Populating surfaces with holes using particle repulsion based on scalar fields

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This paper describes the relaxation of charged particles in order to create a pattern of voids based on a scalar field on any complex polygon mesh. A scalar field representing stress values or a greyscale image, can be used to create void patterns of aesthetic or structural character; all with full awareness of the materiality. Following relaxation, areas with low scalar values consist of large voids with a small distance between them. Areas of high scalar value consist of small voids with a greater distance between them. This research has been applied in the design of a sculpture at Oxford Brookes University, where stress data from Finite Element Software has been used for the automatic and rational distribution of holes.

Keywords: *Surface perforation, Geometry optimisation, Particle repulsion*

INTRODUCTION

Topology and geometry optimisation is a topic of great interest for both Architects and Engineers. The possibility to quickly create complex shapes in freeform 3D-modelling software has evolved with the need for Architects and Engineers to work together in close collaboration to enable high aesthetic value, structural efficiency and buildability. In this matter, topology and geometry have been researched widely (Chen and Kikuchi 2001; Li et al. 2000; Lewis 2011; Bendsøe 1989).

This paper focuses on the use of circular voids. The benefits of using circular voids in topology optimization are many. For example, with steel, holes are easier and cheaper to drill than other more complex subtractive shapes, and the production can be done by automated tools just by providing a cutting or drilling pattern. Circular voids also have the great structural benefit of being without sharp corners which induce local stress concentrations leading to crack propagation (Withey 1997). There have been various previous examples of the use of patterns of circular voids. One example is Mountain Dwellings by BIG Architects, see figure 1.

Figure 1
The Mountain by BIG, photographer Carsten Kring

Figure 2
0-14 tower in Dubai by Reiser + Umemoto, Photo © Reiser + Umemoto

By using a very fine regular grid of points and assigning a radius to each point the perforation pattern creates an image that can be seen from the distance. The 0-14 Tower in Dubai by Reiser Ememoto, see figure 2, has a double layer facade, where a perforated outer layer with huge voids creates an interesting pattern that both aims to strengthen the architectural value while enhancing the environmental performance of the building. Perforations are also used in other contexts such as in gas turbine blades for cooling as seen on figure 3. Even though void distribution is widely used, the materiality is very rarely taken into account. By distributing voids in regards to different factors such as stress values, aesthetics, acoustics, transparency, thermal properties or light distribution while taking density and materiality into account, the distribution can work as a key element for both the Engineer and the Architect.

Our research was applied to a sculpture designed by architecture students and staff at Oxford Brookes University with Ramboll Computational Design providing Structural Engineering advice, geometry analysis and Computational Design. The sculpture consists of 15 steel columns with heights ranging from 3m to 6.5m and a single petal at the top of each column. The diameters of the petals range from 2m to 2.5m, see figure 4. The team wanted to use a perforation pattern to show a natural and logical illustration of the nature of the forces in the structure, without compromising either aesthetics or structural efficiency and to allow light to filter through in a way reminiscent of a forest canopy. Our work focused on the distribution of voids in response to a scalar field representing stress values. Our paper describes this work and outlines how circular voids can be distributed on geometrical meshes in regards to scalar values.

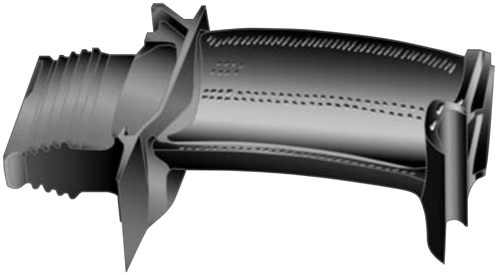


Figure 3
Turbine blade with perforations for cooling(Creative Commons Attribution-Share Alike 3.0 Unported)

Any value can be the key parameter in the distribution of the voids and interpolation of values is also possible, which allow different parameters to influence the distribution of voids at the same time (with different or equal weight). For the sculpture at Oxford Brookes University stress values imported from FEA software were used. The stresses were then converted into scalar values to facilitate the distribution of voids. Instead of using regular arrays or random locations of point, which has been done several times before, we looked into how charged particles could be distributed in regards to any scalar value, on any geometrical mesh no matter what complexity it might consist of. This way the underlying structure is transformed with an efficient and natural looking distribution of voids while taking the materiality into account.

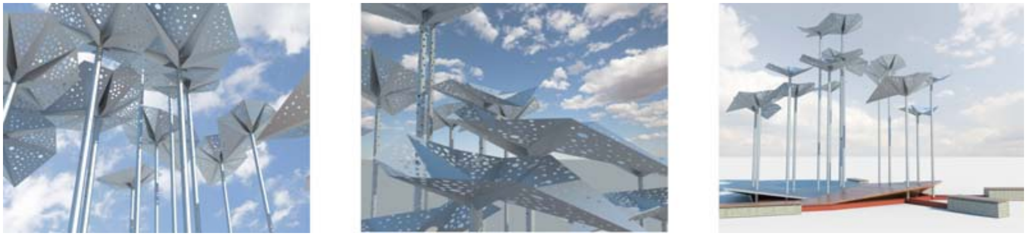


Figure 4
Oxford Brookes
Rain Pavilion

VOID DISTRIBUTION

It is well recognized that by removing material from areas of low stress, and by adding material to areas of high stress, a structure can be highly optimized in regards to material usage, and structural efficiency (Li et. al. 2000; Yang and Chen 1996). The minimum distance between voids and the size of those voids has a great influence on the structural strength (Melander and Ståhlberg 1980; Koss and Dubensky 1987; Benson1993). Inappropriate clustering of just a few particles, can decrease the strength significantly (Ohno and Hutchinson1984). The distribution of voids must be appropriate to the stress present in the surrounding material. The correlation is negative between stress value and void size and positive between between stress value and the minimum distance between voids. Earlier studies have made use of Coloumb's Law to relax a system of particles and create a regular grid and avoid clustering of points (Stowell1950; Macdonald 1957). Coulombs Law (Young and Freedman 2012):

$$F = k \cdot \frac{|q_a \cdot q_b|}{d^2} \tag{1}$$

where$F$ is the size of the force, k is Coulombs constant, q_a and q_b are the charges of the two interacting particles and d is the distance between them. As the force is working in the line between the two particles, formula 1 can be translated into a vector by the following formula

$$\vec{F_a} = k \cdot \frac{|q_a \cdot q_b|}{d^2} \cdot \frac{p_a - p_b}{d} \tag{2}$$

where $\vec{F_a}$ is the force vector working on particle a, p_a is the coordinates of particle a, and p_b is the coordinates of particle b. For realistic behaviour of the particles Newtons Laws of Motion (Young and Freedman 2012) are used.
Newtons Second Law:

$$\vec{F_a} = m \cdot \vec{a} \tag{3}$$

where $\vec{F_a}$ is the force, m is the mass and $\vec{a}$ is the acceleration
Newtons Third Law:

$$\vec{F_a} = -\vec{F_b} \tag{4}$$

where $\vec{F_a}$ is the force working on one particle in an interaction and $\vec{F_b}$ is the force working on the other particle in an interaction.

REPULSION OF PARTICLES ON PLANE

In the following the procedure of creating an even distribution of voids on a plane is covered. To ensure equal distance between different sized voids, particles have a radius assigned, and the charge works from the edge of the void and not the center. If the two voids intersect a punitive force is applied. This can be described by the following definition derived from Coloumb's Law (formula 1):

$$\vec{F_a} = k \cdot \frac{|q_a \cdot q_b|}{(d - r_a - r_b)^2} \cdot \frac{p_a - p_b}{d} \tag{5}$$

$$\vec{F_a} = k \cdot \frac{|q_a \cdot q_b|}{c_f} \cdot \frac{p_a - p_b}{d} \tag{6}$$

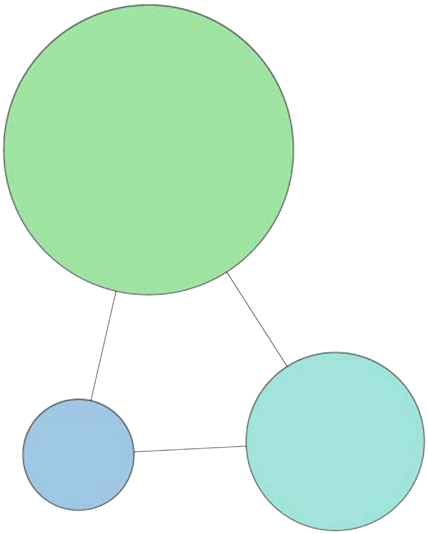
where formula 5 is for $d > r_a + r_b$ and formula 6 is for $d \leq r_a + r_b$. r_a is the radius of particle a, r_b is the radius of particle b, c_f is a correction factor which is set close to zero to ensure that the two voids are pushed apart if intersecting. The value k is set to 1, as the actual size of the force is not important, but only the difference between forces working on individual particles. The total force working on a particle can then be described as follows:

$$\vec{F}_t = \sum_{i=1}^{N} \vec{F}_i \tag{7}$$

where $\vec{F}_t$ is the total force and N is the number of particles in the system.

The same unit mass is used for all particles, without taking the size into account. As the repulsion force is working from the edge of the void, the distance between different sized voids will be equal if the masses and charges are equal. This simplifies formula 3 to:

$$\vec{F}_a = \vec{a} \tag{8}$$



The system is damped so that the velocity of a particle is slowly reduced. The velocity of a particle is described by the following formula (Young and Freedman 2012)

$$\vec{v}_i = (\vec{v}_{i-1} + \vec{a} \cdot t) \cdot k_d \tag{9}$$

where $\vec{v}_i$ is the velocity, $\vec{v}_{i-1}$ is the velocity from previous iteration, t is the time step and k_d is the damping constant. In the initial step $\vec{v}_{i-1}$ is assumed to be zero. An ideal distribution of different sized voids is shown on figure 5.

In figure 6 a system of particles with equal radii is shown. The colours reflect the energy of the particular particle. As the total energy of the system goes towards zero, the system settles in a regular grid. Figure 7 shows the same type of system but with random sized holes. Both figures only show a part of a bigger system, which is why the two pictures of each figure do not necessarily contain the same number or the same size of particles. It is interesting to see how clusters of similar sized voids are created. This is explained by the fact that each void tends to be surrounded by 6 other voids to create a low energy hexagonal grid (as seen on figure 6); a pattern that is seen in many places in nature, such as the structure of graphene. It is not possible to create this structure if the size difference between neighboring voids is too big.

Figure 5
Ideal distribution of voids

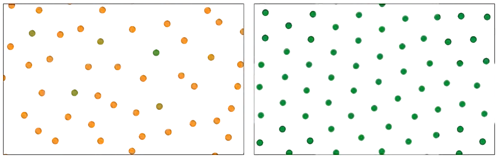


Figure 6
Distribution of equal sized voids.
Left: Initial position.
Right: Settled position

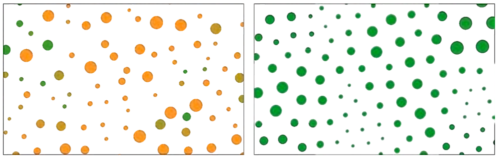


Figure 7
Distribution of random sized voids.
Left: Initial position.
Right: Settled position

DISTRIBUTION OF PARTICLES ON MESH

In the following the procedure of distributing voids in regards to a scalar field is covered. Any value can be used for the scalar field. We used Von Mises Stress Values from FEA Software (Sofistik), based on predefined structural loading values. A surface is imported to Sofistik which can then generate a discretized mesh for finite element analysis. The mesh and scalar values are then exported to Parametric Modelling Software (Grasshopper). The colour property of mesh vertices in Grasshopper can be used to store the scalar field. The vertices are coloured in greyscale values where black (value=0) represents minimum stress and white (value=1) represents maximum stress. The radius and charge of each particle is changed for every iteration with regards to the scalar value of their location. The connection between scalar value and radius and scalar value and charge is assumed to be linear. The weight of colour in regards to radius and charge is manually controlled with two scalar field sensitivity parameters. The calculation of radius and charge can be summarized in the following formulas:

$$r = r_{\text{in}} \cdot ((1 - S_r) + S_r \cdot (1 - C)) \tag{10}$$
$$q = q_{\text{in}} \cdot ((1 - S_q) + S_q \cdot C) \tag{11}$$

where r is the radius, r_{in} is the initial radius, S_r is the radius-field-sensitivity parameter, C is the scalar value at the location, q is the charge, q_{in} is initial charge and S_q is the charge-field-sensitivity parameter.

A software component has been developed for use with Grasshopper, to simulate the natural behaviour of charged particles in regards to Coulombs law and Newtons laws of motion. In the following {X} references to an individual part of the flow chart diagram in Figure 9. The mesh {2} is used as input together with random locations of points {1}. The mesh is coloured in grayscale values, where 0 represents minimum scalar value, and 1 represents maximum scalar value. Every point contains data of what face it is on, the mesh colour of current location {10} and its velocity. The radius and charge is calculated based on the colour of the mesh in the location of the particle. {11}{12} With the use of Coulombs Law the force working on the particle is calculated {14}. On complex 3D shapes the vector of the force is not likely to be in the plane of the face on which the point is located. As the particle is constrained to the mesh, any acceleration added to the particle that is out of the plane of the face, will contribute to, at the current location, non- releaseable energy, which will be added to the velocity every iteration. If the particle moves to a face with a different normal vector, some of the energy that has built up will be released. This makes it impossible to settle the system. To avoid this, the force vector is projected onto the face plane {15} to cull the part of the vector that is not contributing to releaseable energy. See figure 8 where F_t is the total force, F_u is the unreleaseable energy and F_r is the projected vector. For this culling to be appropiate, only small timesteps should be used.

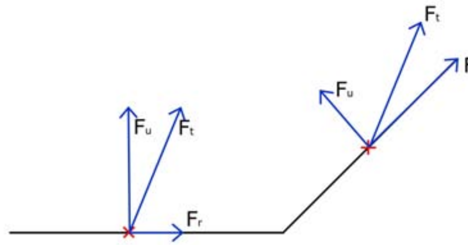


Figure 8
Same total force working on particles on different faces

One of the challenges to be solved is the action to be taken when a particle reaches the edge of a face. For every iteration it must be checked if the particle has reached an edge {19-22}. If the particle is no longer on the face, it should be moved back to the edge of the face and either continue movement on new face or bounce of the boundary maintaining same velocity, depending on whether the boundary of the face is also the boundary of the mesh. See figure 10. {24-31}

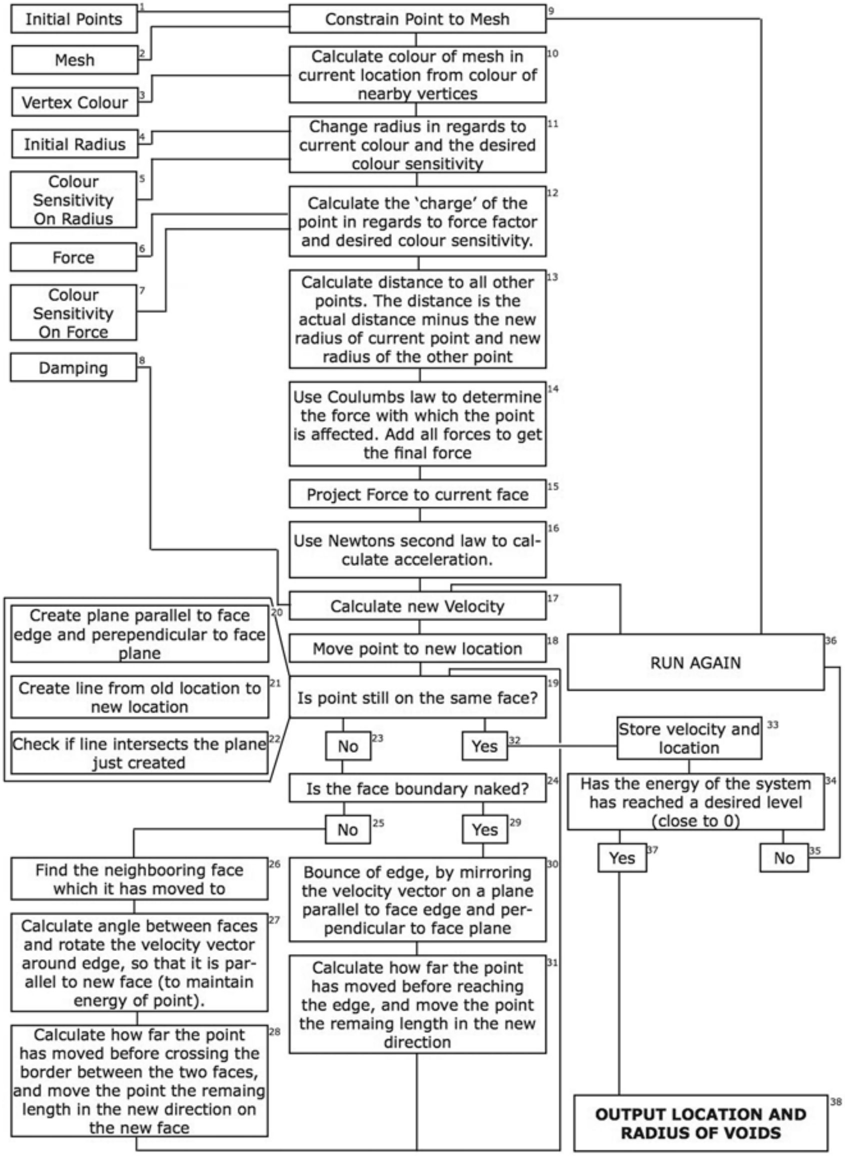
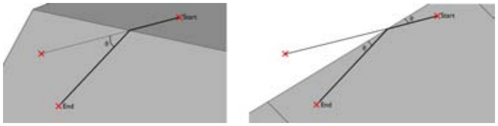


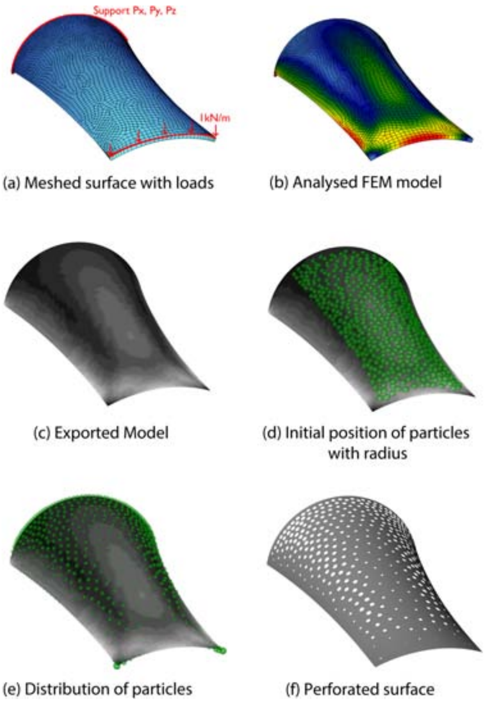
Figure 9
Flow chart diagram

Figure 10
Boundary actions.
Left: Edge between neighbour faces .
Right: Face edge



By damping the system {8} the energy of the particles will slowly reduce. By monitoring the total velocity of the system, the iterations can be stopped when the total energy of system approaches zero {34}. An example of a distribution is shown in figure 11.

Figure 11
Distribution of voids



(a) Meshed surface with loads

(b) Analysed FEM model

(c) Exported Model

(d) Initial position of particles with radius

(e) Distribution of particles

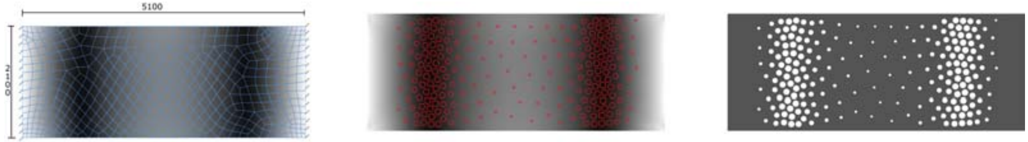
(f) Perforated surface

TEST AND RESULTS

In the following a simple slab is analysed with and without voids. The slab is 2000 x 5100mm, made of 200mm concrete and constrained against movement and rotation in both ends. For the perforated version 210 holes ranging from 25 to 75mm in diameter are created in regards to stress state. Two load combinations are considered. Self-weight and Self-weight + 1 kN/m^2 variable load. The slab is shown in figure 12 This distribution led to a material reduction of 10.9% (from 2,00m^3 to 1.78m^3) and a deflection reduction of 4.0% (from 0.53mm to 0.51mm for self-weight + 1kN/m^2). In the perforated slab the highest value of the Von Mises Stress is 2.52MPa local to the edge of a void. In the unperforated slab the value in the same location is 0.98MPa. This is an increase of 157% (2.57 times the value).

DISCUSSION

The results of this analysis are interesting, yet not surprising compared to earlier studies on topology and geometry optimization (Li et al. 2000; Yang and Chen 1996). By removing material in areas of low stress the self-weight is reduced and the remaining material is being used more efficiently. In the test, stress concentrations at void edges were 2.57 times the value of the same area without voids, which is a little lower than expected the 3 times (Stowell 1950). This can be explained by the fact that the self-weight and thereby a significant proportion of the load, is reduced. An important issue is that the calculation is only done once. As topology is being changed, so is the stress distribution. As most other studies show, an iterative process is needed for proper optimisation (Bendsøe 1989; Schnack 1974}. The distribution of voids creates an interesting natural pattern which can be used as a powerful architectural element. The script created for the distribution allows any value, and not only stress values, to be used for defining the colour of the mesh. If voids are being used as, for instance, natural light sources, an Architect might have preferences for void locations that may differ from the structurally most beneficial location. With very few compromises the mix of interests can be achieved by interpolating two different colour patterns (with equal or different weight). The process of distributing voids only requires coordinates of vertices and colour values for every vertex, which makes further work more doable.



(a) Mesh with dimensions and colours representing scalar values

(b) Particles with radius distributed in regards to stress value

(c) Voids created

Figure 12
Distribution of voids on slab

OXFORD BROOKES RAIN PAVILION

This study was used as a part of the design scheme of the 'Rain Pavilion' at Oxford Brookes University, which is to be built in 2014. See Figure 4. The distribution study was carried out by Ramboll Computational Design to reduce wind loads, increase structural efficiency and create an interesting pattern that reflects the nature of the forces in the structure while allowing dappled light through the otherwise opaque pavilion. The pavilion consists of 15 steel structures with heights ranging from 3m to 6.5m connected by a steel frame base. Each structure consists of a vertical mast and a single petal at the top. The diameters of the petals range from 2m to 2.5m. Each petal has 230 holes with diameters in the range 8mm to 90mm. The largest stem has 200 holes with diameters in the range 6mm to 40mm, and made of 88.9 CHS6.5. The number of holes have been chosen by testing different numbers and then deciding upon the option which was considered to be the best combination of high aesthetical value and structural efficiency, using subjective judgement. As the stresses are higher at the bottom of the stem than at the top, no voids are present in the bottom. From a height of 2m small holes start to appear and gradually become larger and closer towards the tip. This reflects the structural behaviour and creates a visual connection between the ground and the perforated petals on top. In this case random numbers were used as initial radii to create a more vibrant pattern and to strengthen the architectural concept.

FURTHER WORK

This paper has covered how to make a single distribution of voids, but for better optimisation in regards to structural efficiency an iterative loop between void distribution and FEA could be implemented. To allow an iterative loop, the perforated geometry should automatically be imported to FEA. At the current stage the remeshing of the perforated surface is not sufficient for FEA analysis. This restriction must be solved for an automatic iteration process to be possible. The initial mesh should be used for every distribution but the colour can be slightly changed every iteration, depending on the new stress distribution. The next step would be to add and remove voids in regards to the global stress state. Another concern is that of the stress concentration around edges of holes. For optimisation the utilisation level or the maximum deflection should be included in the iteration process. If plastification of small areas at void edge is allowed, the amount of voids could be greatly increased, as stress concentrations are then only expected to be 1.4 times the stress value if no void was present (Stowell 1950).The calculations of the radius/void size could be more accurate with the full use of the research presented by Stowell (1950) and the charge calculations could be made more accurate with the implementation of the information presented by Dubensky and Koss (1987). This implementation would allow an even more optimised distribution but also require a much finer control of the individual particles, which would be interesting for further research. It would also be relevant to carry out a study regarding an implementation of principal stress lines. If voids are created so that they are longer in the direction of the principle stress lines, and narrower in the direction perpendicular to the principle stress lines, it might eventually result in higher structural efficiency, and would also be an interesting architectural element. A computational optimisation would be to change formula 7 so a particle is only affected by other particles within a certain distance, as this would greatly improve the speed with which the software is running. It would also be instructive to look at an automatic time step control to ensure that the system does not become unstable if forces are set too high, and to automatically find a solution as fast as possible.

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