)

The Chicago skyscrapers have built forms that respond, obviously, to these different site configurations (see Willis 1993). In order to maximise floor area they fill as much of the site as possible. They have window walls on the street frontages, and blind facades on the site boundaries within the blocks. Many of these buildings have shops or banks on the ground floor and perhaps also the first floor, which fill the entire site. The shops might be artificially-lit at the centre of the plan. A concourse or banking hall might be top-lit. (These floors are like the lower deep-plan storeys of the archetype.) On the upper floors however the buildings are wholly devoted to offices. Up until the late 1920s or early ’30s, when air-conditioning and fluorescent lighting were introduced, these types of building were heavily reliant on natural light, and were naturally ventilated throughout. They had
tall ceilings and large windows. But even so, they were limited in plan depth by this requirement for daylighting.

Figure 4: (a) Plan of the Columbus Memorial Building (architect W W Boyington 1891), and (b) an approximation of its undimensioned configuration in the 7 x 7 array.

The rule of thumb quoted in the 19th century literature was that an effective limit existed at about 20 to 25 feet (6 to 7.5 metres) from the windows (for example Hill 1893). Beyond this distance, it was believed that office space would be too poorly lit to be lettable – although one does find that in practice some buildings were actually constructed with offices deeper than 7.5 metres. These lighting and ventilation requirements in turn placed further constraints on possible plan shapes.

Let us look at some of the buildings that met these constraints, and see how their forms may be represented using the binary encoding. In each case we will confine consideration to a representative office floor, and approximate its plan configuration by reference to a 7 x 7 array. Figure 4a shows the Columbus Memorial Building, which occupies a corner site. It has an L-shaped plan, with strips of daylit accommodation along the two street facades and along one inner side of the L. Circulation and services form an L-shaped zone in the interior, dependent largely on artificial lighting. Figure 4b shows an approximation of the undimensioned configuration of this plan in the 7 x 7 array. The corresponding binary string is 0001011 0001111. See how this creates the L-shape with its two daylit facades on the streets and two blind sides on the shared site boundaries.

Figure 5a illustrates the plan of the Masonic Temple Building, whose island site has four street frontages. There is a central atrium with daylit offices on three sides, surrounded by a ring of artificially-lit circulation space. Figure 5b shows an approximation of the plan in the 7 x 7 array. The binary encoding is 1101111 1111111. Figure 6 shows the plans of four more buildings with their respective binary encodings. The Old Colony Building has a simple rectangular plan shape, on a three-frontage site at the end of a narrow block in the south of the Loop. The plans of the Marquette and McCormick Buildings are both U-shaped, on three-frontage sites. The Unity Building is also U-shaped, but on a corner site.
Figure 5: (a) Plan of the Masonic Temple Building (architects Burnham & Root, 1891-92), and (b) an approximation of its undimensioned configuration in the 7 x 7 array.

Figure 6: Plans of (a) the Old Colony Building, (b) the Marquette Building, (c) the McCormick Building and (d) the Unity Building, with approximations of their undimensioned configurations in the 7 x 7 array, and their binary codings.
One can conceive of the Chicago architect or developer’s problem, then, as that of putting the maximum amount of office floorspace onto a site, given the combined constraints of site configuration, site size, the maximum depth for daylit office space, and the minimum acceptable width for any interior court or lightwell. In the vertical dimension, there were effective limits on total height fixed by structural considerations and by legislation.

We will come back at the end of the paper to the question of how our method of representation might be used to mimic the process of design of these buildings. Meanwhile notice some special features of this form of binary encoding. For convenience we have started, with the Chicago examples, from a 7 x 7 array representing a single court, giving a 14-digit string. But it would have been equally possible to start from the nine-court arrangement of the archetype in figures 1 and 2. The result would have been to produce 30-digit binary strings, but with all the additional positions filled by zeros.

Whatever the chosen length of string, the positions of all the digits are meaningful. For example, considering the 14-digit strings: the 4th and 11th positions always represent the strips of accommodation that include the court. With 1s in both these positions, the central cuboid in the 7 x 7 array is selected, and the built form has a courtyard. If on the other hand there is a 0 in either of these positions, this means there is no courtyard. Again, the strips of outward-facing daylit accommodation around the four sides of the 7 x 7 array are selected by 1s in the 1st, 7th, 8th and 14th positions. The possible permutations of 0s and 1s in these four positions serve to

Figure 7: Rotations of a configuration. Each rotation has a different binary encoding.
represent different configurations of daylit and blind facades. The reader will appreciate how 1s and 0s in other positions in the string mark the presence or absence of artificially-lit space, or space daylit from the courtyard. We give some formal criteria below for inferring overall plan shapes (I, L, U etc), numbers of courts, and symmetry properties, from the binary strings.

These characteristics of the present representation serve to distinguish it from March’s approach – since in March’s encoding the length of the binary string varies with the complexity of the form, and the positions of digits are not especially meaningful. There is a further difference, which arises from the fact that in March’s method the 1s stand for single cuboids (corresponding to built space), while in our approach the 1s stand for complete rows of cuboids (corresponding to built space, together possibly with courts). The penalty is a certain inflexibility in our method of representation, as we will explain. However the advantage is that, for forms of equivalent complexity, the binary strings are much shorter than March’s. This makes it feasible to enumerate and search all possibilities.

4 GENERATING PERMISSIBLE BINARY ENCODINGS

There are binary codes either that do not correspond to a meaningful configuration of the archetype, or that generate isomorphic configurations by reflection or rotation. A computer program to eliminate these redundant and meaningless configurations for a 7 x 7 array with one court has been written by Jeff Johnson at the Open University using a few simple elimination rules:

![Figure 8: Dimensioning a sidelit cuboid gives equivalent results to strips of adjacent sidelit cuboids.](image)

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• If the first seven digits or the last seven digits of a binary code are all zero, then all the rows of cuboids in the $x$ or $y$ direction are eliminated. Such configurations contain no built space, and are discarded.
• For binary codes that correspond to reflections or rotations of the same configuration, the lowest of the codes is selected. Figure 7 shows four encodings that differ only by their positions relative to the origin.
• Some groups of configurations differ only by having different numbers of adjacent cuboids of the same lighting type - artificial or sidelit - in a particular strip or strips. Such configurations can give equivalent results after dimensioning (figure 8). Again, the lowest binary code is used to stand for these cases.
• If any of the four cuboids that are sidelit from a court are chosen from the archetype, then the court must also be chosen to ensure the cuboids potential for light is preserved (figure 9). This implies that if the court is not chosen, then all strips containing cuboids sidelit from the court must also be eliminated.

The above rules select allowable configurations from within the archetype by first applying them to find a set of code generators. These are 7-digit binary strings. There are 41 distinct 7-digit generators as shown in table 1. The generators are then permuted in pairs in all combinations and the rules applied again. This removes more redundancy of binary codes.

A 7 x 7 array with one court gives 14-digit binary codes, and the rules reduce the number of possibilities here from a theoretical $2^{14} = 16,384$ binary codes to 675 unique configurations. Using the same rules for an 11 x 11 array with four courts gives 208 distinct generators, which permute to give approximately 20,000 unique configurations from a possible $2^{22} = 4,194,304$ binary codes. We estimate that the number of distinct configurations represented by 30-digit binary codes, corresponding to an archetype with nine courts as in figure 1, might be around 30 million. This is a large number. Notice nevertheless how the selection rules pick only a relatively small fraction of all binary codes with a specified number of digits. Other rules could certainly be devised that would select slightly different configurations. We believe

Figure 9: Opposite court lit cuboids without the court lose their potential for light.
however that the present rules pick out the great majority of configurations that are in some sense ‘building-like’, while eliminating all effective duplicates and many configurations that have little architectural meaning.

Table 1: The forty-one 7-digit generators for producing binary codes

| 0000001 | 0000110 | 0011110 | 0111001 | 1001011 |
| 0000010 | 0001110 | 0011111 | 0111011 | 1001101 |
| 0000011 | 0011111 | 0101001 | 0111101 | 1011011 |
| 0001000 | 0111001 | 0101010 | 0111110 | 1101111 |
| 0001001 | 0111010 | 0101011 | 0111111 | 1111111 |
| 0001010 | 0111100 | 0101101 | 1000011 |           |
| 0001011 | 0111110 | 0101110 | 1000101 |           |
| 0001100 | 0111111 | 0101111 | 1011111 |           |

Testing the method of representation against samples of real built forms should serve to show whether this belief is justified.

The legitimate binary codes can be arranged in ascending order, and by the number of courts, to create a catalogue. The first part of this catalogue contains the 14-digit binary codes that correspond to configurations of the 7 x 7 array with one court (including those configurations which do not actually select the court itself). These form a subset of the 22-digit strings, since the 14-digit strings are contractions of those 22-digit strings that have the form *0000****** *0000****** after removing the two blocks of four 0s, illustrated in figure 10. The two-court configurations have 18-digit binary codes comprising a 7-digit binary substring for the \( x \) direction followed by an 11-digit substring for the \( y \) direction. The remainder of the catalogue lists the 22-digit binary codes representing the four-court configurations. The lengths of the binary strings (14, 18 and 22-digit) in the catalogue are thus adjusted to the respective numbers of courts (1, 2 and 4), so that strings for the smaller configurations are not encumbered with many redundant 0s. It will be clear that the same principles could be applied in extending the catalogue to even longer binary strings, representing still larger built forms with yet more courts. Arranging the binary codes in this way allows the catalogue to be searched systematically. At the end of the paper we discuss possible search techniques.

5 CONFIGURATIONAL CHARACTERISTICS

Within the catalogue of binary codes, it is possible to identify some configurational characteristics of the corresponding built forms. The binary codes are descriptors of forms, and it is possible to show that groups of binary codes with a given substring share a particular property, for example a specific number of courts, a common overall plan shape, or the potential for bilateral symmetry.
The number of courts in a built form are represented by the substrings:

- ***0*** from the 7-digit binary strings
- ***1*** from the 7-digit binary strings
- ***1***1*** from the 11-digit binary strings

There are four combinations of these three substrings within the catalogue, giving the number of courts in any binary code as follows:

- ***0***          ***0***   This does not have a court
- ***1***          ***1***   This has one courtyard
- ***1***  ***1***1***   This has two courtyards
- ***1***1***  ***1***1***   This has four courtyards

Figure 10: Contraction of 22-digit binary strings to 14-digit and 18-digit strings. Courts are marked by Xs. Light tone indicates artificially-lit space. Darker tone indicates daylit space.
Forms of similar plan shape can be found clustered together in the catalogue, and underlying substrings emerge which define the various shapes. These substrings are referred to here as ‘shape generators’. They can be combined together to form the plan shapes. Each substring is represented by a letter, allowing shapes to be listed concisely as pairs of letters. There are six shape generators for 7 and 11-digit strings, which are shown with their identifying letters below. The sixteen possible combinations of these generators, with the shapes they produce, are illustrated in figure 11.

```
0001000 I
0001*** L
***|*** U
0001***1000 T
0001***|*** F
***|***|*** E
```

where *** means there must be one or more 1s in these positions

Setting aside shapes generated by IX, all other shapes in figure 11 contain at least one court - where by ‘court’ we mean an open area bordered by built space on four, three, or two adjacent sides. Thus the open area within the two arms of an L-shape would count as a ‘court’ in this sense. The shapes generated by the combination of I with any other generator (IX) comprise in each case what might be called a ‘degenerate’ court or courts, bordered by built space either on two opposite sides, or on one side only. They therefore consist of one, two or three separate rectangles aligned in a row. Such configurations might not seem very ‘building-like’. We have nevertheless retained them in the catalogue, since they approximate for example to the plans of certain types of courtyard house with separate pavilions. In addition to these there are
five binary substrings giving simple rectangular shapes that do not contain a court of any description. These are 0000001, 0000010, 0000011, 1000001 and 1000011. They combine with each other to generate solid blocks of ten different kinds (figure 12). We do not show them combined with any of the other generators, since the results are all equivalents of these ten blocks. Every binary code in the catalogue corresponds then to one of seventeen possible shapes.

Certain symmetry properties of built forms can be determined by inspection of the corresponding binary codes. We refer here to the symmetries of the numbers and types of strips of cuboids in a configuration. These symmetries could be destroyed in the process of dimensioning, by the assignment of unequal dimensions. By ‘symmetry’ then we mean in effect the potential for symmetry in the dimensioned built form. Bilateral symmetry about axes in $x$ or $y$ is indicated by the fact that the respective 7 and 11-digit binary substrings are palindromic. One anomalous substring that also gives bilateral symmetry is 1000011. This corresponds to forms with strips of sidelit space on opposite sides of a central strip of artificially-lit space (and no court). Any binary code with either this or a palindromic substring will correspond then to a configuration with bilateral symmetry. A binary coding with two palindromic substrings represents a form with bilateral symmetry about axes in both $x$ and $y$. If the two substrings are identical palindromes, then the form will show four-way symmetry. In the entire catalogue there are nine 7-digit substrings and twenty-three 11-digit substrings giving bilateral symmetry. Bilateral symmetry about a diagonal axis is shown by binary codes having the same binary substring for both the $x$ and $y$ directions. The substring is not required to be a palindrome in this case.
6 DEFAULT VALUES FOR DIMENSIONS

In principle the dimensional parameters of built forms described within the archetype could take *any* desired values. In practice however, if the forms are to approximate to real buildings, then dimensional values are likely to fall within restricted ranges. Thus storey height would almost always exceed 2m, and for most building types would fall typically between say 2.5 and 5m. We have already discussed the effective limits on the depth of sidelit ‘strips’ of accommodation set by the reliance on natural light (and ventilation) in 19th century American office buildings. Post-war experimental lighting research in the UK serves to confirm those rules of thumb.

Good daylighting was emphasised for example as an important factor in valuing office accommodation in a report of The Lighting Committee of the Building Research Board (1952). This publication quotes 15 - 20 feet (4.5 - 6m) for the desirable depth of small offices lit naturally from one side, and 40 feet (12m) across the full depth of the plan for large offices lit from both sides. That these kinds of values have been applied in practice in Britain is shown by measurements made on a sample of actual office buildings in Swindon, reported in Steadman et al (1993). These show a wide distribution of values for plan depth, but with peaks around 8m (for 6m-deep offices along 2m-wide single-loaded corridors), and at 14m (for two rows of 6m-deep offices along 2m-wide double-loaded corridors). Deeper still are offices dependent on air-conditioning and permanent artificial lighting.

For purposes of illustration, we confine ourselves here to worked examples of office buildings wholly reliant on daylight. We have set default dimensional values for all strips of accommodation of a given type. For sidelit strips, the depth is taken to be 7.5m. We assume that any artificially-lit space, where it occurs, is devoted to circulation. The default depth value is therefore set at 2.5m, to allow for a typical corridor. (Of course in other building types, this artificially-lit depth might be much greater, to accommodate service areas, storage, internal bathrooms etc.) Storey heights are taken as 2.5m throughout.

It is also necessary to consider the widths of enclosed courtyards, since these in turn will determine the widths of daylit or artificially-lit cuboids in the same rows as the courts. The minimum acceptable widths of courts are likely to be set in practice by requirements for visual privacy, and for a minimal level of daylight penetration to the lowest floors. This second requirement would tend to mean that the greater the number of storeys, the wider the court. Many previous studies of built form, and legislative standards for building bulk control, have allowed for this relationship by setting a minimum ‘cut-off angle’ - that is, the vertical angle between the base of one side of a court, and the top of the opposite side. We adopt this same approach, by setting default values for cut-off angles of all courts. Of course any courtyard could acceptably be wider than these minimum angles allow.

Using the default values for dimensions, it is simple to calculate the following attributes for all entries in the catalogue:

- Overall width and depth of the forms in plan
• Minimum site area, calculated from the width and depth
• Total court area
• Footprint area. This is the minimum site area minus the total court area.
• Volume, calculated from the footprint of the form, the height of each storey and the number of storeys
• Exposed wall surface area
• Floor space index (fsi) - the ratio of total floor area to site area
• Circulation length. This calculates the total length of all assumed corridor spaces, measured along their centre lines. Where there is no specific corridor space, the length is calculated along a line that is the light penetration depth away from any sidelit facade.

Other measurements might be made relating to circulation, as for example the distance between the remotest pair of points in the circulation network of a floor plan; or the ratio of daylit floor area to artificially-lit floor area (assumed to be circulation).

7 THE VISUALISATION AND ANALYSIS TOOL: PANGEA

Pangea is a 3-D modelling program, developed at the Barlett School at University College London for visualising and manipulating building and urban designs at the sketch design stage. A 'world' in Pangea consists of a flat rectangular plane floating in space, a yellow light in the sky to illuminate the world, and cameras set at various angles to allow dynamic views of changes in form as variables are altered.

The plane is the base on which models can be built from 3-D objects within a rectangular universe. Objects are created from the toolbar by selecting an appropriate tool, or from scripts, written in an intuitive high-level language, that can be embedded into the plane or into objects. Standard and user-defined attributes of behaviour and data are embedded in the objects. Scripts enable manipulation of the objects through user inputs or via links to other objects, and allow data to be imported or exported through links with external applications like spreadsheets and word processors.

Pangea has some important features for the modelling of the archetype. Through the scripts, suppression and compression of parts of the archetype can be controlled. The resultant forms can be parameterised using the default values for dimensions, and the metric outputs written to a spreadsheet, permitting analysis and comparison of forms. (Alternatively, the metric values can be calculated directly from the binary codes, within the spreadsheet itself.)

An example of a one-storey built form generated from an 11 x 11 array is shown in figure 13. The cuboids are scaled using the default values described above. Each court has fixed plan dimensions of 7.5 x 7.5m. Some of the outputs written to a spreadsheet are also listed in the figure (units are m or m²).
Figure 13: Front and top views of a form generated from an 11 x 11 archetype, with some of the associated metric outputs (units are m or m$^2$).

8 SEARCHING THE CATALOGUE

The catalogue can be searched for built forms corresponding to a specified set of characteristics. These might be configurational characteristics, or metric properties, or some combination of both. We offer two worked examples, one hypothetical, the other an actual design problem.

As a theoretical example, we might want to find U-shaped built forms with bilateral symmetry about the vertical centre line of the U. (Because U shapes are set by convention in the 7 x 7 array on their sides, this implies symmetry in the $y$ direction in the corresponding binary code.) As discussed earlier, U shapes have binary codes of the form 0001*** ***1***. The symmetry requirement means that the $y$ substring must be a palindrome. There are then seven possibilities for the $x$ substring, and seven palindromic possibilities for the $y$ substring. Combining these substrings results in 49 possible binary codes. Since there are 196 binary codes for U shapes in the complete catalogue, the requirement for bilateral symmetry thus reduces the search space by 75%.

Suppose further that a particular configuration is specified for the site which this form is to occupy. Suppose the site has two opposite street frontages, and that the U shape fills the site and is oriented such that the three-sided court is adjacent to one of the boundaries with adjacent sites. There must be sidelong space along the two sides of the form which front onto the streets, and no sideling space on the two sides that lie along the shared site boundaries. This means that the $x$ substring cannot have a 1 in the last position, and the $y$ substring must have 1s in the first and last places. Thus the
desired configuration must have a binary code of the form 0001**0 1**1**1. This reduces the possibilities for the x substring to three, for the y substring to four, and the search space to twelve possible configurations. These are all listed in table 2, together with the metric outputs produced by applying the default values for dimensions described earlier.

A further selection could be made on the basis of these metric properties. It might be required for example that floor space index be maximised. It can be seen from the table that there are four forms with fsi values ≥ 4.5. Figure 14 illustrates these four forms, shaded to show the lighting conditions of the different parts, together with their respective binary codes.

Table 2: U-shaped forms with bilateral symmetry in y, two opposite daylit facades and standard default values for dimensions (units are m or m²)

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<th>Y STRING</th>
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<th>FOOT</th>
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For our second worked example, we go back to the Chicago office buildings described in Section 2. Take the case of the Unity Building (see figure 6d) which occupies a corner site with dimensions 24.5 x 36.5m. The practical challenge facing the building’s designer Clinton J. Warren was to pack as much office accommodation on this site as possible under the constraints already discussed. We can use the catalogue to try to solve Warren’s planning problem automatically. For the sake of illustration, the same default values for plan dimensions are assumed as in the previous example (7.5m depth for sidelit strips, 2.5m width for artificially-lit corridors) since these approximate to the norms which appear to have applied in practice.
Figure 14: **Four forms found in a search of the catalogue, in the first worked example.**

Storey heights are set at 4m, and the maximum number of storeys is set at ten, corresponding to a limit of 130 feet (39.6m) established by the Chicago city council in 1893. The widths of courts are controlled by a cut-off angle of 70°, which seems to approximate the standards actually applying in Chicago at the time. (Of course all these assumptions could be changed if required.)

Figure 15: **Four forms found in a search of the catalogue, in a second worked example.**
No constraints on plan shape or symmetry are set, it being assumed that these will be outcomes of the requirement to maximise floorspace, rather than being predetermined.

Some simple arithmetic shows that, with the given default plan dimensions, no configuration with two courts (or more) can fit the site. The search of the catalogue is therefore confined to 14-digit strings. A search is made to find strings of the form 0*****1 0*****1. These will produce sidelit facades along two adjacent sides of the form, and blind facades on the other two sides. It is further required that the overall width and depth of the form in plan do not exceed 24.5 and 36.5m; and that total floor area is maximised.

The result is to select just four dimensioned configurations, shown in figure 15 along with their binary codes. One plan (a) shows a close resemblance to the Unity Building itself, suggesting that Clinton J. Warren was indeed successful in finding an optimal arrangement in these terms. The other three have the same overall plan dimensions, but the positions of court and daylit strips are rearranged. The (artificially-lit) corridors do not however reach to all office areas in these plans. Only in plan (a) is the circulation arranged satisfactorily and efficiently.

9 CONCLUDING REMARKS

This paper, and the worked examples, are intended to outline an approach which we believe is capable of considerable further development. For the two examples here the searches were made by hand, but clearly this process could be automated - indeed for binary codes longer than 14 digits it would have to be automated. (Notice, all the same, that for certain kinds of search criteria it would not be necessary to examine all codes of a given length. Forms with a specified number of courts are all clustered together in a certain section of the catalogue, so that only that section would need to be searched.)

We may perhaps have given the impression by our examples that the technique is intended to be applied just to the analysis and design of office buildings, and for optimising performance according to one or other simple objective function. We do not however see it as limited in this way - although its greatest usefulness would certainly seem to be for representing built forms on tightly-packed urban sites where the demands of daylighting are especially exigent. A variety of building types lend themselves to representation in these terms, as Steadman (1998) indicates. We see particular promise in relation to residential buildings including houses, public houses (Steadman, in press), flats and hotels. What is more, the process of search might be a much more exploratory and less focussed one than in our office examples. Many formal possibilities might be displayed and compared visually, and the variation in different aspects of performance - fsi, circulation length, surface to volume ratio etc - plotted throughout relevant areas of the space of possible forms.

This said, the representation certainly has its limitations. Our worked examples have been confined to single-storey forms, or prismatic forms with identical plan shapes on every floor level. The archetype itself allows for two shapes of floor plan (on the deep-plan levels and the courtyard levels) to be superimposed. But we
have not discussed any types of form where the rows of cuboids selected are different on successive floor levels - as can often occur in practice. Also, there are certain more complex plan arrangements of sidelit and artificially-lit strips and courts that cannot be produced directly from the archetype. For example, among two-court forms, it is possible to create E shapes but not S shapes. The latter would have to be produced from a pair of U-shapes, one of them mirrored. In general, research is needed to devise sets of rules for the possible combination of built forms, either side by side or on top of each other.

One final limitation is posed by the fact that the representation treats rows of cuboids. When a row is selected, then by implication all cuboids in the row are present, and all share the same dimension. In the forms of many real buildings this is not the case. The built forms produced from the archetype are what one might call 'full' or 'complete' forms. It is generally possible for some individual cuboids to be removed, or to be dimensioned so as to recede or protrude from the remainder of the façade or roof surface. Again it is a matter for further research to explore rules governing these possibilities. In some instances the 'negative form' to be removed from a 'full' built form can itself be directly represented within the archetype.

10 REFERENCES

Hill G. (1893) Some practical conditions in the design of the modern office building, Architectural Record 2, pp.222-68.