Fractal Geometry of Architecture

Implementation of the Box-Counting Method in a CAD-Software

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Abstract: The author describes the basic principles for measuring architecture from the point of view of Fractal Geometry outlining the principle connections between Fractal Geometry and architecture, giving some examples and explaining the Box-Counting Method, which is an easily manageable method that can be applied to elevations. The paper not only deals with problems arising from using the Box-Counting Method but also with its relation to visual perception. It shows how the Box-Counting Dimension DB of façades can be measured with the help of a software program that was written by the author and has been implemented into AutoCAD. Finally, results of different configurations are given for the Koch curve and Robie House by Frank Lloyd Wright, showing the accuracy of this measurement method.

Keywords: Fractal architecture; box-counting dimensions of façades; visual perception; implementation in a CAD-software.

Fractal characteristics and architecture

In the 1970ies the mathematician Benoît Mandelbrot introduced the term Fractal as a possibility to distinguish self-similar – where smaller parts are similar to the whole – natural or artificial structures from smooth Euclidean ones. Since then Fractals have been identified in many fields. In architecture for instance, Mandelbrot points out the difference between buildings of Beaux-Arts, which offer fractal aspects, and buildings by Mies van der Rohe that he calls scale bound throwback to Euclid (Mandelbrot 1991).

On closer observation, taking into account the new Geometry of Fractals, three main types of architectural compositions can be differentiated – with many nuances in between. Any lack of visual depth leads to uninteresting, if not boring buildings – the first extreme – while unstructured buildings which offer disconnected details of different sizes on different scales and therefore lead to confusion, can be seen as the second extreme. Those buildings that offer self-similarity belong to the third group in between, offering details of different sizes on different scales linked with each other. On closer observation buildings of this group consist of a large number of extremely small components that are combined to form a smaller number of larger components and so on, coming up hierarchically to the whole. Fractal Geometry then provides the language in which the connection of architectural composition of such different components can be expressed. In a simplified
way we could say that Fractal Geometry helps to describe complex structures.

There are two important aspects worth considering before such a classification can be made. First, Fractals are defined by their characteristics – self-similarity, generation by iterations, rough surfaces, infinite complexity, dependence on starting parameters and common features with nature –, but often the only way to describe Fractals is through their Fractal Dimension. This means an architecture-specific measurement method of Fractal Dimension has to be developed which is easy to handle and robust (with regard to influences). Second, we have to be aware that buildings are no Fractals in the same sense as mathematical Fractals. Similar to fractals found in nature, fractal characteristics are restricted between a certain upper and lower scale limit.

**Self-similarity as a concept**

With regard to Fractal Geometry, architecture and façades in particular can be described according to visual criteria whether they offer fractal characteristics or not, examining various levels of scale (Lorenz, 2004). Limits are imposed by certain influences that are inherent in buildings, ranging from the materials used over the intended purposes to specific sizes of certain details. In nature such influences are amongst others caused by weather and climate, which change the idealized fractal structure. It is the factor of chance then that turns the outcome of such computerized simulations of nature into realistic models, where the basic self-similar structure is hard to identify any more. Hence fractal characteristics such as self-similarity of natural and artificial objects are difficult to describe precisely.

The idea of self-similarity is not a new aspect in architecture. Many architects asked for an overall form or, in a more abstract sense, for an overall idea that links all smaller scales to each other and to the whole – smaller parts should mirror the whole. The advantage of such a self-similar concept is that it offers the observer a consistent impression of the building while approaching it. Along with variation of the basic theme, which also arises from the material used, the function of single architectural elements and construction constraints, the building remains interesting. Coherence between many scales is the characteristic that distinguishes fractal-like buildings from those which offer details at different scales without any connection between them. No connection means that the observer is confused while approaching the building as he/she is always confronted with completely different impressions.

According to Salingaros (2006), the latter group comprises deconstructivist buildings, which are similar to random forms. But self-similarity is also the characteristic that distinguishes fractal-like buildings from those that have more affinity to Euclidean geometry thus offering large-scale ranges of smooth parts. One might assume that buildings of modern architecture belong to this latter group, but measurements by Ostwald, Vaughan and Tucker (2008) performed on buildings by Le Corbusier refute this.

**Examples of self-similar architecture**

Self-similarity can be detected in many buildings, for example in Gothic cathedrals. Even though they are based on strict Euclidean basic shapes such as equilateral triangles and isosceles triangles, the outcome offers self-similarity from the large to the small. The pointed arch giving an impression of rising up high can be found on many scales at the entrance, at windows – which themselves may consist of combined arches that are again vaulted –, at the costal arch, on turrets and in many details, outside and inside. This leads to a coherent building where all parts offer a similar roughness as the whole achieved by the idea of rising up high, expressed by the pointed arch (Lorenz, 2003). As another example, Robie House by Frank Lloyd Wright offers self-similarity on different scales using the horizontality of the surrounding nature as a concept that is translated into the building. The horizontally stretched façade consists of widely protruding, longitudinally stretched roofs and stretched storeys, where the storeys themselves again consist of level window-strips and horizontal...
Ds is then calculated by the relationship between the number of smaller pieces (N) that the original is divided into and the scale factor (s) that is the reduction of the original line. In case of the Koch curve the original line is divided into four pieces (N=4) of one third (s=1/3) of the original. If we use these values in the first equation (1) the calculation results in the Self-Similarity Dimension Ds of 1.26 – Ds is the linking element between all iterations.

\[ D_s = \frac{\log(N)}{\log(1/s)} \]  

Box-counting dimension Db
Natural, but also artificial structures, do not offer sameness from one iteration to the next – which means they are not divided into equal parts –, but offer a lot of variation. For such objects the calculation of the Self-Similarity Dimension Ds is not possible. This also means that for describing and comparing elevations, belonging to such shapes of variation, another method has to be used instead. The Box-Counting Method for instance is based upon the fact that visually Fractal Dimension is the expression of the degree of roughness, which means how much texture an object has (Bovill, 1996). It compares the roughness of an object over different scales and thus allows measuring the complexity of a structure. With the Box-Counting Method a mesh is laid over the image to be measured and the number of boxes that cover it is counted \( (N_n) \). Then the mesh size \( (s_n) \) – defined by the inverse number of boxes across the bottom of the mesh – is reduced and the number of boxes is counted again (Figure 1). From this
In general an orthogonal frontal approach is not possible, but a restricted street parallel one with natural and artificial barriers (Ostwald and Tucker 2007). From this we can deduce that environment should not be excluded but included into measurements, because the scene as a whole is only perceived by the pedestrian. Due to better comparison and basic testing, the following measurements are again reduced to a frontal, environment-neglecting view.

Another abstraction comes from the fact that, while buildings and their façades are not two-dimensional, the Box-Counting Method as it is presented in this paper only treats façades as flat objects, reduced to those lines that separate fields from one another. Furthermore, the position of the observer is at eye level, from which follows that elements of multi-storey buildings closer to street level are in reality more significant for perception than those at roof level – the eye has to move up and the detail at roof level is distant.

These limits lead to the conclusion that the Box-Counting measurement corresponds only with an idealized view on buildings, but so far it is the most stable one for developing measurement methods for possible classifications of buildings.

### Implementation in AutoCAD

**Influences on the method**

The Box-Counting Method as presented in this paper is based on the assumption that black and white images of elevations or ground plans are measured. As already mentioned this means that the resulting Box-Counting Dimensions $D_b$ are influenced by the illustration, not only by the selection of what is presented but also by line width. The line width affects the smallest mesh-size, because from a certain point on the lines are thicker than the mesh-size and are treated as two-dimensional planes instead of one-dimensional lines, but also the possibility that they are counted twice increases (Ostwald et al., 2008). Other influences on the resulting Box-Counting Dimension $D_b$ come from the measurement.

\[
D_{b(1-2)} = \frac{\log(N_{b(2)}) - \log(N_{b(1)})}{\log(1/s_2) - \log(1/s_1)} \tag{2}
\]
\[
D_{b(1-2)} = \frac{\log(N_{b(2)}/N_{b(1)})}{\log(s_1/s_2)} \tag{3}
\]
method itself. One is based on the section, which means how much white space around the largest extension is defined and included into measurement, giving no more information about the image itself. The selection including white space in turn defines the number of possible starting positions of each single mesh-size – if the whole image is always included and the overall size of selection remains the same. In general the smaller the mesh-size the fewer different starting positions are possible, considering that a box of one by one pixel only has one possibility of different starting position, while a box of two by two pixel has four and so forth. Each starting position can lead to a different number of boxes occupied, which affects the resulting Box-Counting Dimension $D_b$. This influence can be handled by using multiple starting positions for a certain mesh-size. Another aspect is that of starting mesh-size. In many analyses it turned out that one fourth of the smallest side length of the image is a useful size, but it depends on the detail richness of the analyzed image (Foroutan et al., 2000).

**Measuring works of architecture**

Self-similar structures, as they offer a similar roughness on each scale, express this characteristic by a similar Box-Counting Dimension $D_b$ across several scales. Since 1996, when Bovill (1996) presented this method amongst others with Robie House by Frank Lloyd Wright and Villa Savoy by Le Corbusier, some research has been done to apply the Box-Counting Method on different surfaces and to handle several influences arising from the measurement method itself and the preparation of the image. Like other research work on measuring façades with the Box-Counting Method, the program developed by the author, implemented in AutoCAD, is tested comparing results of the measurement of the Koch curve with its Self-Similarity Dimension $D_s$.

The implementation of the Box-Counting Method in AutoCAD with the help of the computer language AutoLisp has some advantages. It not only minimizes influences of the conditioning of the image – no scan preparation is needed – but also of the measurement method itself. Due to the fact that vector-graphics are analyzed an influence of line widths is prevented. Furthermore, existing as well as planned buildings can be analyzed directly, if they have been designed in a CAD package, switching on and off layers of interest – i.e. architectural elements can be turned on or off depending on their size and on viewing distance.

In the first version of the author’s program – version 1 – the number of starting-boxes in x- and y-direction, the rectangular selection (including white space around the image) with the starting point, the reduction factor and the number of iterations can be set manually. Because of quadratic boxes the larger side of the selection is then automatically adapted to the mesh-size of the smaller side length. The second version of the program – version 2 – automatically chooses four as the number of starting mesh-size for the smaller side of rectangular selection. The basic selection itself is now chosen identically to the physical size of the measured image, adding automatically one tenth of the largest mesh-size as white space around the image – the length adaption for the larger distance as a result of quadratic boxes is still done afterwards. Then for each mesh-size 13 different starting points are defined by the program to minimize influence by different starting positions.

**Results**

The basis for analyzing the usability and suitability of the program are different measurements of the Koch curve with 8 iterations, a simple rising line and Robie House by Frank Lloyd Wright using program version 1. For each sample 24 sets of measurements have been performed with 10 to 20 mesh-size decreases, different starting mesh-sizes, starting positions, reduction factors and white space giving a first indication of influences by settings. The resulting average value for the Koch curve is 3.09 percent below the expected 1.2619, defined by the Self-Similarity Dimension $D_s$. The same is true for the single line with 3.97 percent below the expected value 1.
Several influences were then analyzed more closely. For instance, sets of measurements have been performed to prove the influence of the starting position. While the results of one of those sets using 20 single measurements vary from 1.2468 to 1.2887, the average value 1.2664 is only slightly higher than the expected $D_s$. From this follows, that varying starting positions improve the final average value.

The dependence of two values (of a double logarithmic graph) was then examined more closely using an analysis of correlation and regression analysis. This provides information about kind and degree of linear correlation, whose index is called coefficient of correlation ($r$) – with ($r$) between -1 and +1: -1 and +1 indicating a functional linear correlation and 0 no linear correlation. In the following, instead of ($r$), the determination coefficient ($r^2$) is calculated; a ($r^2$) closer to one signifies higher possibility of linear correlation. The determination coefficient is then an index for proper adjustment. For results of Box-Counting measurements, using program version 1, of an overview of Robie House, fading out components of smaller scales, the determination coefficient is above 0.985 for all data. This indicates a nearly linear correlation. Excluding the outliers of the smallest mesh-sizes the index can be improved above 0.995. The reason lies in the fact that from a certain scale on details are larger than the mesh-size and single lines become more important than the whole. The determination coefficient was improved even more by excluding mesh-sizes intuitively. Then it rises above 0.99 for all measurements, indicating a high possibility of linear relation of single points for each data set. The correlation coefficient was then proved with the T-test, indicating a significant statistical correlation. Furthermore, the root mean square deviation of the data set, which is also very small, indicates small statistical spread, which means high accuracy.

For all measurements of Robie House it turned out that the Box-Counting Method is susceptible to fluctuation at the start and the end (smallest and largest mesh-size). Constricting the range then leads to more stable average dimensions. Examining the double-logarithmic graph can identify this: From a certain mesh-size on the double logarithmic graph flattens very fast, while measurements of 8 iterations of the Koch curve remain high. This is because Robie house uses less iteration that means architectural elements of smaller size are faded out. Consequently, in the following measurements such details were incorporated.

Figure 2 shows different measurement sets of Robie House by Frank Lloyd Wright using program version 2. Each set offers different detail-richness. Those measurements of larger mesh-sizes are derived from the overview and those of smaller mesh-sizes from selections of the elevation including stained glass and bricks. Using different elevations and selections of different detail-richness results from the fact that, while approaching the building, different architectural elements with different – even though overlapping – focus ranges come into attention until the elevation limit is reached. Focus ranges are determined by the part of each graph (showing number of boxes versus mesh-size) where single measurement points lie on or at least near the linear replacing line, indicating a connection between scale and occupied boxes. To avoid the influence of starting position, 13 mesh-offsets were calculated (the relatively low number results from limited calculation time). For each mesh offset the smallest number of covered boxes was then taken into account, because only a minimum number for each mesh-size estimates Fractal Dimension (Foroutan et al., 2000). The slopes of the single measurement sets for far distance to middle distance reach from 1.6124 (with a determination coefficient ($r^2$) of 0.995) to 1.6136 (with a determination coefficient ($r^2$) of 0.998), shown in the two upper graphs of figure 2. These results are slightly lower than those of Bovill’s (1996) measurement between first and second mesh-size (1.645) and slightly higher than the result by Ostwald, Vaughan and Tucker (2008) with 1.59 for the whole elevation measured with “Archimage”. Analyzing smaller scales means looking at sections with more details, which was then taken into account with stained glass and bricks leading...
to similar results between 1.63 and 1.66 (with a determination coefficient ($r^2$) from 0.993 to 0.997). All results indicate that single components of the elevation are held together, which is true because of the initial idea of horizontality on certain scales.

**Sloping line**

Analyses of the Box-Counting Dimension $D_B$ between two single mesh-sizes like those performed by Bovill (1996), are less resistant to the influences presented in this paper. This is underlined by many measurements of one and the same mesh-size and size of selection, only changing starting positions, which lead to different numbers of boxes covered – consequently, $D_B$ of the Koch curve varies between 1.2468 and 1.2887. As shown with the Koch curve and Robie House analyses, many measurements and calculations of the Box-Counting Dimension $D_B$ as the slope of the replacing line, excluding outliers, lead to more stable results. In the double logarithmic graph, the number of boxes covered versus mesh-size, it is possible to indicate for which range of mesh-sizes the measurement points are consistent in relation to the replacing line – the smallest and the largest mesh-size for this range is defined by the stage from which on the points of the graph deviate from the straight replacing line. The constant range of little variation from the replacing line, points out that this range offers a similar roughness, indicating a basic requirement for detecting coherence between different scales.

**Conclusions and outlook**

Self-similarity is used by architects to create coherent designs from the whole to the very small detail. The Box-Counting Method is easy to use and an appropriate method for measuring works of architecture with regard to continuity of roughness over a specific scale-range (coherence of scales). Measurements of façades, offering visual coherence between single components across scales, indicate that such connections can be identified whenever points of the double logarithmic graph lie close to their replacing line. Nevertheless certain influences have to be considered that might falsify the results. In terms of exactness and easy handling, further research minimize the influences on the calculation caused by the method itself, but also attempting to define scale ranges of architectural elements, still has to be done. In the final program, a stable automatic analyzing program, implemented in software programs used by architects, the scale range of stable values should then automatically be detected in relation to the replacing line – the largest and the smallest mesh-size of coherence. On the basis of this further research, a unique measurement system can be developed that
enables the user to examine whether it is possible to provide a classification of non-self-similar buildings, over self-similar buildings to confusingly overloaded unstructured ones, connecting self-similarity – similar characteristics on each scale expressed through similar roughness – with the Box-Counting Dimension $D_B$.

**References**

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