

# Estimating the Fractal Dimension of Architecture

## *Using two measurement methods implemented in AutoCAD by VBA*

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**Abstract.** *The concept of describing and analyzing architecture from a fractal point of view, on which this paper is based, can be traced back to Benoît Mandelbrot (1981) and Carl Bovill (1996) to a considerable extent. In particular, this includes the distinction between scalebound (offering a limited number of characteristic elements) and scaling objects (offering many characteristic elements of scale) made by B. Mandelbrot (1981). In the first place such a differentiation is based upon a visual description. This paper explores the possibility of assistance by two measurement methods, first time introduced to architecture by C. Bovill (1996). While the box-counting method measures or more precisely estimates the box-counting dimension  $D_b$  of objects (e.g. facades), range analysis examines the rhythm of a design. As CAD programs are familiar to architects during design processes, the author implemented both methods in AutoCAD using the scripting language VBA. First measurements indicate promising results for indicating the distinction between what B. Mandelbrot called scalebound and scaling buildings.*

**Keywords.** *Box-Counting Method; Range Analysis; Hurst-Exponent; Analyzing Architecture; Scalebound and Scaling objects.*

## BACKGROUND

### ***Scalebound and scaling objects***

Benoît Mandelbrot (1981) who is often called the father of fractal geometry made a distinction between *scalebound* objects and *scaling* objects while at same time taking into account its special significance for buildings. As their main features, examples of the first group display a limited number of characteristic elements of scale that are clearly distinct in their size. In architecture this e.g. corresponds to buildings of the so called *International Style* (if at all applicable to a style), with width and height of the whole representing the elements of scale. Moreover,

*scalebound* objects offer a limited number of smaller components that are distinguishable in their size including windows and doors. As a result, the whole and its parts can be perceived from a certain distance without any difficulty (though this nevertheless depends on the overall size of the building). Furthermore such buildings neglect elements that are of smaller size than the human scale.

In contrast, *scaling* objects display many characteristic elements of scale that cover many different sizes flowing into each other (some of them are even smaller than human scale). As a consequence, single elements can hardly be distinguished any more. In

absence of a typical distance the observer has to move closer, which brings different elements of size into focus. Besides, examples of this second group display characteristics close to fractal geometry, although there are some limits with natural objects as well as with manmade objects like buildings. Those limits concern the range for which fractal characteristics are valid. There the upper bound depends on the outline similar to the canvas of a painting which limits the painting itself. The lower bound, in turn, is defined by material, handling by tools or construction and economic constraints similar to the restrictions of painting by the size of a brush stroke or palette-knife (Mandelbrot 1981).

### ***Fractal geometry as a source for analyzing art and architecture***

Analyzing objects from the point of view of fractal geometry includes observing different parts and different levels of scale, respectively. This is achieved e.g. with a photograph as a two-dimensional representation of a real object by cutting it into pieces each of which is looked at separately (Mandelbrot 1981). The results of such an observation are diversified: Some objects offer the same degree of details on each piece, while others strongly vary with a visual description ranging from nearly empty parts to diversified ones. In this context affinity to fractals is represented by those examples where cuttings are similar to the whole, which means that they offer similar characteristics. Parts need not be identical scaled down copies but should reflect the whole in their characteristic appearance (so they are then called statistically self-similar). In case of architecture this essentially means that each component should mirror the whole, a feature which architects including F.L. Wright intuitively strived for (Evers 2006). Analyzing self-similar objects, however, results in the observation that parts of interest (areas where something can be seen) and those of lower interest (where little information is presented) are alternating. Furthermore when zooming in on parts of interest, the same picture emerges: areas of interest alternate with nearly empty ones. This concept has a

high affinity to nature, where trees, mountain ridges, bushes, courses of rivers, clouds or coastlines display such property.

B. Mandelbrot (1982) basically argues that fractal art is close to nature and, as humans are familiar with such forms, fractals seem to be more easily accepted than forms that are close to the smooth Euclidean geometry. In doing so artwork is not copying natural forms but imitating their rules. These rules are found in characteristics taken from fractal geometry, e.g. self-similarity, roughness and fractal dimension. At this point consequently the question arises, how such an affinity to the fractal concept can be measured in architecture. Self-similarity of facades for instance may be detected by computer programs analyzing proportions of the whole and its parts. This includes elements that stretch forward like balconies, openings like windows and finally ornaments or forms inherent to material, reaching from the scale of general view down to the scale of smaller details. Due to intersections and overlaps, programming this particular analyzing tool is not trivial. Fractal dimension or more precisely its adequate box-counting dimension on the other hand is based on an algorithm that is easy to implement. With the so called *box-counting method* different scales are analyzed putting boxes or, which can even be handled more easily, a grid (consisting of boxes) over the object of interest (similar to pieces of a photograph). This particular manageability is then the reason why – for the moment – the authors' focus of fractal analysis lies on the *box-counting method*. Nevertheless up to now only a few researchers have applied this method to architecture. Some of these results will act as reference values for the author's own measurements (Bovill 1996; Ostwald et al. 2008; Vaughan et al. 2010).

## MOTIVATION

### ***Two methods of estimating fractal dimension***

The fractal dimension  $D_f$  can be interpreted as the dimension of self-similarity of an object (Deussen 2003). Furthermore, it offers a possibility to compare buildings by their characteristics in terms of visual complexity. There are certain methods for measuring  $D_f$ , two of which C. Bovill (1996) introduced into architecture. The first one is called *box-counting method* determining the box-counting dimension  $D_b$ , which is equivalent to  $D_f$  (Mandelbrot 1982). This method enabled C. Bovill to demonstrate the differences in complexity between the main facade of Robie House by Frank Lloyd Wright and that of Villa Savoye by Le Corbusier. Since then this method has been applied to facades nearly exclusively by only a few researchers (e.g. Vaughan et al. 2010, Ostwald et al. 2008). The second is called *range analysis* for measuring the Hurst-Exponent  $H$  of a given rhythm in a floor plan. C. Bovill (1996) described the functionality of this method using Willits House by Frank Lloyd Wright.

The author implemented both methods in AutoCAD using the programming language visual basic for applications (VBA). This has up to best knowledge never been done before, except that the box-counting program basically represents further development of a previous version by the author. In this paper, both programs are analyzed with regard to their suitability as instruments for characterizing architecture, considering, in particular, finding a tool that supports the distinction between *scalebound* and *scaling* buildings. The benefit of integrating the measurement methods in a CAD-software is to allow architects an estimation of box-counting dimensions  $D_b$  and the Hurst-Exponent  $H$ , respectively, of their designs directly in a tool they use during their design process. Furthermore certain parameters can be handled on one's own to detect and limit their influence.

## THE FUNCTIONALITY OF THE COMPUTER-PROGRAMS

### ***Using VBA for AutoCAD***

The fractal dimension  $D_f$  of architecture or more precisely the box-counting dimension  $D_b$  of facades and the Hurst-Exponent  $H$  of a rhythm in floor plans are rarely found in literature. Furthermore, up to now programs for evaluation have been examined separately from their application. The fact that programming and analyzing is done by one person offers the possibility of interaction with the program itself in order to restrict certain influences arising from the respective method and especially from an evaluation of architecture. This includes control over the size of empty space around the measured two dimensional illustration of e.g. a facade, the range of box-sizes that provides stable estimates of  $D_b$ , the reduction factor of box-size and the positioning of the grid over the analyzed illustration (Foroutan-pour et al. 1999). Embedding the algorithm in AutoCAD also offers additional advantages. One results from using vector-based graphics instead of pixel graphics. In this case, the resolution of the picture has no influence on the result. Another advantage is that vector-based designs by architects can immediately be analyzed with regard to the characteristic value of  $D_b$  or  $H$ , respectively.

### ***The box-counting method in detail***

With the *box-counting method* the smallest number of boxes that covers an object is identified, while the box-size approaches zero. The box-counting dimension is then defined as the limes of the  $\log(\text{counted boxes that cover the object})$  divided by the  $\log(\text{reciprocal of box-size})$ . Due to the absence of infinite time that is needed for calculating the number of boxes with infinitely small size, the value can be estimated by the trend occurring with larger box-sizes. This is important for architecture that is strictly speaking not fractal but offers fractal characteristics only for a certain range of scales. Consequently the trend within this range is of interest defining a characteristic value (or range). The algorithm of the simplified

*box-counting method* starts with a grid (instead of single boxes) laid over a two dimensional representation of a real object, for example, an elevation plan representing a real facade. The grid-size (reciprocal of number of boxes at one row) again derives from adjusting a certain number of boxes for the smallest plan side. Then only those boxes of the grid are counted that contain relevant parts as in terms of architecture the outline and openings. In the next step the grid-size is reduced and those boxes covering the curves and lines of the plan are counted again. After several steps the results are printed in a double-logarithmic graph with the log(counted boxes) versus log(reciprocal of grid-size). Finally the slope of the regression line represents the average box-counting dimension for a certain range of grid-size.

The measurement method contains several parameters. Finding out their impact on the result and, as a consequence of that, minimizing their influence on the computer-program for estimating the box-counting dimension in AutoCAD allows the user to adjust them. This is done by a user form including the following points:

1. The number of iterations defines the steps of how often the box-size is reduced by one half.
2. By changing the enlargement factor the user adds a different percentage of empty space in relation to the smallest side around the image.
3. The number of steps in between determines the reduction factor of the box-size between one half.
4. The number of boxes of the smaller side defines the initial grid-size.
5. A number of displacements can be realized in x-direction as well as y-direction. As the algorithm looks for the smallest possibility of covering an image with a certain box-size, accuracy should be improved by analyzing different starting-positions. They are related to the empty space around the image and the number of displacements.
6. One additional box can be added, if the empty space amounts to more than one half of the box size.

Furthermore apart from automatically drawn rectangles that indicate the choice, the minimal outline and the enlarged measuring field, the results can be visualized as well. For doing so two different options are available: either all grids with their covered boxes are visualized or just the largest and the smallest one.

### **Range Analysis**

*Range analysis* as another tool for estimating architecture has rarely been tested. With this method the distances between axes (on a separate layer) determining the design are at first translated into a step-function. Then for different scales of view, the respective average values of the largest differences of distances (maximum fluctuation range) are analyzed, whether they correlate with each other or not. This is done by transforming the data into a double-logarithmic graph with log(maximum fluctuation range) versus log(scale) (Figure 1). The result may offer either no relationship or it signals a trend, i.e. there is a power law relationship. If a relationship exists, this is indicated by a coefficient of determination near one for the average Hurst-Exponent H defined by the slope of the replacing line. As floor-plans are rather experienced by a sequence of rooms, expressed by a rhythm of walls, this seems to be a good addition to the *box-counting method* that is rather suitable for facades. Therefore the method has been embedded in AutoCAD as well. The connection to the fractal dimension  $D_f$  and H is finally given by (Mandelbrot 1982):

$$D_f = 2 - H \quad (1)$$

### **RESULTS OF THE BOX-COUNTING METHOD**

The suitability of the *box-counting method* is tested by analyzing strict self-similar fractal curves of which the self-similar dimension  $d_s$  can be calculated. As it does not matter which measurement is used for calculating the fractal dimensions are all leading to the

Figure 1

Range Analysis: (Left top) Main axis; (Left bottom) Step Function resulting from a planning rhythm, divided into four and eight pieces. (Right) Double logarithmic Graph showing  $H=0.19$  ( $D=1.81$ ).

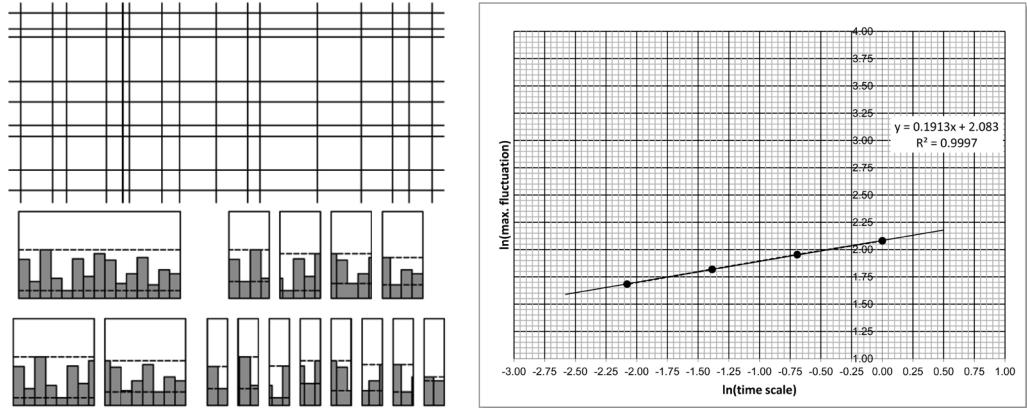
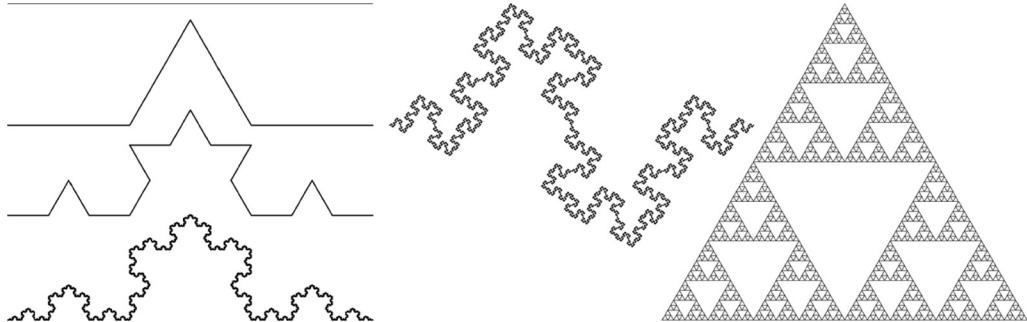


Figure 2

Fractal curves: (Left) Koch Curve: from top to bottom: initiator, generator, 2<sup>nd</sup> iteration, 6<sup>th</sup> iteration; (Middle) 5<sup>th</sup> iteration of Minkowski Curve; (Right) 5<sup>th</sup> iteration of Sierpinski Gasket.



same value (Mandelbrot 1982), both  $D_b$  and  $d_s$  are equal. Constructing a strict self-similar curve, such as the so-called Koch Curve, one starts with an initiator that is replaced by a generator defining the basic construction rule. The generator which determines e.g. the Koch Curve consists of four identical scaled down pieces of the initiator representing a line of a certain distance (Figure 2 left). In the next iteration every new line is replaced by the same rule and so on. The equation

$$d_s = \log(N) / \log(1/s) \quad (2)$$

in which  $N$  is the number of pieces and  $s$  the reduction factor, finally determines the self-similar dimension  $d_s$ . For the Koch Curve, this is  $d_s=1.26$ . In order to generate different fractal curves with different numbers of iterations the author programmed a *Lindenmayer system* in VBA for AutoCAD as well.

### Results of measuring the koch curve

In more precise terms the Koch Curve specifies only the result after infinite iterations, which cannot really be represented in a drawing. Therefore the following measurement using the *box-counting method* is based on an intermediary result of the basic algorithm representing only 6 iterations (this

respectively corresponds to a curve made up of  $4^6=4096$  single lines). The adjustments of the initial experimental arrangement include a reduction factor of one half, a starting box-size determined by two boxes of the smaller side, a white space of one percent of the smaller side and 12 reductions. Calculating the slope of the regression line for all 12 data points finally defines the average box-counting dimension  $D_b$  for this particular range. With 1.19 it deviates from  $d_s$  by 5.72%. Regarding the double-logarithmic graph more closely it can be observed that the left two data points fluctuate before the straight part between the third and 8<sup>th</sup> step indicates a trend (connecting scale and number of covered boxes to each other). After that, for the last four data points the curve changes its direction to a slope of approximately 45 degrees (Figure 3 left), which corresponds to a box-counting dimension  $D_b$  of 1.04. This implies that from a certain grid-size on, only the single one-dimensional lines are measured, out of which the one presented starting of the Koch Curve (with 6 iterations) is constructed. However, for the first straight part  $D_b$  equals 1.27, with a calculated difference from  $d_s$  of only minus 0.65%. Furthermore, for this arrangement a coefficient of determination very close to one (0.999) indicates a significant correlation. Consequently, the lower and higher bound of the range is determined by the area where data points are following a trend. This is in

turn characterized by a coefficient of determination near one, which means that data points are close to the regression line in the double logarithmic graph.

### Influences

Comparing the previous measurement with an initial construction after 8 iterations (the resulting curve is then made up of  $4^8=65536$  single lines) using the same adjustments shows how the range of correlation is enlarged (Figure 3 right). On closer analysis of the double-logarithmic graph this time only the first two data points deriving from the two largest grid-sizes fluctuate to some amount from the regression line, while the others are almost placed on it. Excluding these two points and the last one (deriving from smallest grid-size) finally improves the deviation from  $d_s$  by then being only minus 0.35% ( $D_b = 1.266$ ). The larger range of correlation results from the influence of the relationship between the smallest detail of the object and the smallest grid-size. In the first case (6 iterations) the length of the distinguishable straight lines equals 6.3 units, with the smallest grid-size being 10.6 (for  $D_b=1.27$ ), while in the second case (8 iterations) these are 0.7 and 1.3 units, respectively. Following from that, the smallest grid-size should be greater than the smallest detail.

Different measurements of fractal curves (more precisely of their initial constructions) indicate some variety affected by influences of white space (in

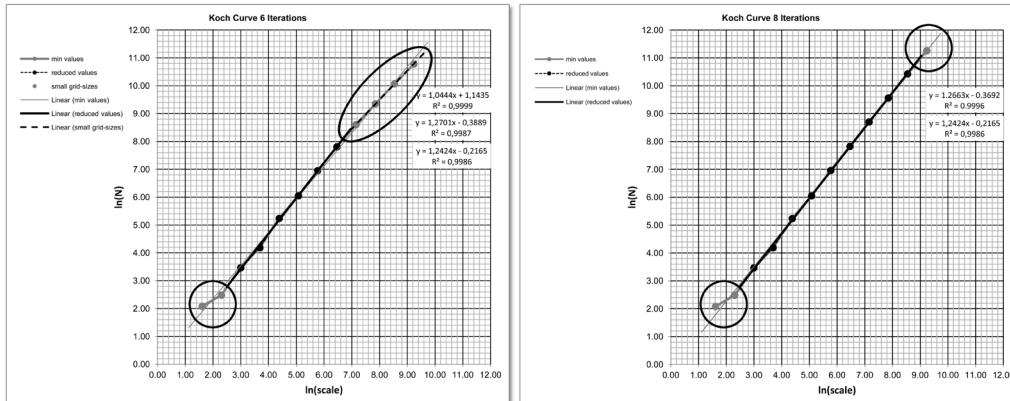


Figure 3  
Box-Counting Dimension:  
(Left) Koch Curve with 6 iterations  $D_b = 1.27$  for range 340 units to 10.6 units. (Right) Koch Curve with 8 iterations  $D_b = 1.266$  for range 340 units to 1.3 units.

percentage), the number of displacements and the degree of fineness given by the number of steps in between. However, with Koch Curve (8 Iterations), Minkowski Curve (7 Iterations; figure 2 middle) and Sierpinski Gasket (Figure 2 right) additional steps in between do not improve the results, but extend measurement time. The same is true with changing the position, which seems to have an even negative impact. As a consequence starting values for a first overview are recommended as follows:

1. Number of iterations depends on the smallest detail on the plan, as the smallest grid-size should not fall below its size.
2. The set enlargement factor equals one percentage.
3. With initial adjustment there are no steps in between.
4. Two boxes at the smaller side define the starting grid-size.
5. Initial adjustment is performed without displacements.
6. No additional box is added (would enlarge the white space as well).

In the double-logarithmic graph the straight part of the data curve then visually defines the initial range of coherence, with a calculated coefficient of determination close to one (greater or equal 0.998). Subsequently for this particular range more detailed measurements with different adjustments achieve several results. Finally the average box-counting dimension and its bandwidth act as a characteristic. With Koch Curve (8 Iteration) this is 1.26 to 1.27 ( $d = 1.262$ ), with Minkowski (7 iterations) this is 1.47 to 1.5 ( $d = 1.5$ ) and with Sierpinski Gasket (8 Iterations) this is 1.56 to 1.58 ( $d_s = 1.585$ ).

## RESULTS OF FACADES

Analyzing facades with the help of the *box-counting method* yields a great diversity of results. The south-east elevation of Villa Savoye (1930) by Le Corbusier e.g. does not display one clear linear part of the data curve, leading to the assumption of a weaker correlation. But on closer observation two different areas become prominent. The first range reaches from 0.6

to 4. meters with a box-counting dimension around 1.66 indicating a higher visual complexity. However, then an area from 1.0 meters down to 0.03 meters with an average box-counting dimension around 1.25 follows. This result underlines the tendency of modern architecture towards a clear expression with details on small scales being reduced to a minimum: After higher complexity at the beginning, the data curve quickly flattens, but remains constant. This particular break-point may also be the reason for different results given in literature (Vaughan et al. 2010).

In contrast to that, Peter Behrens' Turbine Factory in Berlin (1910) displays a strong correlation for a wider range of grid-sizes reaching from 0.2 meters up to 7.0 meters with box-counting dimensions between 1.64 and 1.68. This is similar to starting values of Villa Savoye but for a wider range of scales, both up and down (the facade width being twice the size). With 1.63 a slightly smaller average value is obtained for Robie House by Frank Lloyd Wright, which confirms other measurements (Ostwald et al. 2008, Lorenz 2011). This time grid-size ranges from 0.4 to 8.7 meters. Considering that on a smaller scale window design comes into focus, which is not shown on this particular plan, the range even extends below 0.4, underlining the architect's effort for consistently providing further details from large to small scale. In turn the measurement of the south east elevation (the smallest facade width of presented examples) of Gerrit Rietveld's Schröder house (1924) indicates strong correlation for a smaller range from 0.04 to 1.8 meters with values between 1.48 and 1.55. This, however, shows that even at first sight smooth modern architecture may offer complexity for smaller scales.

The results are presented as box plot showing the median, the lower and upper quartile (defining the area of 50% of data points), the largest observation and outliers (Figure 4 left). Moreover, facade widths define the scale of each building and their relation to range of box-sizes (Figure 4 left). To express a clear statement still more measurements for each building are, however, necessary. Nevertheless,

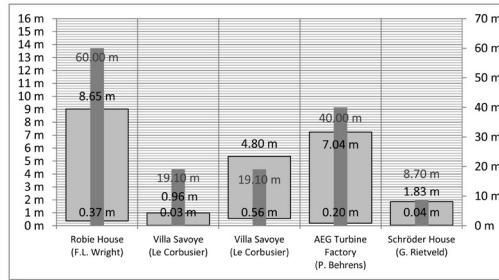
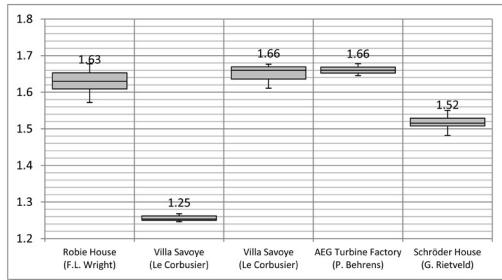


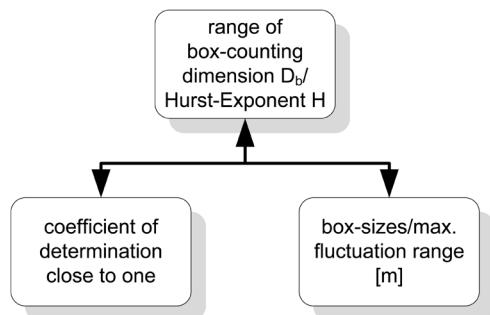
Figure 4  
Results of measurements:  
(Left) Range of box-counting dimension. (Right) Range of box-sizes and facade widths of facades.

up to now the results indicate that scaling buildings like Robie House display a wider range of correlation with a higher average value. In contrast to that the rather scalebound building Villa Savoye offers a break-point, with a restricted area of a higher value followed by an area of an average value close to one.

## CONCLUSION

### Box-counting and range-analysis as instruments of comparison

Both the *box-counting method* (for facades) and the *range-analysis* (for rhythms of the floor plan) offer a possibility to measure architecture with regard to its visual complexity. Thus  $D_b$  and  $H$  provide a computable value for comparison and, as a further consequence, for classification with respect to what B. Mandelbrot called *scalebound* and *scaling* buildings. However, a couple of measurements have to be done with the *box-counting method* for each building, before it is possible to state the characteristic



roughness over a range of scales. In any case, for comparison the result of measurement has to include the range of box-counting dimension  $D_b$  and/or fractal dimension  $D_f=2-H$ , the range of box-sizes and/or the maximum fluctuation range in meters, and finally a coefficient of determination close to one (Figure 5).

## OUTLOOK

In comparison to the vector based measuring programs presented in this paper, both methods will be implemented in *NetLogo* (an agent-based programming language and integrated modeling environment) looking for differences in comparison to pixel graphics. By not referencing on already existing similar programs this means one maintains the possibility of interacting and using the same adjustments of parameters. Furthermore, another focus will be put on the automation of finding the range of significant correlations for standardizing the comparison of the results. Finally, different measurements will be performed with both, the *box-counting method* and the *range analysis*, on outstanding buildings of the twentieth century looking at the consistency between facades and rhythm, but also in order to be able to compare various buildings. Another possible field of application concerns the comparison of buildings with their natural or man-made neighborhood. This means that the visual complexity of a mountain ridge behind the building or a wood nearby is evaluated and compared with that of the building itself (Bovill 1996).

Figure 5  
Important values for comparison of buildings.

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