Materializing the mathematical is, advertently or inadvertently, a fundamental procedure in the production of architecture. Pedagogical models such as H.A. Schwarz’s copper plate engravings from 1890 documenting minimal surface solutions as well as an extensive collection of plaster models assembled by Schwarz and Felix Klein in Gottingen in the early 1900’s are seminal examples. In architecture, works such as Corbusiers’ and Xenakis’ Philips Pavilion or the details of Gaudi’s Sagrada Familia display the overt presence of materialized mathematical models.

Our work focused on a basic design problem: how to produce an enclosure system that maximizes cavities and niches as opportunities for moving across a threshold. Conventionally mitigated by the goal of producing enclosure, porosity was used as a means to dematerialize and make more a intelligent (bi-directional/permeable) enclosure system. Repetition, modularity and the presence of cavities – all conventional aspects of masonry systems of construction – were incorporated into the design of prototypes for a small-scale building enclosure.

**Keywords.** Minimal surfaces: porosity; aggregation; prototype manufacturing.

---

**Minimal surfaces & architecture**

Materializing the mathematical is, advertently or inadvertently, a fundamental procedure in the production of architecture. Historically this materialization was, on the advertent side of things, deployed for the communication of theoretical and abstract solutions within the discipline of mathematics. Pedagogical models such as H.A. Schwarz’s copper plate engravings from 1890 documenting minimal surface solutions (fig. 1) as well as an extensive collection of plaster models...
assembled by Schwarz and Felix Klein in Gottingen in the early 1900’s are seminal examples.

A minimal surface is a surface that is locally area-minimizing, that is, a small piece having the smallest possible area for a surface spanning the boundary of that piece. Soap films are minimal surfaces.

In architecture, examples of larger shell and long span roof structures such as Corbusiers’ and Xenakis’ Philips Pavilion (fig. 2) or the details of Gaudi’s Sagra-da Familia (fig. 3) display the overt presence of materialized mathematical models. Projects by Frei Otto like the Stuttgart train station (fig. 4), demonstrate the formal & structural novelty that derives from experiments with minimal surfaces.

**Intenions**

Our interest in minimal surfaces is divided in two areas. First, investigating the architectural/structural and programmatic qualities that aggregations/transformations of a minimal surface could produce. Other than the formal ‘newness’, or the ambiance & atmosphere that minimal surfaces could create on an architectural level, there also could be space for innovative structural solutions or programmatic differentiation/variation and diverse spatial conditions. What’s specifically interesting is the continuity between inside and outside space that can be achieved.
by simply copying, pasting and mirroring a minimal surface, something that is visible in one of Toyo Ito’s latest projects, the Taichung Metropolitan opera in Taiwan (fig. 5).

Exploring the fundamental exchange between the mathematical realm – in the form of digital modeling – and the material realm – in the form of material prototype fabrication and manufacturing was the second goal of our study.

In order to become more site specific, we focused on how to produce an enclosure system that maximizes cavities and niches as opportunities for moving across a threshold. In contradistinction to either the case study logic of threshold, where glass is employed to erode wall systems while maintaining environmental enclosure, or the logics of composite lamination where the layering of the material allows for adjacent apertures and construction systems to exfoliate, our study focused on porosity as a new approach to the issue of threshold.

Conventionally mitigated by the goal of producing enclosure, porosity was used as a means to dematerialize and make more a intelligent (bi-directional/permeable) enclosure system. Repetition, modularity and the presence of cavities – all conventional aspects of masonry systems of construction – were incorporated into the design of prototypes for a small-scale building enclosure.

Digital modeling of a minimal surface

Particularly fascinating are minimal surfaces that have a crystalline structure, in the sense of repeating themselves in three dimensions, in other words being triply periodic. The inherent ability of three dimensional aggregations of such surfaces was what triggered our study.

We chose to investigate one of the Schoen’s hybrid minimal surfaces, called $S'$-$S''$, due to its relatively simple geometry and possible orthogonal aggregation that could be structurally sound using contemporary building techniques. Minimal surfaces can be categorized in families depending on their characteristics. Since it’s a combination of families of other minimal surfaces we wanted to explore the diverse spatial conditions such a surface could produce.
The $S'$-$S''$ surface, can be inscribed in a square box with tunnels to the top and bottom of the box and to the vertical edges, allowing vertical and horizontal stacking (fig. 6).

Another problem we had to face was the software we would use in order to digitally model the surface. Minimal surfaces are generally made by defining and evolving the fundamental region of the surface, which is usually very simple due to the high symmetry, and then displaying many copies of it, suitably transformed. The $S'$-$S''$ surface was made of a fundamental region, or “geometric primitive” which is defined by five curves. These curves derive from an intersection of 2 toruses and a cube (fig. 7).

Maya, like most of the 3D modeling software can ‘birail’ or ‘loft’ curves in order to produce a surface that is inscribed by them. Lofting these 5 curves produces almost exactly the fundamental region from which the Schoen hybrid surface can be digitally modeled. In order to create the designated curvature on the surface, two secondary parabolas are designed as 3 point NURBS curves and then biraled altogether with the initial 5 curves (fig. 8). Using NURBS curves were crucial in order to create the exact digital model of the surface due to the specific equation that defines such curves. Due to the crystalline and symmetrical structure of the Schoen hybrid surface, the fundamental region is
the 1/16 of it. Simple copy/paste/mirror commands put together the pieces of the puzzle (or more likely 16 identical pieces of the fundamental region) and provide the whole digital model of the minimal surface (fig. 9).

**Mutations & aggregations of the Schoen hybrid minimal surface**

The small scale building enclosure was to have gradient porosity from 90% to 20% in selected areas, in order to allow for diverse kinds of apertures, passageways and enclosures. In order to achieve that, we experimented with generic site aggregations of the fundamental region, manipulating its scale. We came to realize that scaling could provide us with flatter areas that could serve as floor plates but would not provide us an efficient number of different spatial conditions (fig. 10).

**Fabrication matters**

Another issue we had to deal with was the material we would use in order to fabricate the physical model. So far, we had been digitally modeling using surfaces, so one of our choices was to build a physical model out of something extremely thin, like vacuum formed PVC plastic. We knew that such a material in the scaled physical model would stand for some kind of glass or EFTE (ethylene tetrafluoroethylene) membrane in the real world, like the Allianz arena in Munich. It could even be reinforced polymer, made with liquid/ injection molding techniques. Meaning, it would need some additional structure, like mullions or a steel lattice in order to be able to stand.

Another idea was to continue the digital modeling with a thickened version of the fundamental region. That would allow us to build a mock up using 3D printed pieces, which we could duplicate using
rubber moulds and plaster. This way, if the facade was to be constructed, it could be made of concrete or steel members that would make it structurally sound, without needing a secondary support system. That was what convinced us to experiment with the thickened surface scheme.

Thus, we digitally modeled five modified and thickened parts by simply subtracting or adding areas of the fundamental region which were more or less solid/permeable (fig. 11).

Fabricating these structural components would allow us to explore topics such as the non-uniform aggregation, the maximization of internal cavity networking and the continuity of material as it moves from an exterior to an interior. Assembling them in different configurations, we achieved different degrees of porosity for the epidermis of the building façade (fig. 12).

In order to study as many spatial configurations as possible, the digital aggregations where random, making groups of 8 to 12 parts each time, more like putting together pieces in a puzzle. The next thing was to combine the different groups together in order to build up the digital model of the façade (fig. 13,14).

We produced a large number of 3Dprinted parts from which we made replicas using rubber molds in order to produce the physical model. The mock up
was a façade for a 2-storey building which came to be 1.2m high X 0.80m wide. With a scale of 1:5, we had the chance to test its structural abilities as well as the ambiance produced within such a structure (fig. 15).

Assessments

The exchange between the mathematical realm/idea and physical three dimensional outputs by prototype fabrication & manufacturing was irreplaceable in order to understand and explore the possibilities of such an experiment. In the scale used for this specific model, aggregations between a few pieces of the fundamental region allow for a clear conception of different degrees of porosity, whereas further stacking creates a more ‘blurry’ and organic effect. At the same time, the way light is diffused within the multiple parts of the model creates a certain ambiance (fig. 16).

Structurally, all configurations allow for an open space between 4 parts, transferring loads to the perimeter, like a vaulted ceiling. At the same time, another configuration allows for an opening at the center of it, allowing vertical movement (fig. 17.)

The same configurations of the fundamental region could be used for projects of totally different scale since each spatial condition is generic and is not designed for a specific purpose; it’s not about form following function, it’s rather about form being able to host more than just one conditions. (fig.18)
The spatial and structural conditions/transformations in such an aggregate system of enclosure are innumerable, especially if they are combined with transformations of other minimal surfaces. I left for last something that might be just as - if not more - important than all the issues discussed above. The first thing someone realizes by observing the physical model is the huge impact such a structure would have within the real world. Its form alone would surely raise discussions about the cultural context/significance and precedents of such an architectural gesture. Reminiscent of forms from gothic architecture, Gaudi's Sagrada Familia or even human bones, it would trigger discussions that move way beyond minimal surfaces, digital modeling or prototype fabrication, proving that architecture is highly connected to cultural production of an era.

Acknowledgements

Michael Leaveck and Manzer Mirkar for their cooperation ideas and labour in bringing together this project for the technology seminal @ UCLA AUD in 2005. David Erdman and Marcelyn Gow for all the input and help during and after the technology seminar 289.2 @ UCLA AUD.

References

Alexandra & Andreas Papadakis: 2003, New Arch07:Innovation from experimentation to realization, Papadakis publisher, London.
Smythe, J. S. (ed.): 1990, Applications of Artificial Intelligence to Communication, CMP and Springer-Ve-