Assisting Early Architectural Planning Using a Geometry-Based Graph Search

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In early design phases of architecture ideas exist mostly on a vague level concerning the expectations for the building plan and the respective design parameters. One established method is to examine and develop ideas through existing designs, and to use these to clarify design parameters and be further inspired. Thus, the aim is a computer-based system like sketch-based query approach to show similar floor plans using semantic building fingerprints. During the search floor plans are compared in form of graphs, which means that the sketch-based floor plans are converted to graphs together with the existing floor plans. Herewith, a gradual condensation of the request is possible. The entry is condensed continuously through the repetitive process of entry and search. The challenges with this approach lie in the following mathematical model behind similar floor plans, Queries that satisfy complexity of the data and optimal way for the user to engage in search process.

Keywords: Semantic fingerprints, early architectural planning, geometry-based graph search, adjustment theory

INTRODUCTION

Against the backdrop of growth in global construction projects with soaring complexity levels, the development and transfer of architectural knowledge is becoming essential. The use of powerful IT infrastructure, as well as design and planning processes that are based on semantic models (e.g. BIM/IFC and GIS/CityGML), unleashes enormous potential - but also increasingly generates large quantities of digital data that are more and more frequently stored on the Internet in cloud solutions, model servers and the like. This stored data contains "inherent" knowledge which can be used as a "pool of ideas/inspiration" for future projects. To do so, the "inherent" knowledge must be made accessible and processed into a searchable format. Information query capabilities are a prerequisite for developing knowledge. To that end there is a need for knowledge management systems in which information can be mapped and harnessed through knowledge representation and engineering methods.

In early design phases, architects utilize references; this is a recognized design method employed to investigate and develop ideas through existing plans, clarify design parameters and find new inspiration. Alongside mainstream media such as books and magazines, there is also growing use of digital catalogs in this regard, although searches mostly in-
volve key words or fixed categories. The objective of the approach described in the paper is to develop innovative search methods to support design activities during the early stages, the concept phase.

**Metis research project and the significance of fingerprints**

The aim of the research project METIS (Langenhan, Weber, Liwicki, Petzold and Dengel 2013) (supported by the DFG) is to develop innovative IT-based search methods to support design activities in the concept phase. The iterative nature of the design process requires continuous switching between creative, analytical and investigative steps in order to choose the best design approach by selecting from the most promising alternatives. We therefore recommend utilizing semantic fingerprints to formalize aspects of creative and analytical issues. Thus formalized, the issues can be interpreted by computer and related solutions can be displayed as references. Buildings and plans serve as a knowledge base, encompassing not only spatial situations but also solutions for specific architectural forms. Reference usage is an efficient method in both design work and downstream activity fields.

In order to facilitate the comparability of floor plan solutions (full or partial) and aspects thereof, consideration is paid to specific floor plan characteristics, such as spatial topology properties, which are called semantic fingerprints (Figure 1). Semantic fingerprints should be understood as separate levels of abstraction containing various aspects or details of a spatial arrangement. A number of different fingerprints are defined for a reference in the form of a constructed or planned building, and derived using computer-based processes. These fingerprints are represented mathematically in graph form, allowing floor plan solutions - graphs - to be investigated with regard to their similarity. Examples of fingerprints include:

- The number of rooms on a floor plan
- Accessibility and non-accessibility between rooms
- Adjacency relationships between rooms
- Size ratios
- The geometry of room boundaries

In early design phases, concepts - usually in the form of sketches representing spatial arrangements and configurations with textual annotations through to sketched scale drawings - are used as a means of reflecting, visualizing and investigating mental design ideas. During the design process, more concrete specification is given to the spatial model, i.e. the rooms and their relationships to each other. The level of abstraction is reduced, with geometric and topological details fleshed out. Sketched graphical representation is used as an input metaphor in the research project. From there, "query graphs" are formalized in accordance with the defined fingerprints. Search queries are submitted with the help of these graphs, and the results obtained serve either as inspiration for (or more concrete specification of) design concepts, or as feedback for changing the search queries.

The approach used in the METIS project can be regarded as a multi-stage formalization of conceptual thought. The conceptual thoughts will be sketched out and topological relationships between these sketches will be formalized in computer-based graphs.

From a user’s perspective, the process is as such: a basic layout will be sketched out with the properties of spaces and connections such as geometry, the function of spaces, accessibility, special properties such as the orientation of the windows, etc. Different types of fingerprints will be derived from this information and examined on the basis of these datasets. The types of fingerprints vary widely from room functions, throughway graphs, orientation relative to the cardinal directions, to geometrical properties. In the following, specific geometrical properties will now be specifically considered as a subset of fingerprints.
**Geometrical fingerprints**

In order to support a more finely or coarsely granulated exploration, a new approach to interaction is proposed below. In addition to classic input regarding the shape of rooms and the definition of the relationship between rooms such as adjacency and circulation, the entered room geometry can be modified semi-automatically. Thus the room proportions or the spatial arrangement can be modified by entering the distances and angles within spatial boundaries, i.e. the incremental generation of a scaled drawing from sketched input. Since the corner points of the room do not need to be individually selected and displaced by the user, an increase in efficiency can be assumed.

**From the sketch to the scaled drawing as a basis for deriving geometrical fingerprints**

An algorithm was developed for the search application, which supports the incremental generation of a scaled drawing from sketched input. As such, the rooms may not be to scale and can be entered using approximate room cubature (sketch) as well as the topological relationships between them. With this input, the user is already indirectly providing diverse information such as the topology of the outline of rooms, edges and outlines that extend parallel to one another, etc. The geometrical dependencies that are later explicitly entered by the user, such as linear dimension, perpendicularity, parallelism, etc., supplement this information. In the event of inconsistencies, these will be meaningfully resolved. The algorithm "adjusts" the layout in accordance with the input values and, in addition to the direct dimensional input, also shows the user the dimensions that are automatically determined. A scaled drawing is incrementally generated from sketched input by indicating dimensions. This drawing can be subsequently modified, concretized and prioritized at any time (see Figure 2).

In the case of sketch-based input, the geometry is concretized by inputting concrete measurements such as linear dimension, diagonal dimension, perpendicularity and parallelism, including pre-defined accuracies or those that can be defined on the user side, as appropriate. The chapter, "Concretization of the search query with the aid of geodetic adjustment computation", shows the internal numerical implementation of this functional principle.

A "geometrical" fingerprint is derived from this input, which implicitly uses the adjusted room geometry, and which explicitly uses the dimensions for search and similarity measures.

**Comparison on the basis of geometrical fingerprints**

The comparison of geometries is characterized by various problems. Humans divide image information in various ways and compare it against memory. Humans specifically manage geometric comparisons impressively well. Currently, these skills can only be transferred to the computer to a limited degree. One
of the main issues is the mathematical definition of similarity. Humans filter out a lot of detail differences and thus are able to correlate geometric objects that are roughly similar. Following this approach, a comparison algorithm is presented, which can recognize similar geometries even in the case of locally insignificant, topological differences. Humans not only distinguish between the same and different, but also have a "feel" for greater or lesser degrees of correlation. This approach has also been transmitted in the form of a similarity measure.

Thus one problem is the recognition of correlations when geometries are not oriented identically, i.e., they are skewed relative to one another. Additionally, geometries often have symmetries, for example in the case of a square or octagon. In this case, the different alignment options need to be tested.

The chapter, "Comparison with the database", presents a possible implementation of a corresponding similarity comparison. It is based on the specific allocation of the geometrical topography of a sketch entered by the user (query) to rooms in a database. Geometrical dependencies that are directly defined by the user are tested against the database. Other approaches are also possible, and a free approach is presented in the outlook. Regardless of the implementation, this fingerprint considers the similarity of the three-dimensional shape as a criterion of the overall similarity comparison. For example, the fingerprints can be combined for the throughways, room type and the similarity of the room shape.
CONCRETIZATION OF THE SEARCH QUERY WITH THE HELP OF GEODETIC ADJUSTMENT COMPUTATION

The method used for the geometrical concretization of the search queries will now be explained on the basis of this introduction. To this end, as described here, we will expand here on a model of geodetic adjustment computation. Adjustment computation is a mathematical method of optimization in order to determine the unknown parameters of the geometrical, physical model for an array of measurement data, or the parameters of a predetermined function. In the approach described here, the sketch-based geometry incrementally adjusts according to the input by the user, and connects this geometry with different geometrical abstractions and specifications. The Gauss-Markov model is used as a basis therefore. Consequently, the modifications are traced in the sequence of the individual steps of the model. In the interest of simplification, the mathematical notation was selected such that it is analogous to the standard for geodetic adjustment computation.

The primary goal of the Gauss-Markov is to calculate the adjusted unknowns $\hat{X}$. The explanation of the developed method is explained at the end of the adjustment computation, in other words according to the following points:

- Determining the approximate values
- Linearization of the observation equations
- Determining the functional and the stochastic model
- Determining the system of equations, regularization and solution
- Testing

These individual steps are then incorporated into the method of the discrete observation groups.

**Determination of the approximate values and pseudo-unknowns**

The geometry of the layout is defined to a large extent by corner points such as those of the individual rooms. The sketch-based input of the point coordinates therefore allows said coordinates to be used as approximate values for $X^0$. In addition, in the case of the incremental creation of the sketch, a reduction of the unknowns is already carried out. To this end, pseudo-unknowns are introduced. Pseudo-unknowns may be used as unknowns, however they are not variables but instead, reference functions, which in turn reference unknowns and/or additional functions. Pseudovectors are constructed on the basis thereof, the components of which (the entry of the vector, for example the individual coordinates $x$, $y$ or $z$ in the case of position vectors) pseudo-unknowns. These references may continue recursively (Figure 3).

$$f_1(\bar{v}_1) = f_1(v_1x) \rightarrow \begin{pmatrix} f_2(...) \end{pmatrix} \rightarrow ...$$

The simplest and most frequent case is a multiple reference to unknowns in the case of perpendicular geometries (Figure 4).

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$$P_1 = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}, P_2 = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix}, P_3 = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}, P_4 = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix}$$

(Note: $x_i$ always represents an unknown $i$ according to geodetic notation)

Thus the number of unknowns is reduced and
at the same time, hard constraints are modelled. Other hard constraints must be modelled more complexly. Here is an example of a direction vector \( \mathbf{r}_1 \) as a pseudovector:

\[
\begin{bmatrix}
    f_1(x_1, x_2, \alpha) \\
    f_2(x_1, x_2, \alpha)
\end{bmatrix} = 
\begin{bmatrix}
    \cos(\alpha)x_1 - \sin(\alpha)x_2 \\
    \cos(\alpha)x_1 + \sin(\alpha)x_2
\end{bmatrix},
\]

which is constructed on the basis of the unknowns \( x_1, x_2 \) and the scalar \( \alpha \). From an information-technological point of view, the result is function blocks that can be flexibly interconnected. They have standardized inputs and outputs, for example as transformation in \( R^2 \) with an input and an output vector. By solving the references, it is ultimately possible to determine the actual quantity of the true unknowns of a pipeline, which form all pseudo-unknowns.

**Adding observations and linearization**

Geometric values such as distances are introduced as observation and observation equation. By way of simplification, these are regarded as uncorrelated. The weight matrix \( P \) can therefore be easily formed from the dialogue of precision. The model matrix \( A \) is formed through linearization on the basis of the central difference quotient [1]

\[
\frac{\Delta f}{\Delta x_i} = \frac{f(x_1, \ldots, x_i + \epsilon, \ldots) - f(x_1, \ldots, x_i - \epsilon, \ldots)}{2 \epsilon}.
\]

In the case of a meaningful selection from \( \epsilon \) (see Holzer 2002), no disadvantages were identified as compared to analytic differentiation but rather, a higher degree of robustness was found (extreme value digits etc.). The author used values such as those \( 2^{-20} \), which are coded in machine-internal floating-point representation \( 2^{-20} \cdot 1 \). The calculation times are negligible in the overall process. The complexity is in linear proportion to the number of equations of conditions since an equation of condition always uses only a small number of constants. The approach also solves the problem of linearization of the pseudo-unknowns. These are solved by the quantity of unknowns formed thereby.

**Regularization and solution of the system of equations**

Thus the matrices \( A \) and \( P \) and the shortened observation vector \( l \) arise in a linear calculation period. \( A \) is extremely sparsely populated, \( P \) a diagonal matrix, which allows for very efficient calculation \( N = A^T PA \) and \( n = A^T Pl \). Though the approach of first creating a sketch, and then supplementing said sketch with geometrical values, there are, as a rule, far more unknowns available than observations, and thus \( N \) is not regular. In order to achieve rapid regularization, continuous regularization \( \|Nx - n\|^2 + \alpha|x|^2 = \min \) was used, since this yields the generalized inverse \( N^{-1} = (N + \alpha I)^{-1} \), and this simply needs to be added \( \alpha \) to each element of the diagonals of \( N \). This approach improves the condition of the matrix in the normal case, however is ill suited for singular matrices with many iterative solvers, while direct solvers have excessive calculation times and produce fill in. An appropriately selected Krylow subspace method also produced good results with good parallelization.

**Testing and discreet observation groups**

Both poor approximate values in the numeric sense at startup and the rounding error of the iterative solver determine even more the necessity of testing, without which the system would quickly destabilize during the startup process or would oscillate toward the end with increasing precision. Like the overall process, testing must also occur in a short period of time, and thus must remain linear in complexity. The mean unit of weight error \( m_0 \) cannot be used since, as described, \( u > n \) generally arises. Instead, \( \Omega = v^T Pv = \sum_{i=1}^{n} (v_i v_i P_{i,i}) \) is directly drawn on for the assessment. A reweighting of the improvement vector \( \hat{x} \), has proven effective in countering the issues initially described. Different \( \alpha \) for \( \hat{x}' = \alpha \hat{x} \) are tested, both for \( \alpha > 1, \alpha < 1 \) and \( \alpha = 1 \). The calculation of \( \Omega \) can thus be easily parallelized; all \( v_i v_i P_{i,i} \) can be calculated in parallel.

In the case of inconsistencies in the geometrical
input conditions, there are different conceivable possible solutions. Discrete observation groups were introduced in order to support monitoring by the user during solution. As such, observations will be allocated by the user-managed groups. A group is represented by a level. The adjusted observations $\hat{L}$ of a superordinate group are thereby attributed to the next subordinate group as equations of condition.

**COMPARISON WITH THE DATABASE**

As described in the chapter, "Introduction," the comparison of search query and database must account for imprecisions. As explained, these are both geometrical and topological. The steps for the comparisons with each entry in the database are shown below. Insofar as it is not clear whether the entry matches, said entry will be referred to as a candidate in the following. The steps are performed for each new candidate.

**Prior transformations**

The prior transformation corresponds to the classic, known method referred to as the Helmert transformation (Luhmann 2000, Niemeier 2008, Wolf 1968), which prepares a candidate for the subsequent steps. Following this step, a candidate that is oriented roughly parallel to the coordinate axis should be available.

**Reduction and consolidation equation**

Now the topological description of the geometry of the candidates will be simplified at the same time, in that local manifestations are removed and the number of geometrical parameters are reduced in a manner analogous to the chapter, "Concretization of the search query with the aid of geodetic adjustment computation". The method is directly derived from the determination of the pseudo-unknowns. A conformance test is performed for all (vector) components of point coordinates within a delta defined by the user. The equivalent value of the sketch input (capture) can be used as an initial value for delta, however it may be modified by the user as desired.

In Figure 5, delta is symbolized as a gray rectangle.

In this way, quantities of (vector) components arise, which are combined with each other. Any quantity is thereby assigned to an individual variable, which represents the pendant to the pseudo-unknowns in the chapter, "Concretization of the search query with the aid of geodetic adjustment computation".

In Figure 5, $P_1$ is symbolized as a gray rectangle. In this way, quantities of (vector) components arise, which are combined with each other. Any quantity is thereby assigned to an individual variable, which represents the pendant to the pseudo-unknowns in the chapter, "Concretization of the search query with the aid of geodetic adjustment computation".

From a purely topological standpoint, points are thus recognized, which can be consolidated. The identification only takes into account the consolidation of the (vector) components. If all components are consolidated, this means that it is a reduction down to just one single point. In Figure 5, $P_{3x} = P_{4x} = P_{5x}$ and $P_{3y} = P_{4y} = P_{5y}$ are thus reduced to a single point. In this way, the loops are reduced. After reduction, they are therefore freed from small local characteristics (Figure 6).

What is essential in this strategy is the defini-
tion of the consolidation equation. This equation describes how, as a rule, similar but non-identical values for a quantity of (vector) components can be allocated to the initial value of their substitution variables. Approaches such as the sole average of the values fail to take into account the different influence of the components on the overall geometry. More suitable are variants, which take into account the element of a characterized environment such as the element ratios of the length of the adjacent edges as weight, for example. Thus \( P_{1x} \) and \( P_{3x} \) have substantially more influence on \( x_3 \) than \( P_{4x} \) and \( P_{5x} \) and \( P_{6y} \) have substantially more influence on \( x_4 \) than \( P_{3y} \) and \( P_{4y} \).

\[ P_{1x} = P_{6x} = x_1 \]
\[ P_{1y} = P_{2y} = x_2 \]
\[ P_{2x} = P_{3x} = x_3 \]
\[ P_{3y} = P_{4y} = x_4 \]
\[ P_{4x} = P_{5x} = x_5 \]
\[ P_{5y} = P_{6y} = x_6 \]

**Variant test**

After the reduction, only those candidates with a corresponding topological description are considered. This refers to the number of points in the loops (Samet 2006) that form the geometry and the reduction of the point elements by means of common variables. The more symmetrical a geometry is in the topological sense, the more variants that can be found for a candidate that correspond with the query. In the case of Figure 6, the conditions according to the block of formulas (2) are met by all rotations and different definitions of the starting point in the loop. These variants are now further tested and reduced.

The first test radically reduces the number of variants and is structured very simply. In addition, each corner point is examined from its "perspective" to determine whether the further edge contour now extends to the left (\( \sin (P_{i-1}, P_i, P_{i+1}) > 0 \)) or right (\( \sin (P_{i-1}, P_i, P_{i+1}) < 0 \)). If the change in direction is very small, it is flagged as such so that it is not taken into consideration in the comparison. The example yields the allocation \( P_1 \rightarrow R, P_2 \rightarrow R, P_3 \rightarrow R, P_4 \rightarrow R, P_5 \rightarrow L, P_6 \rightarrow R \). In this way, the variants are immediately reduced to 1, albeit mirrored geometries are immediately seen as equal to 2.

**Figure 6**
Candidate after reduction.

**Figure 7**
Exemple of finger print.

\[ P_{Ax} = P_{Bx} = x_a \]
\[ P_{Ay} = P_{Fy} = x_b \]
\[ P_{By} = P_{Cy} = x_c \]
\[ P_{Cx} = P_{Dx} = x_d \]
\[ P_{Dy} = P_{Ey} = x_e \]
\[ P_{Ex} = P_{Fx} = x_f \]
The allocation thus gives rise to the allocation $x_a \rightarrow x_5$, $x_b \rightarrow x_4$, $x_c \rightarrow x_6$, $x_d \rightarrow x_1$, $x_e \rightarrow x_2$ und $x_f \rightarrow x_3$.

Once the reduced variables of the query and variants have been allocated, the geometrical dependencies and values entered by the user can be directly transferred. Figure 7 shows a distance of 6 m between the points $P_D$ and $P_E$. This is modelled as $6m = \sqrt{(P_{E_x} - P_{D_x})^2 + (P_{E_y} - P_{D_y})^2} = \sqrt{(x_f - x_d)^2 + (x_e - x_e)^2}$. Thus ... yields the allocation in the variant $l = \sqrt{(x_3 - x_1)^2 + (x_2 - x_2)^2}$.

As a rule, $l$ will not precisely correspond to the desired 6 m. After the weighting, which was indicated in the $P$ matrix in the chapter, "Concretization of the search query with the aid of geodetic adjustment computation", the difference is therefore weighted. This is done using all of the geometrical dependencies that have been directly or indirectly indicated by the user, which are modelled as observations. The overall dimension of the deviations is determined on the basis of the weighted differences.

OUTLOOK

The variant of geometric comparison presented here looks for geometrical dependencies that are directly and indirectly fixed by the user, on the basis of correlating geographic topologies. Other approaches allow more freedom in the definition of geometric similarity. As such, the approach pursued is similar to hash values. Different topological and geometrical properties are projected as a floating point number in a function. This function is defined in such a way that similar geometries also yield similar floating-point numbers. Although statistically rare, roughly similar geometries may have similar values, however, due to the loss of information.

In this article, all examples for geometric properties that are directly entered by the user are consistently considered in direct relation to corner points. But it is also possible to introduce points as pure geometric reference points similar to construction aids. To this end, additional steps must be added. In this article, a distinction was therefore already made between points and corner points in the mathematical description.

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