Augmented Visibility Graph Analysis

Mixed-directionality graph structure for analysing architectural space

Tasos Varoudis
Bartlett School of Graduate Studies, University College London, UK
http://www.bartlett.ucl.ac.uk
t.varoudis@ucl.ac.uk

This paper introduces a new spatial analysis methodology based on visibility graphs. Through various design elements architects can create configurations where our visual field and the relations of spaces can be challenged in a combinatorial game beyond the easily accessible and understandable boundaries of the environment. This research explores the trans-spatial architectural elements, transparencies and projected realities that playfully challenge visibility, permeability and accessibility in built environment. The paper presents the computational problem of analysing spaces that include 'augmented visibilities' and areas with 'inaccessible but visible' locations, where dislocated multi-dimensional overlaps frequently occur. Furthermore, proposes a new 'mixed-directionality graph structure' and the definition of 'Augmented Visibility Graph Analysis' (AVGA) as a new spatial analysis methodology. AVGA overcomes limitations of current visibility graph analysis theories and allows the analysis of architectural and urban space that includes visuo-spatial overlaps, hybrid configurations and multi-dimensional information. Finally, a series of mathematical graph analysis measures and test cases associated with observations and experimental data from real spaces is presented in order to demonstrate AVGA.

Keywords: *Visibility graph analysis, spatial analysis, architectural space, visual accessibility-permeability, mixed-directionality graph*

INTRODUCTION

In architecture and urban design, visibility plays an important role in defining relationships of spatial elements, influencing movement and understanding of space around us. The concept of visibility graph analysis (VGA) and isovist analysis has had a long history in architecture and other disciplines. Turner et. al. (2001) presented visibility graphs as a method of taking away from build environment a permanent record of spatial configurations and relationships. VGA revealed a series of meaningful characteristics and correlations about architectural space, morphology, movement and social engagement.

While the analysis of a graph constructed by inter-visible locations in space is widely used in spatial analysis techniques, it has a number of limitations in order to enable the analysis of environments. Spaces need to satisfy a number of constrains in order to computationally and methodologically be analysed correctly. That generates a number of situations where without a radical simplification of the input plan, it is impossible to analyse a space. Firstly, this can occur when, transparencies, half-height partitions and furniture can create spatial morphologies where a direct visible connection exists but the equivalent movement route differs from the visible line. Secondly, more recently visual augmentations produced by ambient projections, displays and other digital elements, not only create pairs of locations with non-matching visual and movement routes, but in most cases generate dislocated spatial realities that distort the architectural morphology of the perceived surrounding space.

Initially, this paper will demonstrate a number of situations that challenge the mapping of architectural visuo-spatial relations and morphologies to mathematical graph models. Using the new set of visuo-graph relations a new 'mixed-directionality graph' analysis methodology and the 'Augmented Visibility Graph Analysis' (AVGA) will be introduced enabling the analysis of many possible combinations of complex architectural typologies or multi-dimensional models in both building and urban scale. Finally, the new methodological and computational advances will be used in two test cases that borrow spatial characteristics from a hybrid architectural typology that challenged the visibility analysis of the experimental space.

BACKGROUND

Visibility graph analysis (VGA) was developed by Turner et. al (2001) based on space syntax theory (Hillier and Hanson, 1984) and early foundation work, such as that carried out by Thiel (1961), who attempted to record the details of the visual experience through buildings or urban environments by analysing the properties of spatial paths.

The concept of an 'isovist' (name originated from Benedikt, 1979), which has had a long history in various fields of research including architecture, geography and mathematics, is central to visibility analysis. An isovist is 'the set of all points visible from a given vantage point in space and with respect to an environment (Benedikt,1979, p. 47). Turner (2001) argues that Isovists are an intuitively attractive way of thinking about a spatial environment because they provide a description of the space 'from inside', from the point of view of users as they perceive, interact with it, and move through it.

Tandy (1967) introduced the concept of isovists for the analysis of landscape but it was Benedikt (1979) who developed the method for the consideration of architectural space. Tandy used isovists as a way to, '[take] away from the architectural space a permanent record of what would otherwise be dependent on either memory or upon an unwieldy number of annotated photographs' (Tandy, 1967, p. 9). A similar concept has a long history in the form of the 'viewshed' in the field of landscape architecture and planning (Amidon and Elsner, 1968; Lynch, 1976) and 'inter-visibility' in computer topographic models (Gallagher, 1972).

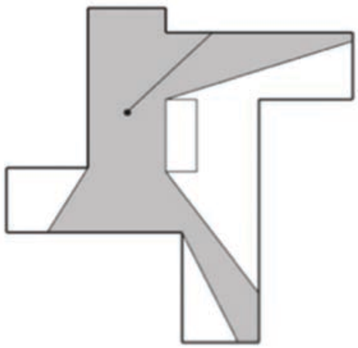


Figure 1
Example of an isovist, showing visible space from a single point.

Benedikt starts by considering the volume visible from a location and then simplifies this representation by taking a horizontal slice (two dimensional) through the 'isovist polyhedron'. The resulting 'isovist' is a single polygon without holes, as shown in figure 1. The geometric properties, such as area and perimeter, are then considered and through this process the qualities of space, and its potential, are quantified.

Benedikt noted that analysis of multiple isovists is required in order to quantify a whole configuration and suggested that the way in which we experience a space, and how we use it, is related to the interplay of isovists. This led to the development of methods to calculate `isovist fields' which record the individual isovist's properties for all locations in a configuration by using contours to plot the way those features vary through space. The closeness of the contours shows how quickly the isovist properties are changing and, according to Benedikt, this relates to Gibson's (1979) conception of ecological visual perception with 'textured gradients'.

Configuration is defined in general as, at least, the relation between two spaces taking into account a third, and, at most, as the relations among spaces in a complex taking into account all other spaces in the complex' (Hillier et al 1987, p 363). Turner et al (2002) argue that despite the appeal of Benedikt's isovist methodology its use in architectural analysis has been limited due to two reasons. The first is that the geometric formulation of isovist measures means that they record only local properties of space, and that the visual relationship between the current location and whole spatial environment is missed, including the isovist's internal visual relationships. The second is that Benedikt did not develop any guidelines on how to usefully interpret the results of the analysis, meaning that there is no framework to show how isovists relate to social or aesthetic factors.

Turner et al. (2001) developed the Visibility Graph Analysis (VGA) methodology that overcomes the limitations they reported with Benedikt's theory. The method draws from space syntax theory (Hillier and Hanson, 1984) and small worlds analysis (Watts and Strogatz, 1998) and produces a graph of mutually visible locations in a spatial layout termed visibility graph. VGA (figure 2) was firstly implemented in Turner's 'Depthmap' software and is now widely used by both academics and practitioners through the open source and multi-platform 'depthmapX' spatial network analysis software (Varoudis, 2012).

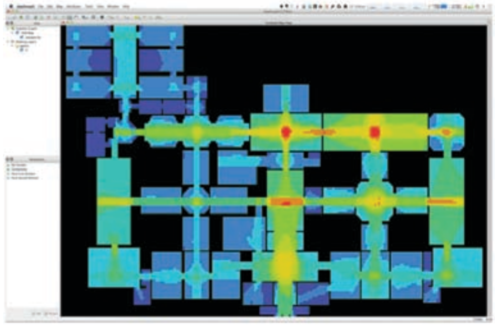


Figure 2
Typical VGA performed by depthmapX depicting 'Visual Integration' values (Red=High, Blue=Low).

Turner et al. (2001) suggest that through this graph numerous local and global measures of spatial properties that are likely to relate to perception of the built environment can be taken and compared with real life data of usage to 'shed light on the effects of spatial structure on social function in architectural spaces' (p. 104). Moreover, many studies have demonstrated a significant correlation between visibility analysis measures and the way people move. (Desyllas and Duxbury, 2001; Turner and Penn, 1999).

While VGA overcame limitations of older studies of isovists, it is restricted to analyse spaces that only include fully obstructive walls or simple openings. In practice, researchers and practitioners in most cases have to remove elements from the input drawings or extend and block other elements before performing an analysis. While architecture is full of complex and interesting visuo-spatial phenomena like, depth augmenting hybrid configurations (Varoudis, 2011, Varoudis et al., 2011), transparent or reflective elements (Psarra, 2009), digitally linked office environments (Schnadelbach, 2007, 2012) and urban displays (Fatah gen. Schieck 2005), the systematic analysis of settings that include these elements is not possible with current theories and techniques. Such elements introduce an added layer of a dislocated visual depth or destination information that translate to a set of new spatial, visual and isovist relations. Here we introduce a new methodology that describes these complex relations and a visibility graph analysis of architectural and urban space that can fully analyse them.

AUGMENTED VISIBILITY GRAPH ANALYSIS (AVGA)

Visuo-Spatial Relationships, Overlaps and Augmented Visibilities

In order to describe the different visibility relations, the problem of visuo-spatial overlaps and multi-dimensionality of information in the visibility analysis techniques, we first need to introduce the basic concept and steps required in order to perform a visibility graph analysis.

Constructing a visibility graph is a two-step procedure. Firstly, we select an appropriate set of location in space, according to some criteria, to generate the isovists locations. These locations will form potential nodes of the graph (based on the selection process that follows). The most obvious approach to construct the isovists is to generate them at some regularly spaced intervals (figure 3). This implies that the generating locations will be at points defined by some sort of grid. In practice we try to select a set of generating locations that provides an acceptable `near-full' description of the space. Turner et. al. (2001) argues that if analysis is to relate to human perception of an environment, then the resolution of this grid must be fine enough to capture meaningful features of the environment in human movement scale. In Turner's et. al. VGA, we then select the physical or walkable space to be analysed. This continuous set of grid-cells depicted in figure 3 represent the nodes (vertices) of the resulting visibility graph. Secondly, given the set of final nodes, we must determine the direct visibility relations between them to form links (edges) in the graph. In order to add an edge, between two locations, to the graph, the two locations must be directly inter-visible.

The edge formed in the traditional VGA analysis can only be an undirected link that represents a symmetrical visibility and accessibility relation between the two locations. For any given pair of linked locations it is required that both ends of the link can be origins and destinations of movement and visual rays. In addition the movement vector from and to the two locations must coincide with the visual ray that connects the two. Simply, you cannot form a link with a location if you can see that location directly but cannot approach it using the same route as the visual ray connecting the locations.

In reality, a number of different scenarios can occur that cannot satisfy the origin-destination or directionality restrictions of Turner's et. al. VGA. A number of different scenario will be discussed here that form an example set of core ideas of visuo-spatial relations that can occur in space. These examples were chosen because they can form the basis for more complex combinations.

Figure 4 demonstrates a common layout inside a building. The first part depicts the un-directional visibility relations between four locations in space. This is a typical graph representation in Turner's et.

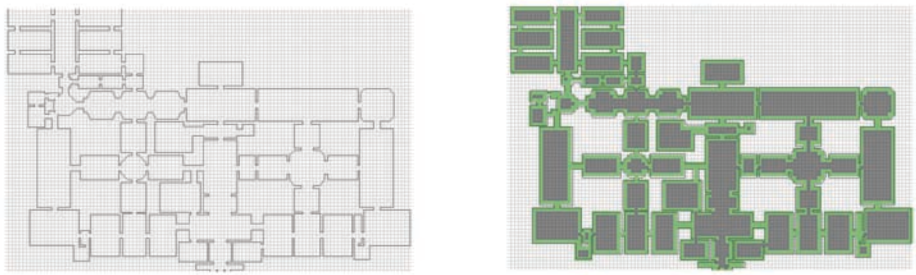


Figure 3
Example of grip (left) and selected locations for analysis (right) in depthmapX.

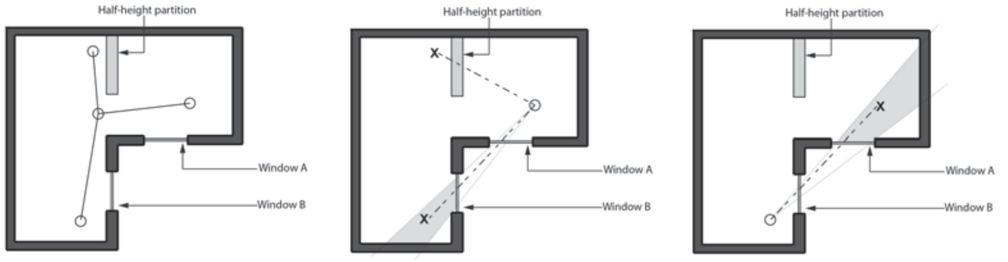


Figure 4
Example of complex visuo-spatial relationships ('accessible' = O, 'augmented' = X)

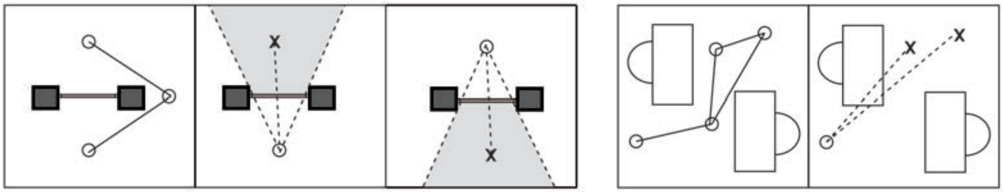


Figure 5
Examples of complex visuo-spatial relationships. ('accessible' = O, 'augmented' = X)

al. VGA, though the environment's affordances are more complex in this occasion. While in the left part we see the un-directed origin-destination relations which form the foundation of a graph analysis, the next two illustrations demonstrate the multi-layered information that is omitted in traditional analyses. Multi-dimensional overlaps (grey regions) are generated when movement is obstructed (or not possible) but a direct visible link is possible. In this case the two windows generate an overlapping space for each of the location. The half-height partition also generates a 'direct-inaccessibility' but gives a direct visual hint about the destination. The same situation can be described for the open space beyond the building (through the windows) but its omitted here because of the increased complexity. Other examples can include virtual spaces, CCTV cameras, ambient interfaces that produce visual depth, and other physical of hybrid layouts that produce augmented visibilities. A new 'augmented isovist' definition should include complex visual setting like the in the figures.

An important difference between the original visibility graph analysis and AVGA is that habitable or accessible space can act as an origin and destination of a visibility relation, while the augmented and multi-dimensional location can only be a destination of visibility links. In AVGA we divide space into 'accessible' and 'augmented' locations (or nodes of the analysis). The relation always originates from some 'accessible' location. Locations in the 'augmented' space, marked with 'X', are not allowed to form outgoing connections (become origins) with any other node in the systems. Some simpler examples are depicted in figure 5 where detours are required to reach the destination but a direct visual hint is given. These relations generate the need for directional link representations in visibility graph analysis. When a visibility ray passes through or over certain elements the field of view is transformed to an added 'augmented' layer of information. The bases of AVGA is a multi-dimensional definition of space with asymmetrical relations. As illustrated in figures 4 and 5, a location can exist in both 'accessible' (marked as 'O') and 'augmented' (marked as 'X') space, layered on top of each other.

Based on this set of possible relation the AVGA algorithm generates all possible layers of information in a form of overlapping multi-dimensional grids (ex. 'accessible' physical space and 'augmented' space) and then selects the appropriate sets of locations in order to form the nodes of the analysis. Nodes can be marked as origin-destinations or just destinations. The process of generating the nodes and the spatial categorisation is done through a series of ray-casting analyses and algorithms in order to find all possible visible augmented isovist locations in space and their relations to others (i.e a location is only visible through glass and not accessible etc). Material and object dimensions (i.e. height) play an important role here.

Mixed-directionality Graph

The set of un-directed and directed relations and the different sets of locations are then encoded into a graph. The new graph representation is a mixed-directionality graph that has vertices (nodes) linked with both undirected and directed edges (links). In AVGA the graph can hold a number of complimentary information about the locations (vertices) and the links (edges) and can also form a multi-dimensional graph with vertices (or edges) that exist on the same location or overlapping in other dimensions. A typical AVGA graph has nodes that can be identified by their 'location tag', like 'accessible' or 'augmented' and special tags like 'hybrid', 'virtual', 'reflected' or other. The edges apart from the directionality information can have added information like an 'edge description' tag. These added information help to form a more complete description of space and is relevant with future work, outside the scope of this paper, but demonstrates the need for more open spatial representations in the analysis of architectural space.

Analysis and Measures

With the graph representation of all spatial and visual relations built we can now perform a set of analysis in order to extract some meaningful measures that take advantage of the mixed directionality visibility graph.

The two most widely used visibility graph analysis measures are the node's 'Connectivity' and 'Visual Integration' values. The VGA measure of connectivity is equivalent with the degree of the node in graph theory, thus represents the number of connections (simple undirected links in Turner et. al. case) that the node has with other nodes in the system. Connectivity also relates to the size of the isovist at that particular location. Because space in VGA is quantised based on a grid, connectivity can only approximate the isovist size but the relation between the connectivity value and the isovist size is linear. Visual integration in VGA is directly linked with 'mean shortest path' of a node. Turner et al (2001) describe the mean shortest path in the original implementation of visibility graph analysis and its connection to Hillier and Hanson's 'integration'. Hillier and Hanson relate the visual accessibility of spaces, with the number of changes in direction, whereas in a visibility graph we can describe the visual accessibility of every location in the spatial system through the number of steps. Visual mean shortest path is a representation that quantifies the visual accessibility of every location in a spatial system and it has a significant advantage over other analysis of spatial configuration. As the mean shortest path length measures configuration by considering all locations with respect to each other in the system, global relationships between locations in the system can be explored. This is a noteworthy difference to the measure of connectivity. Users of spatial analysis techniques extensively use this significant feature of the visibility graph to obtain an alternative spatial and morphological description of the build environment that departs for the previously available technics of partitioning in terms of local geometric properties of visual fields as Benedikt does.

For this paper, instead of Hillier and Hanson's Integration formula that is used by Turner et. al.

(2001) and is implemented in Depthmap (Turner, 2001) and depthmapX (Varoudis, 2012), we use the directly equivalent measure of graph closeness centrality (Opsahl et. at., 2010) in order to relate better with existing graph analysis theories. AVGA uses the term Visual Closeness Centrality when we want to identify and systematically describe global visuo-spatial relations and mean visual distances in architectural space.

Moreover, in the new augmented visibility graph analysis, 'connectivity' is specified as the total number of links (graph edges) a node has, this number includes links formed with both 'accessible' and 'augmented' nodes (locations). Connectivity here represents the total visual information presented to the user at a location but due to the more complex nature of the new graph representation connectivity is a complex measure. In order to describe the added spatial, transpatial or hybrid information in a location we need to define the measure of 'Hybrid Connectivity' for a node. Hybrid connectivity is the number of links formed with 'hybrid' nodes or locations and it is directly linked with the amount of 'directed' added information that the new mixed-directionality graph analysis produced.

With all connectivity and directionality information assigned to each location in space we can now determine all possible shortest paths in the graph. Shortest paths will then be used for the calculation of Visual Closeness Centrality (Opsahl et. at., 2010). While in Turner's et. al. analysis, all possible pairs of locations are used as origins and destinations of a shortest-path search, AVGA uses only a subset of the locations as an origin, the set of locations in 'accessible' space, and the full set of locations (nodes) as a destination. This is easily understandable as nodes that represent 'augmented' locations are dead-ends of the graph, with incoming connections (graph 'in-degree' greater that zero) and zero outgoing connections (graph 'out-degree'), and thus any path search is useless.

All three AVGA measures are demonstrated below using test scenarios that derived from an established series of hybrid-space experiments (Varoudis, 2011 and Varoudis et. al. 2011).

TEST SCENARIOS

The novel test scenarios presented here derive from a series of experiments and research outputs about the influence of visual depth augmentation in architectural space. A detailed description and analysis of the significant results can be found in Varoudis (2011) and Varoudis et. al. (2011). These spaces were used because they exposed a phenomenon that demonstrated the limitations of current spatial analysis and understudying of complex spatial relations through graph analysis. The layouts include dislocated hybrid morphologies, spatial overlaps between layers of visuo-spatial information and restrictive one-way visual relations (through a digital link) , as depicted in the figure 6.

The first layout hypothesises that, placing ambient projections so that they virtually link (one-way) and extend one physical space towards another real or virtual space and thus augmenting the visual depth, will influence the topological and visual rela-

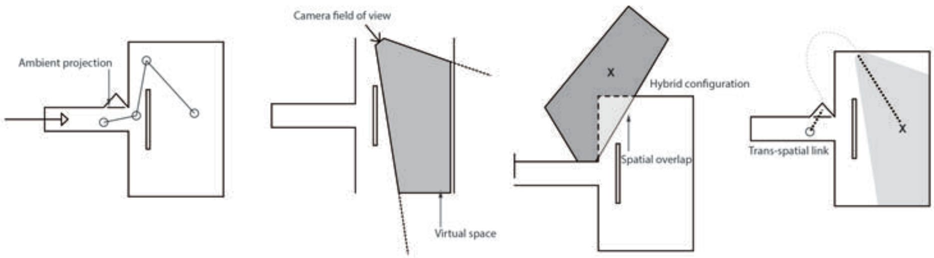


Figure 6
First scenario - Visual link, hybrid configuration and spatial overlaps

tions between spaces and as a result will affect the distribution of people's movement. The ambient projection is located either left or right at the corridor setting as figure 6 illustrates. While the results of the experiment clearly showed a significant change in movement patterns as a result of the virtual added depth (Varoudis, 2011); if we use depthmapX to perform a VGA analysis the values of connectivity and visual integration will be symmetrically distributed, with no clues about the visuo-morphological differences between the physical and the hybrid topology.

Figure 7 depicts the AVGA Connectivity and Hybrid Connectivity of the space. A grey outline is used to indicate the hybrid information space and the overlap produced by the video link and orientation of the projection. In this analysis the location of the ambient projection is the cross-over barrier between the physical and the augmented space. The characteristics of the new multi-directional graph model, in terms of Connectivity and Hybrid Connectivity, are evident here, with the distribution of values concentrated in areas of high hybrid visual interaction near the end of the corridor and more specifically the left side.

The AVGA Visual Closeness Centrality that encapsulates the global visibility graph relation of this space is presented in figure 7 and depict slight asymmetry to the left, but in this particularly small setting, global relations cannot add significant amount of information over the local Connectivity values. The exploratory power of AVGA Visual Closeness Centrality is more suitable for more complex and multi layered environments where local relations between spaces are combined to construct complex integration or segregation relation. Overall the AVGA results of this space depict a similar trend with the results presented in Varoudis (2011), where there was a significant shift of movement towards the left side of the 'T' shape although the purpose of this paper is not to statistically compare the two results but present the new graph analysis.

The AVGA results of the second experimental layout are presented in the figure 9, and are based on the hybrid architectural space used in the Varoudis et. al. (2011) study. In this case the ambient projection generates a digital augmentation through a skewed perspective projection. This hybrid setting challenges the visuo-topological relations by digitally bending the line of sight towards the skewed dislocation environment 'behind' the wall. The complex visual relations and the equivalent directional graph structure of this space are illustrated (simplified) in figure 8. These relations are encoded into a multi-directional visual link in the AVGA graph and the result of the Connectivity and Hybrid Connectivity is presented in the figure 9. This scenario presents a stronger challenge in terms of the hybrid spatial overlap that extents the end of the corridor 'through' the physical wall and over the room behind it. AVGA describes the complex morphological shift and what the added analysed information produces. The Visual Closeness Centrality in figure 9 depicts the focus point of the hybrid setting near the end of the corridor. This location becomes the dominant location not only in local (immediate) properties but also in global relations that extent behind the physical and the virtual barrier. Similarly with the first case study, the AVGA results of the second space depict similarities with the results presented in Varoudis et. al. (2011), where there was a significant shift of movement towards the right side of the 'T' shape. The hybrid spatial setting seems to work in favour of the right hand side of the corridor.

The two test cases clearly demonstrate the analysis of the hybrid architectural spaces with the Augmented Visibility Graph Analysis. The augmented realities transform the architectural morphology and produce fluid visuo-spatial relations. While the hybrid layouts can be seen as simple this was needed in order to better describe the new idea. Another reason for using these two studies is that the experimental results showed significantly different emerging movement patterns, where a simplistic interpretation of a traditional VGA of the space would give a 50/50 distribution of movement.

Figure 7
First scenario - AVGA measures, Visual Closeness Centrality, Connectivity and Hybrid Connectivity. (Red = High, Blue = Low)

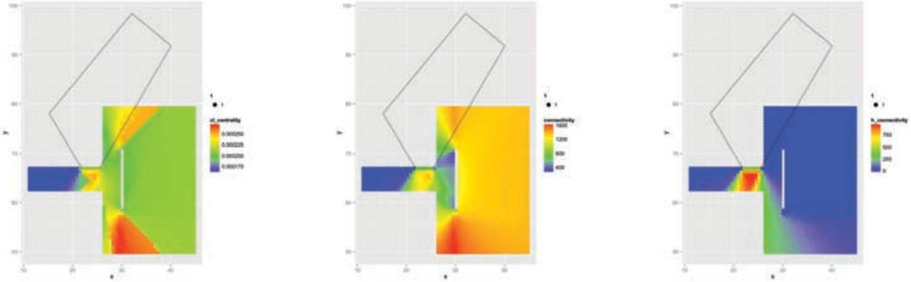


Figure 8
Second test scenario - Visual link, hybrid configuration and spatial overlaps

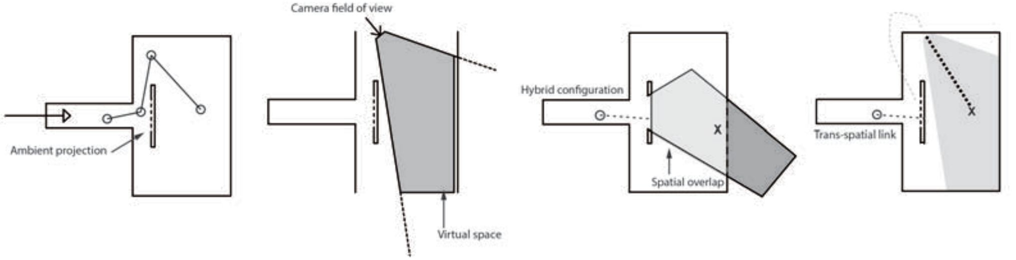
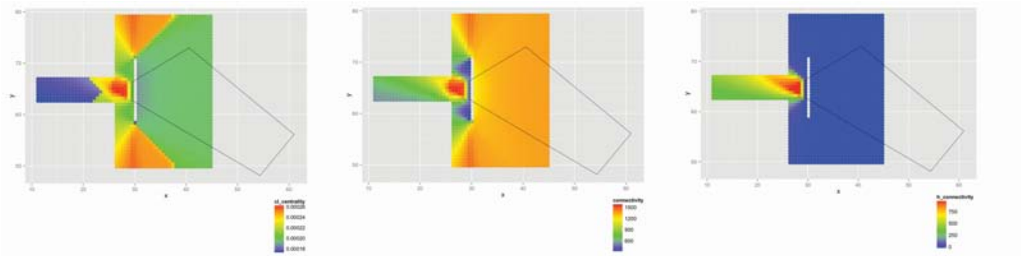


Figure 9
Second scenario - AVGA measures, Visual Closeness Centrality, Connectivity and Hybrid Connectivity.



CONCLUSION

The paper presented a new methodology and application of visibility analysis in architectural space. Rather than investigating the properties of a space that is only surrounded by physical boundaries, as has been considered in the literature, the 'Augmented Visibility Graph Analysis' is introduced and a new mixed-directionality graph that enables the visibility analysis of virtually any spatial configuration or multi-dimensional space.

Architectural and urban settings engage in complex relations challenged by visibility, permeability-accessibility overlaps, discontinuities and trans-spatial morphologies. Current visibility or spatial analysis techniques use simple symmetrical relations in order to analyse space with reduced complexity. The presented AVGA eliminates the need for simplifying building models before analysis. This improved methodology was demonstrated with the two hybrid architectural scenarios and the measures of Connectivity, Hybrid Connectivity and Visual Closeness Centrality. The scenarios had a number of complex visuo-spatial topologies, in which directed visibility graph edges and 'augmented' multi-layered graph locations were used.

The new mixed-directionality graph model can be further used for the analysis of not only two-dimensional complex environment with transparencies, reflations, digital augmentations and barriers, but also for the analysis of multi-dimensional spaces and spatio-temporal models. Further work that makes use of this new methodology is currently in development in order to answer questions about, three-dimensional architectural configurations, data-driven spatio-temporal models in urban analysis and other accessibility-permeability paradoxes.

Finally, the mixed-directionality graphs and the AVGA can impact upon architectural space design featuring visually complex elements or topologies, as well as navigation in space. In the fast-growing field of digital augmentation in architecture, understanding and acknowledging people's movement, proximity and navigation in space can give new ways of managing and directing movement towards desired places or interfaces. Examples within this area include the analysis subliminal visual nudges ('augmented' location is visible, but not directly accessible) for accessibility of remote or 'hidden' spaces as well as alternative and more efficient methods to assist way-finding.

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