Fuzzy Approach to the Analysis of Architectural Composition

As applied to villa design by Adolf Loos

Zuzana Talašová
Czech Technical University in Prague, Faculty of Architecture, Cabinet of Architectural Modelling, Thákurova 2700/9, 16634 Praha 6, Czech Republic.
zuza.talasova@post.cz

Abstract. The fuzzy sets theory enables to mathematically process uncertain information (uncertain data, uncertain relations). Natural language is also capable of dealing with this type of information. It was this capability of natural language that inspired Lotfi Zadeh in creating the concept of fuzzy sets. Linguistic fuzzy models are models with two levels of description: linguistic (intelligible to humans) and fuzzy, i.e. mathematical (intelligible to computers). They operate on linguistic variables and linguistically defined functions, rule bases. Substitution of input values into a linguistically defined function, and calculation of output values are performed through approximate reasoning. The application of linguistic fuzzy models to the analysis of A. Loos’s architectural work is described in the dissertation thesis of the present author. The aim of this contribution is to apply linguistic variables in describing architectural composition.

Keywords. Fuzzy sets; architectonic composition; geometrical analysis; A. Loos villas.

TOOLS OF FUZZY MODELING AND THEIR APPLICATION TO ARCHITECTURE
At the moment when we start to consider CAAD not only in its initial stage, i.e., as mere translation of a geometric model of a building into a computer, but in a higher stage of its development (Achten, 2007) where also the expert knowledge on the process of architectural design should be fed into a computer, the tools of standard mathematics are no longer sufficient to provide an appropriate model. While the defining feature of classical mathematics is accuracy, an important feature of human reasoning is ambiguity of concepts with which a human operates. This ambiguity of notions that is present in human language is not an imperfection but, quite the contrary, is a major advantage of human thinking. Essential features of a very complicated phenomenon can be described in rather few words. A natural language, by means of ambiguous concepts, is well suited to describe relations among the values of variables of interest, i.e. of behavior, even with very complicated systems.

The tools of standard mathematics are well applicable in modeling usual physical or technological systems. However, in modeling very complicated, especially so-called soft systems (biological, sociological, political, economic systems or, as is the case here, system of architectural design) the tools of fuzzy mathematics are more appropriate (Dubois and Prade, 2000; Zadeh 1965; Zadeh 1975; Talašová 2003). Fuzzy mathematics represents a mathematical tool for conceptualizing uncertainty. The mode-
ling based on fuzzy mathematics is close to a human way of thinking. The use of fuzzy mathematics is justified whenever it is necessary to model expertly-set (often verbal) data or expertly-defined knowledge (described through natural language) about relations among the values of variables.

A study of A. Loos’s work simultaneously with the study of the fuzzy sets theory has taken me to the idea of using the mathematical tools of this theory to describe compositions of Loos’s buildings, localization of particular elements within these buildings, and to describe the principles that A. Loos adhered to while designing his villas. After having searched the literature, having browsed the Internet, and having consulted with experts on the theory and applications of fuzzy sets, I have reached the following conclusion. Although there are a number of applications of the fuzzy sets theory in the field of architectural work (Oguntade and Gero, 1981; 1983; Gero and Volfneuk, 1980; Colajanni and De Grassi, 1989; Cao and Protzen, 1997; Koutamenis 2001; Durmisevic and Ciftcioglu and Sariyildiz, 2001; Durmisevic 2002), when compared with other applications of the fuzzy sets theory the applications in architecture are not very frequent. The available sources have not mentioned any use of fuzzy tools in describing architectural composition, neither have they mentioned any application of a fuzzy expert system to identify the principles of an architect’s work.

A concept employed – linguistic variables
Linguistic variables are variables (e.g. Size, Shape) the values of which denote uncertain quantities or uncertain degrees of a characteristic (“small”, “approximately square”). It is important to note that the meanings of these words are modeled through fuzzy numbers, i.e. special fuzzy sets defined on a given interval of real numbers (e.g. as regards the floorage of A. Loos’s villas, on the interval of all possible floorage sizes in m2, e.g. [0, 200]). A fuzzy number is represented by a real function (a membership function) that assigns to each number from the given interval (e.g. to each size of floorage) a number from the interval [0, 1] with the meaning how well the given value corresponds to a given word (e.g. to what a degree a villa with the floorage of 95m2 is a “medium-sized villa”). The most commonly used fuzzy numbers in applications are the simplest ones, so-called linear fuzzy numbers, which are defined through ordered quadruples of numbers \(x_1 \leq x_2 \leq x_3 \leq x_4\). The interval \([x_2, x_3]\) represents those values that are in perfect correspondence with a given word (e.g. floorage from the interval [100, 130] certainly corresponds to a medium-sized villa), the membership function being equal to 1. The values smaller than \(x_1\) (e.g. 70m2) and larger than \(x_4\) (e.g. 160 m2) do not correspond to a given word at all (the membership function being equal to 0 there). A shift from a zero value at \(x_1\) to a unit value at \(x_2\) is linear; analogously for the points \(x_3\) and \(x_4\). Fig. 1 illustrates linguistic scale Floorage of a villa.

The values of linguistic variables, which are used in linguistic fuzzy modeling, usually form linguistic scales (e.g. linguistic variable Floorage with values Small, Medium, and Large). The linguistic scales are mathematically modeled by fuzzy scales. A fuzzy scale is a set of linearly ordered fuzzy numbers with the following special property: the way the membership function of the fuzzy number decreases from one to zero is analogous to the way the membership function of an adjacent fuzzy number increases from zero to one. Sometimes the basic elements of a linguistic scale are not sufficient for a proper description. Then it is better to use some new linguistic terms derived from the basic ones (e.g. “more or less medium” or “definitely small floorage” in case of an enriched linguistic scale, “small to medium floorage” in case of an extended linguistic scale, or “floorage between medium and large” for a scale with in-between values).

THE POTENTIAL OF LINGUISTIC VARIABLES IN A GEOMETRICAL ANALYSIS OF A BUILDING AND IN ARCHITECTURAL COMPOSITION
In usual applications, linguistic variables are employed to replace the intervals of real numbers (with infinite number of values) with their finite represen-
tations, i.e. fuzzy scales. In a large majority of cases what we have in mind is a linguistic description of univariate mathematical variables.

The objects that are of interest to architecture are either two-dimensional (ground plan, facade) or three-dimensional (villa). In this contribution it will be argued that linguistic variables can be employed to localize an architectural object in a plane (an oriel in a facade) or in a space (a room in a house); to characterize the shape of an object (oblong or square shape); and to capture the overall impression of a building (dynamic appearance, open plan).

A fuzzy approach to localize an architectural element within a building

In geometrical analysis of a building verbal description is useful. Figure 2a takes as an example the Moller Villa designed by Loos and shows how to localize an object (e.g. an oriel) within a facade by means of a pair of linguistic variables “Height” (with linguistic values Bottom, Center, Top) and “Width” (with linguistic values Left, Center, Right) whose values are modeled through fuzzy scales on the intervals $[0,v]$ and $[0,s]$, respectively, where $v$ is the height of facade and $s$ is the width of facade. Figure 2b shows analogously how to use three linguistic variables to localize rooms within a villa.

A fuzzy approach to determine a shape of an architectural element

A proportion is defined as a ratio of magnitude of two or three basic dimensions (height, width, depth), or as a relation between parts in a whole, or as a relation between a part and a whole. The well-known proportion is that of a so-called golden section. Through identical proportions the elements can acquire the quality of relatedness, association.
Fuzzy sets can use proportions to provide a rough description of shapes. In Figure 3a rectangle (ground plan, window) is described through the ratio of height to width. If the ratio is greater than one, we have a horizontal rectangle, if smaller than one, approximately a square. The same principle can be applied in a three-dimensional space. A building can be of a horizontal, vertical, or cubic shape.

A fuzzy approach to describe composition of a building

The notion of composition (structure, configuration) means, in architectural terminology, the configuration of individual architectural elements in a space. In this sense, the above-mentioned applications of linguistic variables to describe a building's geometry can be subsumed under the analysis of its composition. However, falling under the head of architectural composition is also the overall impression of a building in relation to its shape and configuration of individual elements. Notions as compactness of a building, its symmetry, dynamics, openness vs. closedness etc. have been introduced (Tichý and Dvořák, 1986; Krier 1989).

In what follows I will outline the potential of fuzzy sets and linguistic variables in modeling some of these notions. The designs of Loos’s villas will serve as examples.

For example, the measure of façade symmetry with respect to a vertical axis running through the center of a façade can be defined as a ratio of the sum total of areas of all elements and parts in a façade that are symmetric with respect to this axis, to the sum total of all areas on a given façade (see Figure 4a).
The compactness of Loos’s villas, that contain mainly rectangular elements, can be defined, for example, as a ratio of a building’s volume to the volume of the nearest prism (Figure 4b). (Minor protrusions may be neglected.)

Another important feature of a building is the position of windows in the facade. Loos’s facades possess either concentric or sparse pattern of windows. The type of pattern is determined through the ratio of window area in the center of the facade to the total window area (Figure 5).

Other architectonic features, like closedness and openness, can be assessed in the same manner. The measure of façade openness can be defined through the ratio of the sum total of all openings to the total area of the facade. The area of openings of the street-facing facade of the last Loos’s villa is very small compared to the total area – the facade gives a closed impression; on the contrary, the ratio of openings of the backyard facade of the Moller Villa is large (Figure 6).

One possibility is to conceive of the degree of a given quality as a degree of membership of a building to the fuzzy set of buildings possessing this quality. The degree of a quality can be expressed through the points on a linguistic fuzzy scale, so that instead of the classical degrees of membership we work with the fuzzy degrees of membership to a fuzzy set of buildings with a given quality. For example, dynamics is a rather complicated notion to be measured through a mathematical formula – the dynamics of a building can be better assessed expertly by a linguistic value, e.g. “highly dynamic facade”. This verbal expression will be modeled by a fuzzy number on the interval <0,1>. That is, instead of the degree of a quality from the interval <0,1> we get a fuzzy degree of dynamics – a fuzzy number on <0,1> (Figure 7a).

Contrast, in architectural terminology, denotes any difference in a whole or in a detail. It runs through all components of architectural composition, reflecting their variety and change. We speak of the contrast of size, shape, location, direction, scale, proportion, material, color, light etc. We also discern the scale of contrast, ranging from radical contrast to nuance contrast. This very subtle minute contrast represents transition to unity (Figure 7b).

Colorfulness is a very subjective component of architectural composition. A characteristic feature of Loos’s buildings is the utmost neutrality of exterior (white plaster) contrasting with a large variety of colors and materials of interior that mark individual rooms. Again, the colorfulness of interior will be assessed expertly (Figure 7c).

Structure is the pattern of interior elements that are not perceived separately, due to their subtlety, but have merged into an overall, unified pattern. In Loos’s interiors we can find a host of distinct materials and structures: roughcast plasters, soft carpets, greenish Cippolino marble with red and blue and yellow streaks, precious woods highly polished to imitate the reflection effect. The surface structure of Loos’s villas will also be assessed expertly, through values of a linguistic variable (Figure 7d).

APPLICATIONS

These linguistic variables can be employed directly in fuzzy rule bases that describe the work of an architect. The analysis of a set of designs of Loos’s villas in Matlab’s Fuzzy Logic Toolbox has yielded a fuzzy model consisting of four partly interrelated bases of fuzzy rules (Talašová, 2011). Figure 8 (next page) shows the structure and relations among these fuzzy systems.
The **first model** deals with decisions about the overall appearance of a villa. On the one hand, there are requirements of a client on the size of the villa (number of bedrooms, area of living rooms, another apartment inside the villa), the type of terrain available (sloping, flat), and the overall conception of the villa (early period in Loos's work, transitional period, second period); on the other hand, there are other qualities of a villa (ground plan layout and size, number of floors, number of Loosian levels, shape of the roof, overall shape of the building, its compactness, façade finish, extent of classicist elements). The model reflects the relations among these requirements.

The **second model** deals with size and localization of rooms of the principal living area in a villa.
with respect to the overall appearance of the villa that has already been agreed on (ground plan layout and size, number of floors). The input variables to the second model are selected from among the output variables from the first one. This second fuzzy model describes, through a fuzzy-rule base derived from Loos’s villas, a relation between ground plan size and layout and number of floors on one hand, and size and disposition of rooms in the principal living area on the other. For each room that is situated on the most important (social) floor in a Loos’s villa, three coordinates are introduced: \( x \) (“left”, “center”, “right”, “left to center”, “right to center”, “anywhere”); \( y \) (“front”, “center”, “back”, “front to back”, etc.); and \( z \) (a Loosian level of the room or space relative to the level of the hall, e.g. 0, 1, 2, ..., -1, -2, ...). For a given overall appearance of the building, this fuzzy expert system offers alternatives how to design the primary social floor so as to best suit the specified input.

The third model helps with decisions on the type of façade. The input variables to this model are the same as the output variables from the first model. The output variables here are façade symmetry, its openness, configuration of openings in the façade, their prevailing shape, and presence of three-dimensional elements like niches, orielts or balconies (terraces).

Of relative autonomy is the fourth model that takes into consideration the overall interior design of individual rooms, their colorfulness, lightness, contrast, and surface structure in relation to materials used.

CONCLUSION
The research focus is the application of linguistic variables in describing architectural work. Beginning with application of linguistic variables in describing the shape of objects and localizing an architectural element in two- and three-dimensional spaces, we have proceeded to applications in describing the qualities of buildings in terms of architectural composition. Some of these linguistic variables (Compactness, Symmetry, Contrast, Colorfulness and the like) were employed in fuzzy-rule bases to capture the principles of Loos’s villa design.

REFERENCES


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