Parametric modeling applied to the virtual reconstruction of the damaged sculpture of St John Nepomuk in Petrovaradin

Stojakovic Vesna¹, Igor Budak², Ratko Obradovic³, Daniela Korolija-Crkvenjakov⁴, Zeljko Santosi⁵
¹,²,³,⁵ Faculty of Technical Sciences, University of Novi Sad ⁴ The Gallery of Matica Srpska
¹,²,³,⁵ {vesna100|budaki|obrad_r|zeljkos}@uns.ac.rs ⁴ daniela.korolija@gmail.com

Valuable cultural heritage may be permanently damaged as a consequence of different man-made influences, as well as natural erosion. In cases where no documentation is available about the original shape, an analysis of the typological group of similar objects is commonly used as a base for reconstruction. The interpretation of typology can be based on explicit method relied on relevant shape data. The aim of this paper is to propose a method which may be used in the virtual reconstruction of the damaged sculpture of St John Nepomuk in Petrovaradin, Serbia. Virtual reconstruction presented in this paper was based on parametric analysis of the typological group. The spatial characteristics of the sculptures were transformed into measurable parameters which were further used to determine the missing shape such that the reconstruction has the highest probability of representing the true shape of the damaged sculpture based on attributes the same typographical group. We propose three different ways based on how the most likely shape of the missing part may be computed: i) the reconstruction in which average values for each parameter are used; ii) the reconstruction in which average values are used for numerical parameters while spatial parameters were selected from the sculpture with the highest probability for these parameters; iii) the reconstruction in which all of the characteristics are inherited from one real sculpture, closest to the average shape.

Keywords: virtual reconstruction, parametric modeling, image-based modeling, cultural heritage
INTRODUCTION
Valuable cultural heritage may be permanently damaged as a consequence of different man-made influences, as well as natural erosion. Several different approaches and digital tools may be used for surveying, virtual reconstruction, and restoration of cultural heritage. One approach to cultural heritage analysis is parametric modeling, which was used in the process of restoration (Ma et al. 2015; Michel et al. 2014), virtual simulations and shape analysis (Varinlioglu et al. 2014; Tepavcevic and Stojakovic, 2013; Coutinho et al. 2011) and reproduction (Slawomir et al. 2015; Scopigno et al. 2014).

Regardless of the quality and quantity of documentation, the reconstruction of missing parts of permanently damaged cultural heritage is usually the result of an artist's interpretation (Pereira et al. 2009). The individuality incorporated by an artist necessarily includes his/her interpretation of the documented influences, and his/her perception and understanding of the cultural heritage. In cases where no documentation is available about the original shape of the cultural heritage, an analysis of the typological group of similar objects is commonly used as a base for reconstruction. However, the reconstruction does not necessarily have to be based purely on artistic interpretation. Instead of an artist's interpretation of typology, type analysis can be based on explicit method relied on relevant shape data.

The aim of this paper is to propose a method which may be used in the virtual reconstruction of the damaged sculpture of St John Nepomuk in Petrovaradin, Serbia. Sculptures of St John Nepomuk within other symbolic portrayals are usually depicted wearing a biretta. This detail is highly common in local sculptures from the same typological group (Roth, 2014). We will focus on the virtual reconstruction of the biretta that was likely to cover the top of the head of damaged St John's Nepomuk sculpture in Petrovaradin. Virtual reconstruction of the biretta presented in this paper was based on parametric analysis of the typological group - sculptures that share similarities in shape and style with the damaged sculpture. For the purposes of the analysis presented here the spatial characteristics of the sculptures were transformed into measurable parameters. Parameters were further used to determine the shape of the biretta such that the reconstruction has the highest probability of representing the true shape of the missing part of the damaged sculpture based on attributes of the same typographical group.

METHODS AND MATERIALS
The approach for virtual reconstruction of the damaged sculpture consisted of several phases:

i) The representative sculptures were selected. Selection criteria consisted of a similar style and geographic area.
ii) The relevant characteristics of different birettas’ shapes were determined in order to make comparable 3D models of the representative shapes.

iii) Single image-based modeling of the selected representative sculptures was used for biretta’s shape reconstruction.

iv) Parametric models of the selected representative sculptures were made and the relevant parameters were analyzed and compared.

In textual descriptions St John Nepomuk is depicted to wear a biretta as an iconographic characteristic of a saint (Roth, 2014). Photographs of the free standing sculptures and sculptures in church niches of St John Nepomuk are used as an input data for the analysis.

SCULPTURES OF ST JOHN NEPOMUK
The sculpture of St John Nepomuk in the niche of the St Juraj’s church in Petrovaradin, Serbia, has been highly damaged. There is no data (photographs, drawings or descriptions) about the original shape of the sculpture. The top of the head has completely eroded, and the original shape of the head is unrecognizable (Figure 1).

The sculpture is unapproachable, located in facade niche at a height approximately 7m. The sculpture was surveyed by structure from motion photogrammetric method supported with an elevator bucket truck. A precise 3D model of the sculpture was reconstructed from these images (Figure 2).

As inputs for the analysis of the potential shapes of the missing top of the sculpture’s head, we used photo documentation, available online, of the sculptures of St John Nepomuk. We made further selection of these images in order to choose a group of sculptures with similar geographic area, material of which the sculpture is made and style. We choose only sculptures made out of stone, and omitted the ones made out of bronze or wood. Considering the style and shape, we selected sculptures with a biretta and omitted sculptures which used modern styles of sculpting (characterized by primitive based shapes, different proportions and hard surfaces). According to the described criteria a group of six sculptures was selected (Figure 3) to be used as the input for the shape analysis.

SINGLE IMAGE-BASED MODELING OF THE SCULPTURES
For each of the six selected sculptures, a single photograph was used to reconstruct the 3D shape of the biretta. Single image-based modeling considers 3D reconstruction based on a single photograph as an input. Single image-based modeling produces models of low accuracy (compared to photogrammetry or laser scanning) (Styliadis and Sechidis, 2011), especially in unfavorable cases such as the reconstruction of a complex sculptural shape. However, single image-based modeling does not require on-site work and additional surveying costs, and high accuracy is
Figure 3
Sculptures of St John Nepomuk in Trieste, Rijeka, Pancevo, Osijek, Subotica and Krapina

not obligatory for parametric analysis, which is why single image-based modeling was selected as a suitable method for this research.

In single image-based modeling the most critical part is photo orientation because it depends on the constraints (Hauvel van den, 1998). It is necessary to use constraints in the image because of the single image ambiguity, in order to find the position of the camera’s optical center (Stojakovic et al. 2013). In photographs in which complex free-form shapes occupy the most of the image, it is not possible to use common constraints, such as perpendicularity and equal distances for orientation.

The orientation problem requires a specific approach to be used for single-image sculpture reconstruction. The head of the sculpture, which is present in each photo, is used for photo orientation. One constraint that was used is the symmetry of the face. Another aspect that can be used to solve the orientation problem is that the symmetry plane is perpendicular to the line which connects eyes and to the line of the mouth. Beside these, at least one more constraint is needed for photo orientation. Since there were no other reliable constraints, available assumptions about common proportions of the face were used. Here, the common distances and distributions of facial elements and face contours were used to perform photo orientation. The 3D model of the average head (Burt and Perrett, 1995) was imported as a reference and was matched to the oriented photo (Figure 4 a).

Because of the complexity of the model and limitations of single-image based modeling in the reconstruction of complex shapes, the biretta had to be simplified to basic lines and spatial relationships. The simplification was performed by taking into account the capabilities of parametric analysis that followed.

The shape of the biretta was simplified in order to be represented by angles, lengths and curves, which are suitable for the latter parametric analysis. Data to be compared were parameters that define:

- the position of the biretta on the head
- the longitudinal curve of the biretta (the section that is the symmetry plane of the head)
- the position and shape of the cross curve of the biretta (through the plane in which the characteristic seam is)
- the dominant spatial shape of the biretta.

In single-image based modeling of complex shapes it is important to follow the workflow in which the single image ambiguity problem is solved during the modeling phase (Stojakovic and Tepavcevic, 2011). To determine the position of the biretta on the head, the automatic texture extraction from the photograph to the imported 3D model of the head was used (Figure 4 b). In that way the line where the biretta meets the head was reconstructed in 3D.

The longitudinal curve of the biretta was in the symmetry plane of the face, and therefore the contour curve of the longitudinal shape could be reconstructed in that plane. In the same plane the section of the longitudinal and cross curve can be noted in the photo, and therefore its 3D position can be re-
constructed. When the position of that section was computed, the plane perpendicular to face symmetry plane was placed. That plane was used for the reconstruction of the cross section of the biretta, and it is constructed in the same way as the longitudinal curve. In that way all of the characteristic curves of the biretta were reconstructed in 3D (Figure 4c).

Other visible parts and symmetry constraints were used to check the model’s accuracy. As expected, due to the quality of input data and limitations of the single image-based modeling of free-form shapes, the models do not have high accuracy. We note that deviations go up to 5% of absolute magnitude.

**PARAMETRIC ANALYSIS**

After the characteristic shape parameters are recovered in 3D for all birettas, they had to be analyzed in order to find which features have the highest probability of occurrence in the typographical group. 3D models of the heads with the reconstructed curves that define biretta’s shape were scaled and rotated to match size and position in order to gather the required measurements.

The shape of the biretta was interpreted in a way suitable for parametric analysis. Angles and distances, which are numerical values, were measured. All distances were represented as a percentage of the head's height, and angles were measured in degrees. The average curve was constructed so that all curves have same horizontal length. All of the curves were valued in comparison to the average curve. The height of curve points was calculated as the average height of all points on curves that share the same horizontal coordinates.

Shape parameters which were measured for the analysis are (Table 1, Figure 5):

1. Position of the biretta on the head - defines the position of contact curve of biretta and the head
   a) Angle between the horizontal plane and the touching plane of the biretta and the head
   b) Distance of the front part of the biretta to the top of the head
2. Longitudinal curve - defines position and shape of longitudinal curve
   a) Angle between normal to touching plane of biretta and head and the front linear part of the longitudinal curve
   b) The length of the front linear part of the longitudinal curve in the vertical direction
   c) Average vertical distance between the longitudinal curve and average longitudinal curve
3. Cross curve - defines position and shape of the cross curve
   a) Horizontal distance from the point at the forehead to the cross curve plane
   b) Angle between normal to the touching plane of biretta and head, and side linear part of the cross curve
   c) The length of the side linear part of the cross curve in the vertical direction
Table 1
Values of the biretta’s shape parameters for each sculpture.

<table>
<thead>
<tr>
<th>Sculpture</th>
<th>$1_a$</th>
<th>$1_b$</th>
<th>$2_a$</th>
<th>$2_b$</th>
<th>$2_c$</th>
<th>$3_a$</th>
<th>$3_b$</th>
<th>$3_c$</th>
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<td>0.56</td>
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Figure 5
Parameters of the biretta shape in the right and front view. Average curve is in green color.

Table 2
K factors for all relevant parameters used for the generation of the resulting model.

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<tr>
<th>Sculpture</th>
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<th>$K_{1b}$</th>
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<th>$K_{4a}$</th>
<th>$K_{5a}$</th>
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<td>Krapina</td>
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<td>0.19</td>
<td>0.14</td>
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<td>0.36</td>
<td>0.141</td>
<td>0.142</td>
<td></td>
</tr>
</tbody>
</table>
d) Average vertical distance between the cross curve and average cross curve

3. Shape of the spatial curve - defines overall shape of the biretta
   a) General shape of the curve in the middle of the biretta (for an elliptical shape the value is 0, for other shapes the value is 1)

5. Spatial relationships - analysis of some parameters makes for a more logical final shape if the relationships between parameters are analyzed together rather than independently
   a) Variation in the angle of the lower conical part of the biretta: the difference between the angle between normal to touching plane of biretta and head, and the front linear part of the longitudinal curve (2a), and angle between normal to touching plane of biretta and head and side linear part of the cross curve (3b)
   b) Variation in length in the two lower conical part of the biretta (in the longitudinal plane and in the cross plane): length of the front linear part of the longitudinal curve (2b) and the length of the side linear part of the cross curve in the vertical direction (3c)

In order to find the most likely shape of the biretta based on the typographical group, we compared the numerical values \( m \) of the six selected sculptures \( s = 6 \) to the average value for each parameter. For the numerical data, average values may not be used in reconstruction. This is because using the average value in the shape analysis of realistic sculptures can sometimes lead to illogical results (e.g. in the analysis of curve shapes). Instead of this approach, we used the existing shape of the curve which is closest to the average shape across all sculptures. In this way we can measure the deviation of any particular sculpture or its specific parameter from the average shape.

For each parameter \( i \) influence factors \( K_i \) were assigned to each sculpture \( j \) according to their difference from the average value. Differences from the average value \( D \) were calculated as

\[
D_{i,j} = \left| \left( \sum_{j=1}^{s} m_{i,j}/s \right) - m_{i,j} \right|. \]

That means that sculptures which have smaller values of \( D \) are closer to the average value in that parameter. In order to get values comparable for the different parameters, the influence factors were scaled to between 0-1, \( K_{i,j} = D_{i,j} \sum_{j=1}^{s} D_{i,j} \).

The introduction of \( K \) factors (Table 2) allows measuring of how typical a biretta’s shape is, according to a multiple parameters. By adding all factors \( K_i \), we can compare how much the specific biretta reflects the average biretta shape (it is the one with the smallest \( \sum K_i \)). Or we can add \( K \) factors representing a group of parameters (e.g. just the cross curve) and find which cross curve is closest to the average. The comparison of \( K \) factors allows us to generate the average shape of the biretta in several different ways.

**RESULTS**

The most likely shape of the biretta may be computed in several different ways based on different starting assumptions. We propose three different approaches for the virtual reconstruction.

i) Since the biretta is presented as a logical spatial formation, separate analysis of each parameter may not be an adequate statistic by itself. This is because when all the parameters are independently averaged across all of the birettas, the resulting parametric model may not conform to the original logical spatial formation because of potential discordance. The first proposed reconstruction used average values for each parameter, adjusted (scaled and rotated) so that the curves meet in one central point and outline the logical shape (Figure 6a). In the figure dark purple/blue curves are calculated values, and green are adjusted curves.

ii) The second proposed virtual reconstruction uses the average values of independent numerical parameters (angles, lengths and relationships). Other parameters, such as curves that have to correspond to one another, were selected from the sculpture with the smallest \( K \) factor for the sum of curve parameters \( K_2c, K_3d, K_4a \). So, in this example, the parameters have average values, except for the
Figure 6
Results generated by different approach to parameter analysis presented in right, front and perspective view; a) all average values, b) average numerical parameters, curves with minimal K factor, c) shape that has minimal total K factor.
curves, which are same as the sculpture in Pancevo (Figure 6b).

iii) The third way is to select all of the characteristics of one real biretta, the one with the smallest $\sum K_i$ factor when all of the criteria are considered, ($K_{3b}$ and $K_{3c}$ were omitted, since they are already included in relationships $K_{5a}$ and $K_{5b}$). In this way we can detect which biretta is closest to the average shape (Figure 6c) across the typographic group. In this research, the biretta on the sculpture in Pancevo was found to have the smallest $K$, although the one in Krapina has almost the same $K$ factor.

CONCLUSION
In this paper parametric spatial data analysis is proposed and applied on a case study of the damaged sculpture of St John Nepomuk to show a statistical approach to the virtual reconstruction of cultural heritage. Single image-based modeling and parametric modeling were used for 3D data representation. Parametric interpretation of the spatial structure was made and the values were analyzed statistically. The analysis determined the shape most likely to appear in selected typological group. Variation in the analytical approach can cause small discrepancies in the resulting shape. The results demonstrate that it is possible to generate a statistically representative average of the typological group which can be used for the virtual reconstruction of damaged heritage, when no data about the original shape is available.

Since the resulting shape is sensitive to input data and preferred statistical approach, it should not be treated as imperative for real reconstructions, but more as a useful guideline. Engagement of cultural heritage professionals is advisable during the input and approach selection. The limitations of the presented study are a small number of analyzed sculptures and a variation in the quality of input photographs. Future research includes applying a similar methodology to a larger set of data, and the use more automatic steps in the analysis.

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