Fuzzy Rule Bases as a Tool of the History of Architecture

Application to the Analysis of Villas Designed by Adolf Loos

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Abstract: This article explores possibilities of using fuzzy models as a new research tool in the history of architecture. Namely, Adolf Loos’s villas are studied using these models. Fuzzy expert models will be used to analyze the principles of Adolf Loos’s work. Rules applied by A. Loos in villa design will be expressed in the form of fuzzy rule bases. The formulation of rules will result from the analysis of individual villa designs and will be based on theoretical concepts pertaining to Loos’s work -- in particular, on his designing process drawing on the concept of a Raumplan. Further, the concepts of the theory of architecture, related to a general procedure of construction design, will be utilized. The article also gives examples how to employ fuzzy sets in describing a building and its characteristics.

Keywords: Fuzzy models; expert systems; architecture.

Architectural work of A. Loos

Architect Adolf Loos (born 1870 in Brno -- died 1933 in Vienna) ranks, besides Le Corbusier and Mies van der Rohe, among prominent representatives of contemporary world architecture of the 20th century (see Kulka, 1931, Rukschcio, 1987, Risselada 1988, Kühn, 1989). Contrary to functionalist architects who draw on human collectivism, the point of departure for Loos’s work is the individuality of a man and respect to tradition. He participated in urban planning, civil engineering and social housing projects, and designed interiors. Of greatest importance for the history of modern architecture are his projects of villas from 1920’s and 30’s where his brand new idea of a so-called Raumplan (space plan) found its expression. The principle of Raumplan, in Loos’s own words, meant that he did not design ground plans, facades and cross sections of villas. He designed spaces of different heights, depending on their function, and situated them in villas in different levels. There are dozens of his villas, designed in this way, both constructed and unrealized. The masterpiece is the Müller villa in Prague (Kühn, 1989; Ksandr, 2000).

Loos considers interiors of his houses as impossible to photograph, he deems drawings and sketches as unimportant; he thinks that a good architecture can be described in words (Kühn, 1989). Configuration of individual rooms respects functional and economical requirements of everyday life, and transcends these functions. The outer form of a house is a framework for complex interplay of Raumplan. The outer form of a house says nothing of its inner life. The interior of a building is special and personal; the
exterior is just expressive, yet unforceful work of art.

The principles of Raumplan can be deduced from the design of Loos's villas. His villa is always an object of a simple cubic form, having a vertical structure and four functional levels. The attic and cellar are reserved for maintenance equipment; the middle part is intended for living. A day-time zone is located downstairs, a night-time zone upstairs. These zones are strictly separated, the day-time zone through its height never encroaches on the night-time one. There is a marked difference between the left side and right side of a house. If a view is on the right, then Loos situates a cloakroom here, a kitchen being on the left. The left side of a house is reserved for servicing staff. The front giving out onto a street is public and compact, the backyard front is private and terraced. Typical is a centrifugal use of the space, with the furniture being placed on circumference and the center remaining unoccupied. Various niches and recesses of different heights are also characteristic.

Employing fuzzy expert systems to analyze architectural work of Adolf Loos

A specific aim of the current research is to create a fuzzy expert system to describe general rules adhered to by A. Loos when designing his villas. A more general aim is to explore the possibilities of using mathematical models based on fuzzy rule bases as a tool of analysis of architectural work of individual architects and for description and analysis of individual concepts of construction design. A study of literature and Internet sources supports the following conclusion: Although there do exist applications of the fuzzy sets theory in architectural work, their focus is rather on practical problems of design. The use of a fuzzy expert system to describe working rules of an architect has not been noticed. However, there is no doubt (Von Altrock, 1995) that a fuzzy expert system can be used to record expert knowledge in many rather different areas of human activity (in automobiles, photographic cameras, electric home appliances with a “sixth sense”, even in devices ensuring life functions of patients at units of intensive care, or as a support for decision making in large institutions).

Because a fuzzy expert system describing the process of architectural work presents a completely new solution (architectural work is not only a new but also specific area of application; initial information is represented graphically; the process of design is not simple and clearly defined), completion of such a fuzzy expert system will not be easy. It will be necessary to proceed by trial and error, to begin with very simple partial models, only gradually making them more complex. Indeed, this is the usual process with these types of model.

On the other hand, the use of fuzzy expert systems in architecture seems promising and reasonable. As mentioned above, the task need not be restricted to an attempt at finding working rules of one architect. For example, a specific theoretical approach to design can be expressed in the form of interconnected fuzzy rule bases, and its validity can be subsequently tested through computer simulations.

The fuzzy expert system used to describe the rules of Loos's work will represent a formalized account of knowledge gained both from analyzing a selected group of existing Loos's villas, and through study of his concept of Raumplan and applying the general theory of design. The analysis of villas will bring, first of all, information on a typical form of a building, on arrangement of basic units in a house, on the way of their connecting, and on conception of a facade. The structure of rule bases and interconnections will also reflect general principles of creating architectural design (i.e. investor's requirements and site characteristics are noted, key elements and spaces are dealt with first, then secondary ones etc.). Of course, the specific features of Loos's Raumplan are also taken into account -- the point of departure being the basic units of a house: a hall, dining room, kitchen, other rooms), these are conceived of as separate rooms and connected through artistically remarkable communication channels into a single
Linguistic models (Talašová, 2003) are models with two levels of description: linguistic (intelligible to a human) and fuzzy, i.e. mathematical (intelligible to a computer). They operate on linguistic variables and linguistically defined functions, rule bases. Substitution of input values in a linguistically defined function, and calculation of output values are performed through approximate reasoning.

**Linguistic variables**

Linguistic variables are variables (e.g. size, shape) the values of which denote uncertain quantities or uncertain degree of a characteristic (small, roughly squared). What is important here, the meanings of these words are modeled through fuzzy numbers, i.e. special fuzzy sets defined on a given interval of real numbers (e.g. as regards the size of a room in a villa, on the interval of all possible floorages in m², e.g. [0, 100]). A fuzzy number is represented by a real function (a membership function) that assigns to each number from the given interval (e.g. to each size of floorage) a number from the interval [0, 1] that means how well the given value corresponds to a given word (e.g. to what a degree a room with the size of 18m² is a medium-sized room). The most commonly used fuzzy numbers in applications are the simplest ones, so-called linear fuzzy numbers which are defined through ordered quadruples of numbers \( x_1 \leq x_2 \leq x_3 \leq x_4 \). The interval \([x_2, x_3]\) represents those values that are in perfect correspondence with a given word (e.g. to what a degree a room with the size of 18m² is a medium-sized room). The most commonly used fuzzy numbers in applications are the simplest ones, so-called linear fuzzy numbers which are defined through ordered quadruples of numbers \( x_1 \leq x_2 \leq x_3 \leq x_4 \). The interval \([x_2, x_3]\) represents those values that are in perfect correspondence with a given word (e.g. to what a degree a room with the size of 18m² is a medium-sized room).

The fuzzy sets theory (Dubois, Prade, 2000) enables to mathematically process uncertain information (uncertain data, uncertain relations). A natural language is also capable of dealing with uncertain data and uncertain relations. It was this capability of a natural language that inspired Lotfi Zadeh when creating the concept of fuzzy sets. Fuzzy expert systems that will be used to model the rules of architectural work of A. Loos represent so-called linguistic fuzzy models. Linguistic fuzzy models (Talašová, 2003) are models with two levels of description: linguistic (intelligible to a human) and fuzzy, i.e. mathematical (intelligible to a computer). They operate on linguistic variables and linguistically defined functions, rule bases. Substitution of input values in a linguistically defined function, and calculation of output values are performed through approximate reasoning.

**Used concepts of the fuzzy sets theory and linguistic fuzzy modeling**

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real number from this interval should have the sum total of its degrees of membership to all these fuzzy numbers equal to 1. Rule bases describe relations among linguistic variables. A rule base is generally of the following form:

If $X_1$ is $A_{1,1}$ and ... and $X_n$ is $A_{1,n}$, then $Y_1$ is $B_{1,1}$ and ... and $Y_m$ is $B_{1,m}$

... If $X_1$ is $A_{k,1}$ and ... and $X_n$ is $A_{k,n}$, then $Y_1$ is $B_{k,1}$ and ... and $Y_m$ is $B_{k,m}$

Variables on the left-hand sides are inputs, conditions, assumptions (e.g. a required size of a villa, area and slope of the building site). Variables on the right-hand sides are outputs, conclusions (size and shape of the ground plan, number of floors). The rules of a base are either determined by an expert, or deduced from data (in this case e.g. from existing Loos's villas).

The rule base is modeled by a fuzzy relation. This fuzzy relation is again a fuzzy set and is characterized by a membership function that assigns to each $(n+m)$-tuple of real numbers $(x_1, \ldots, x_n, y_1, \ldots, y_m)$ a number from the interval $[0, 1]$ with the meaning how much possible this combination of values is with respect to the information contained in the rule base. We are able to construct this fuzzy relation if we know the meanings of linguistic values of the variables that appear in the rules. Each rule is modeled

**Rule bases**

Fuzzy expert systems use those linguistic variables whose linguistic values make up linguistic scales, e.g. a linguistic variable *Size of a room* with values *small*, *medium*, *large*. Linguistic scales are mathematically modeled by fuzzy scales. A fuzzy scale is a set of fuzzy numbers, defined on a given closed interval, that are ordered and satisfy the property that each
as an uncertain point in a (n+m)-dimensional space (a Cartesian product of n+m fuzzy numbers representing meanings of linguistic values). The whole rule base is then given as a union of these uncertain points.

**Approximate reasoning**

If the relation among variables is given through a rule base and if the values of input variables are set, we can calculate the values of output variables, similarly to substituting numeric values into a mathematical function. The calculation is carried out as a composition of a fuzzy relation modeling input with a fuzzy relation modeling a rule base. From a practical point of view (Mamdani’s approach), the procedure determines the extent of fulfillment of each rule of the base by a given input, and the resulting value denotes the height at which the fuzzy relation modeling the right-hand side of the rule is being cut-off. The union of these results for individual rules gives the uncertain information on potential values of output. This computational procedure is known as approximate reasoning. The calculated fuzzy values are usually approximated by real numbers (defuzzified by the method of center of gravity).

**The example of creating a fuzzy expert system with Fuzzy Toolbox of the MATLAB software**

Now, we will show a simple example of creating a fuzzy expert system from information on selected Loos’s villas (Table 1). First, we define input and output variables. In this case, input variables are *Cubature of a villa* and *Number of Floors*. Output variables are *Size of Ground Plan* and *Shape of Ground Plan* (see Figure 3). Let us determine the linguistic scales of these variables (for *Cubature* see Figure 4). The variables *Cubature*, *Number of Floors* and *Size of Ground Plan* can take on values of *small*, *medium*, and *large*;
the variable *Form of Ground Plan* can take on values *square*, *rectangular*, and *elongated*. Figure 5 shows the course of fuzzy inference -- individual rows represent graphically visualized rules. Colored areas show how much a rule is fulfilled for a given numeric input. For input values 2250 m$^3$ of *Cubature* (a larger villa) and 4.5 of *Number of Floors* (four floors and a roof terrace), the Mamdani inference gives the output values of the *Size* and *Form of Ground Plan* expressed as filled-out areas on the right-hand side of Figure 5. After defuzzification, we get the result that the corresponding size of the ground plan is roughly 180 m$^2$. 

![Fuzzy Toolbox of the MATLAB software -- the linguistic scale for the variable Cubature](image1.png)

![Fuzzy Toolbox of the MATLAB software -- approximate reasoning](image2.png)
and the form of the ground plan is rectangular with the ratio of the two sides being about 1.62. Figure 6 shows in three dimensions the dependence of size of ground plan on cubature and number of floors.

**Conclusion**

The aim of presented research is to create a linguistic fuzzy model that would characterize the designing process of a “Loosian” villa, in terms of both its inner spatial organization, a so-called Raumplan, and its outer appearance. The model will be composed from several interconnected rule bases (the process of design is not, in general, firmly set – at any point the architect can go back and change any of his previous decisions; the designing process is a process of experimentation with decision alternatives). Each of the rule bases will help the architect, who is designing a Loosian villa for a given customer and his specific requirements and conditions, with one of the decisions that have to be made during the designing process. At the beginning, the basic characteristics of the building will have to be determined (size and form of ground plan, number of floors) according to basic initial information (the size and type of premises, requirement on the villa size). Following is the decision on location and shape of the prominent unit in a Loosian villa (a hall), related units (dining room, kitchen) and on the ways of their connection. This decision depends on a previous decision on building’s characteristics, and also on characteristics of the surroundings (hall’s orientation onto a recreational zone, possibility of gaining a view). Similarly, the decision on location and connection of units in the house is conditional on previous decisions. The principal choice of the shape of the building and location of units within the house then influences the design of the facade, etc.
Another research goal is the use of fuzzy sets in describing graphical information (location of an element in a given space, description of an object’s shape and characteristic features). The examples provided in this paper demonstrate that it is possible to make fuzzy statements about issues such as position and size of elements. This makes it possible that the large stock of graphical information available in architectural resources may be used to create fuzzy sets.

References