Who cares about right angles?

Overcoming barriers in creating rectangularity in layout structures

Sven Schneider¹, Reinhard König², Robert Pohle³
¹,²,³Bauhaus-University Weimar, Germany
¹,²http://infar.architektur.uni-weimar.de
¹sven.schneider@uni-weimar.de, ²reinhard.koenig@uni-weimar.de, ³robert.pohle@uni-weimar.de

Abstract. This paper examines methods for the generation of structures that exhibit rectangularity. Rectangularity in architectural and urban structures can be traced to various reasons, including facilitating the design process, since the use of rectangular geometry limits both the space of possible solutions and the operations necessary to search the solution space. With the help of computer-based methods it becomes possible to explore huge solution spaces, however most existing methods stick to traditional concepts for the generation of geometric structures, such as the use of predefined elements (rectangles). These approaches do not take into account geometric irregularities which the structure to be generated may be subject to. In this paper we present a method that makes it possible to create a nearly rectangular structure within a freely definable boundary.

Keywords. Rectangularity; Structures; Design Tool; Design Process; Evolutionary Optimization.

RECTANGULARITY IN BUILDINGS

Architectural design is a process of systematic exploration in a potentially infinite space of possible solutions. The result of this process is an abstract representation of a spatial configuration (e.g. floor plans or urban plans). Rectangularity is a common characteristic of such configurations (in the following we use the more general term ‘structure’ to stand for the two-dimensional aspects of a configuration). Precisely which factors have actually led to the use of rectangular forms is still not fully understood (Bafna & Shah, 2007). However, numerous advantages can be mentioned which are related to the use of right angles. The right angle turns out to be practical in many ways, for example for the furnishing of layouts (due to rectangular furniture) or as a means of simplifying the construction (impact of the available materials and construction methods). For movement in space, it has been found that right angles also facilitate spatial orientation. Through experiments Sadalla and Montello (1989) figured out that, during pathway traversal, people tend to remember changes of direction better, when they are a multiple of 90°.

In addition, it is often good to create spaces with a convex shape so that they can be overseen by an inhabitant from any point in the room because this creates a sense of security and well-being (Alexander et al, 1977). If many such (convex) spaces are put together, the use of straight lines and right angles helps to keep the influence of the shape of one room on the shape of another as low as possible (see figure 2).
This raises a further important aspect, which is the handling of rectangular structures during the design process. Handling means on the one hand the ability to create formally consistent structures (aesthetically resolved through geometric similarity) and on the other, the ease of use of the tools needed to create such structures. This aspect will be examined in more detail in this paper and serves as a reference point for considering the development of new design tools.

TOOLS AND THEIR IMPACT ON DESIGN
The tools used in the design process have a significant impact on the resulting design. This is due to the fact that each tool allows only certain kinds of operations. This in turn constrains the scope of the user’s actions to those offered by the respective tool (in other words, you can’t draw a curve with a straight-edge). The German architect Günter Behnisch remarks that: “Cardboard models give rise to stodgy, flat incorporeal buildings: wooden blocks produce wooden block architecture, and modelling clay produces relatively free plastic structures.” (Kaeppel, Kandzia, Behnisch, & Partner, 1987). Such limitations can be inspiring, leading to a specific design methodology, but it can also be constraining, leading to monotonous results. In the late 19th century, Sitte (1911) was concerned about the widespread use of the drawing board, which in his opinion had led to largely simplified forms in urban planning. For Steadman (2006), however, tools play only a secondary role with regard to the emergence of rectangular shapes (see p. 120). He argues that it is the flexibility of the shape of the rectangle itself which allows one to create many variations of formally consistent, rectangular structures based on simple operations, such as packing and scaling (see figure 1). The fact that a rectangle still remains a rectangle after scaling helps in arranging elements within a given outline (which once again should ideally be a rectangle).

![Figure 1](image1.png)

*Figure 1*  
Transformation of packed rectangles compared with that of packed triangles (from Steadman, 2006)
However, the assertion that the impact of the tools is not as significant as previously thought can be qualified by the realisation that the notion of tools encompasses more than just the drawing instruments themselves, but also the mental concepts and methods used when designing. If one considers the concept of the rectangle as a “device” or tool itself employed by the designer as a means to support mental work, then Steadman’s explanation for rectangularity can also be understood as a result of a tool: in this case the mental concept of working with rectangles.

CONSTRUCTING DESIGN WORLDS
Mitchell (1990) writes that during the design process designers are moving within an implicitly created design world, which is determined by the choice of design tools and methods. According to Steadman’s argumentation the design world employed consists of elements (rectangles), which can be combined to form rectangular structures through simple operations, such as translation and scaling. Due to our limited mental capacity (Miller, 1956) such a design world makes sense, because on the one hand it limits the solution space, and on the other, provides simple operations for searching through this space effectively.

Through the use of computer-based design tools we have reached a previously unprecedented level of freedom in designing that allows us firstly to control forms of considerable technical complexity and secondly makes it possible to very quickly explore large solution spaces. The pinnacle of such systems is the automation of design processes. These systems allow designers to pass complex evaluation and generation mechanisms to the computer in order to let him independently calculate optimal solutions. One prominent example of such design automation is the generation of layouts, which, since the early 1960s, has been a recurring theme in the application of artificial intelligence in the field of architecture. Considering the topic of rectangularity, it can be noted, that the systems developed for generating layouts more or less follow the prototypical design world described above. By way of example, the work of Elezkurtaj (2004), Arvin and House (2002) as well as Schneider, Fischer and König (2010) can be mentioned. As an elementary form all projects use rectangles, which are joined together without gaps or densely packed within a rectangular boundary. This can be described as an additive approach. Such simplified assumptions, of course, also simplify the algorithms used for their generation and evaluation. This reduces the computational time required to reach a solution, while the use of basic forms and known operations facilitates the interaction of the user with the system.

An alternative approach to the additive method is to subdivide, i.e. to generate a structure by subdividing a given surface. The works of Flemming (1977), Duarte (2005) or Knecht and König (2010) are examples of projects which work with subdivision methods. With regard to the forms produced, however, all these projects are limited in the exploration of the solution space to purely rectangular structures. The work of Doulgerakis (2007) is an exception in that it does allow non-orthogonal structures. However, it can be noted here that the angle occurring within the structure is not taken into account in the generation process which often leads to poorly usable or uncomfortable rooms.

But how can a system, or more precisely a design world, be constructed that makes it possible to create rectangular structures without using predefined elements (rectangles) or limiting the design space a priori through geometrical restrictions (such as the solely use of right angles)? This will be discussed in the following by developing a method for generating nearly rectangular structures.

GENERATING NEARLY RECTANGULAR STRUCTURES
Despite the frequency of right angles in architecture and the built environment, pure orthogonality in cities and buildings is rare. In general, most building sites have geometric irregularities (obtuse or acute angles). If these need to be taken
into account in the design of a spatial configuration, this inevitably leads to non-rectangular shapes. Using rectangles alone one is unable to respond to such irregularities. Here we should note that neither the shape of a rectangle is a prerequisite for architectural spaces, nor is a perfect right angle absolutely necessary: “There are reasons why they [the walls] should be nearly straight; [...] But there is no evidence that their sides need to be exactly the same nor its corners absolutely rectangular” (Alexander et al, 1977; see figure 2). To consider geometric irregularities and at the same time obtain predominantly rectangular structures, angles need to be handled more flexibly. Rather than ignoring angles (which is the case with rectangles as all its four angles are equal), the following system generates structures that adhere wherever possible to the principle of right angles – in short: who cares about rectangles if you can use right angles!

A PARAMETRIC APPROACH
A simple algorithm for generating nearly rectangular structures in a given polygon is based on its division starting from its centre point (cp) to the midpoints of its bounding edges (mp). The resulting quadrangles must then be divided again using the same method. The result is a grid formed of compact, nearly rectangular quadrangles (see figure 3).

A problem with this method is, firstly, that polygons with a high number of edges result in the increasing occurrence of acute angles, and secondly, that it is restricted to convex polygons. Furthermore, it always creates the same grid-like structures, as determined by the linear sequence of the algorithm. One can also speak of a parametric approach because the results are completely dependent on the input parameters. To create structures that satisfy the requirement of right angles but otherwise “develop” freely, a different, less linear approach needs to be found, which can generate a wider range of possible solutions.

AN EMERGENT APPROACH
For dealing with problems where the solution is not already conditioned in part by the definition of the problem (e.g. where the direction is not clear), Evolutionary Strategies (ES) are a suitable approach. ES, which are inspired by the process of biological evolution, create solutions in an iterative process of trial and error, which gradually adapt to specific requirements (fitness criteria) without defining a specific direction of improvement (Rechenberg, 1994). Crucial for the successful application of such strategies are the generative mechanism and evaluation mechanism. The generative mechanism frames the solution space within which the best solutions are searched for. It is important, therefore, that this mechanism can generate a large number of different solutions.
Consequently, we have chosen to subdivide the given area by Voronoi tessellation. Voronoi tessellation is suitable for the generation of structures because it is able to generate a wide range of possible solutions (from a rectangular grid to a honeycomb grid to totally free allocations). The evaluation mechanism checks the angles occurring in a structure according to their deviation from the right angle.

To create a structure that is as rectangular as possible within a given boundary the algorithm works as follows: in the first generation, a random solution is generated for every individual. This happens by randomly distributing points, from which the Voronoi diagram is calculated. The resulting Voronoi diagram (which consists of centres, regions and their edges) is then examined to determine which edge points are located within the boundary polygon and which intersections occur between the Voronoi edges and the polygon. For the interior points, each of the three interrelated angles is calculated. These three angles arise from the fact that in Voronoi diagrams three edges always meet at one point. For the intersections of the Voronoi edges with the boundary, the angle of intersection is calculated (see figure 4). Afterwards the deviations of the occurring angles from the right angle (90°) or multiples of right angles (e.g. 180°) are totalled. The various possible solutions (individuals) are sorted in terms of fitness, and the best solutions are then carried forward to the next generation and the rest discarded. New solutions are generated through the combination or mutation of the best solutions. Based on this simple generation, evaluation and selection rules, the structure is progressively optimized in an iterative process.

The software tool we developed to demonstrate the functionality of the algorithm described above (see figure 5) is interactive, allowing the user to intervene during the optimization process. For example, the boundary shape can be changed at any time, causing the structure to adapt to the changes in real time. The designer therefore receives immediate feedback on his actions and can explore the space of possible solutions in dialogue with the tool (Schön, 1992).

Using this tool, we tested the algorithm in a variety of test cases. From the results, we can infer that specific types of solutions recur again and again, most notably the creation of continuous lines (see figure 6: 1b, 1c, 2a, 2b, 3a). These lines are usually located centrally within the boundary and trace the form of the boundary (see 1a, 1b, 2b, 3a).
The algorithm implemented at present is still very simple and is not yet optimized for performance. The calculation of the examples above takes at least 2-3 seconds (about 30 generations) to complete. To obtain more responsive user interaction it is necessary to reduce the speed of calculation to a fraction of a second. This would then make it possible, for example, to add new rooms or to move existing rooms in real time, whereby the angles occurring within the structure always remain approximately at right angles.

To improve the results, the evaluation mechanism also needs to be extended (see Figure 7). For example, it would be conceivable to introduce additional criteria, such as the convexity of the resulting regions (7a), the proximity of the edge of a region to the corners of the boundary (7b) or parallelism (7c). Similarly, approximately rectangular structures could be corrected subsequent to their evolution, for example, to straighten crooked lines (7d). A combination of the iterative evolutionary algorithm and a working method for directly improving problematic cases could also reduce the calculation time.

To arrange a large number of regions within the boundary, it is also worth examining whether hierarchizing the subdivision algorithm (i.e. the re-subdivision of the approximately rectangular regions) could bring further benefits. When creating densely-packed sets of rectangles it has been shown that such an approach can have a tremendous impact on the speed of calculating a solution, since the number of elements that have to be optimized is simultaneously reduced (Koenig & Schneider, submitted).

CONCLUSION & OUTLOOK
The dominance of the right angle in architecture has many causes. This in itself is not the focus of this paper, likewise the issue that new design tools have made geometric construction less dependent on rectangles. However, computer-based methods are particularly useful in the design process (for example, as a complement to existing CAD systems) and open up new ways of generating structures. In this paper, the right angle served as a starting point for the generation of structures. We discussed methods for dealing with rectangularity that are not dependent on predefined elements (rectangles) or a priori defined restrictions for the occurring angles.

The division of an area and its optimization for near-rectangular structures represents one promising approach. The implemented evolutionary algorithm can be enhanced further and alternative subdivision algorithms are currently being investigated. In a later stage, the evaluation mechanisms will be extended to include functional constraints (such as topological relations and spatial variables).

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