APART but TOGETHER

The Interplay of Geometric Relationships in Aggregated Interlocking Systems

Alireza Borhani¹, Negar Kalantar²
¹,²Texas A&M University
¹,²{borhani|kalantar}@tamu.edu

In this research, the authors discuss multiple design process criteria, fabrication methods, and assembly workflows for covering spaces using discrete pieces of material shorter than the space's span, otherwise known as topologically interlocking structures. To expand this line of research, the study challenges the interplay of geometric relationships in the assembly of unreinforced and mortar-less structures that work purely under compressive forces. This work opens with a review of studies concerning topological interlocking, a unique type of material and structural system. Then, through a description of two design projects - an interlocking footbridge and a vaulted structure - the authors demonstrate how they encouraged students to engage in a systematic exploration of the generative relationships among surface geometry, the configuration and formal variations of its subdividing cells, and the stability of the final interlocking assembly. In this fashion, the authors argue that there is hope for carrying the design criteria of topological interlocking systems into the production of precast concrete structures.

Keywords: Topological Interlocking Assembly, Digital Stereotomy, Compression-Only Vaulted Structures, Surface Tessellation, Digital Materiality.

A NEW CONSTRUCTIVE LANGUAGE
An aggregated interlocking system

An interlocking structure establishes equilibrium through compression forces; the weight of each heavy block is used against itself to maintain it in the air (Sakarovitch 2003). Being in a static state of equilibrium, the interlocking structure is carefully balanced under its own weight, without any additional binding materials (such as mortar joints) between the disparate blocks. In general, the stability of such a structural system depends on the location of its modules within the overall assembly and the adjacent contact faces of those modules. Specifically, the final module locks all of the other elements into position (see Figure 1).

FROM STEREOTOMIC STRUCTURES TO INTERLOCKING SYSTEMS

In light of the relevant precedents for the geometrical complexity of topological interlocking systems,
this section limits its scope to the historical development of stereotomy (Evans 1995). As an introduction to the nesting components of a volumetric block of material, this section not only targets certain historical approaches to digital fabrication, but also reviews recent state-of-the-art advances in this field.

Although history is rich with examples of interlocking stone structures, stonecutting is not the only way that geometrically complex building blocks can be created. Topological interlocking is rooted in stereotomy, but presumably stemmed from the field of reciprocal frames or nexorades (Baverel et al. 2000), such as in ancient Asian forms of timber construction (Brocato et al. 2015). Compressive strength and bilateral contact conditions are the threads that bind reciprocal frame and interlocking structures. The most remarkable example of a topological interlocking structure is the Abeille flat vault, in which identical truncated tetrahedron-shaped stones were assembled in a two-directional woven pattern that spans the curvature of the vault (Tessmann et al. 2013), providing an “all in one ceiling for the lower storey, and a pavement for the upper storey” (Fleury 2009). The rationale behind Abeille’s system is similar to the principle of reciprocity employed by medieval building masters and found in Villard de Honnecout’s fylfot grillage assemblies, Leonardo da Vinci’s spatial structures, Sebastiano Serlio’s planar floors, and John Wallis’s scholarly work (Yeomans 1997). In such constructions, a discrete loadbearing element supports two neighboring components, and is mutually supported by two others to span distances longer than their length (Pugnale et al. 2011, Brocato et al. 2012). Patented at the end of the 17th century, Abeille’s vault overcomes the structural instability of reciprocal frames when loads are not applied in a fashion perpendicular to their planes (Weizmann et al. 2016).

Based on a more complex version of a similar organizational pattern, Sébastien Truchet presented an improved variant of Abeille’s design with the same cyclic distribution of loads. In Truchet’s enterprise, similar to Abeille’s, the entire bi-directional assembly is erected with identical blocks (Fallacara 2009) that lean on two adjacent blocks and endure two others. While Abeille’s polyhedron-shaped blocks are composed of four intersecting flat surfaces in the shape of isosceles trapezia, Truchet’s blocks have concave and convex surfaces made out of ruled sides. Along with the saddle-like joint surfaces of Truchet’s vault blocks, their section profiles are different so as to be able to touch at all points.

In the projects put forth by Abeille and Truchet, the structural elements are combined to show the natural aesthetics of a stereotomic system. In both, each block is in contact with four others, and thus is a part of a squared homogeneous net reflecting its own inherent decorative pattern. As an improvement on Abeille’s design, Truchet used no square

Figure 1
A stereotomic-like vaulted structure illustrating the division of a massive body into topologically interlocked elements.
Freshman Design Studio, Texas A&M University, 2016.
voids on either the extrados or intrados sides of the ceiling, while Abeille left pyramidal holes on the underside of the vault to provide a coffered structure.

Although both of these flat, vaulted structures partake of the complex geometrical configurations of reciprocal frames in order to withstand their own weight, they behave differently from reciprocal frame structures (Miodragovic Vella et al. 2016). Contrary to reciprocal frames, one of the advantages of both Abeille’s and Truchet’s vaulted patents is their ability to sustain loads and control the displacement of their blocks. The loads cannot be discharged because the blocks are interwoven. After resolving their boundary constraints, both of these structural systems are capable of tolerating orthogonal and transverse forces (Brocato et al. 2014).

Unfortunately, the great geometry of these vaults only comes into play once the whole assembly process is completed. Additionally, the substantial horizontal thrust within the assembly requires a strong boundary condition to support the structure, such as buttresses or hefty walls (Miodragovic Vella et al. 2016). Thus, both of these flat vaults are not often applied in an architectural format as a successful construction method. Moreover, since Truchet’s block has proven difficult to execute with an analogue tool, the vault does not offer many practical advantages.

From the 17th century onward, science and the art of stereotomy fell into general disfavor, due to issues of labor intensity, atypical craftsmanship, and geometrical complexity. After the first half of the eighteenth century, the industrial revolution and the introduction of new materials caused a sudden break away from stereotomy (Fallacara 2009, Rippmann et al. 2011, Miodragovic Vella 2016). A major shift in contemporary architecture, nearly two and half centuries after its demise the principles of stereotomy are now being revived. The applications of Abeille’s stereotomic system have redirected attention to the development of new materials. Therefore, on a different scale, the potential for his system is being re-explored by materials scientists (Dyskin et al. 2001, Estrin et al. 2011, Khandelwal et al. 2012, Carlesso et al. 2012), and has become a research topic for academics and industry professionals alike. Syskin’s team coined the term “topological interlocking assembly” in the course of their research on the stiffness, deformation, bearing capacity, and even sound absorption of planar configurations comparable to Abeille’s core idea. There is now a great deal of spec-
ulation surrounding the concept of topological interlocking systems and their promise as a materials design principle.

As a basis for developing a paving system, Glickman (1984) proposed a set of interlocking tetrahedral units that could link together due to the inclination of their neighboring sides. In recent years, with so much attention being paid to assembling discrete components in complex loadbearing formations, many scholars (Tessmann et al. 2013, Miodragovic Vella et al. 2016, Weizmann et al. 2016) have attempted to incorporate stereotomy, at that point a vanishing building discipline, into their procedures for designing and fabricating freeform geometries (Andriaenssens et al. 2014). Additionally, due to technological advancements in digital design and fabrication, the associative rules of topological interlocking connections have motivated designers to test those rules on large, non-planar assemblies. The work of these researchers and designers is still in progress. It is worth mentioning that contemporary stereotomy-inspired studies have been developed to benefit contemporary architecture rather than to serve as a nostalgic replication of an isolated chapter in the history of construction and archaeology. For instance, the Brocato team's research has contributed to a better understanding of how Abeille's system could be adapted to designs for geodesic spherical assemblies (Brocato et al. 2010, Schwartz et al. 2014). These researchers also developed a prefabricated stone wall based on stereotomic designs (Brocato et al. 2014). The ever-consistent Fallacara has also transferred the geometric rules of Abeille's flat vault into geometrically expressive non-planar structures such as vaults and domes (Etlin et al. 2008, Fallacara 2009). Clifford and McGee updated stereotomic techniques through their translation of Inca stonework techniques into contemporary practice (Clifford et al. 2014, 2015). In the past few years, several academic institutions and research labs have investigated robotic approaches to self-supporting, compression-only, unreinforced, mortar-less structures; these include the Hyperbody Research Group at TU Delft University (McGee et al. 2013), and the Robotics Research Group at the University of Sydney (Fernando et al. 2017).

Fortunately, a continuous increase can be seen in the formal complexity of more varied stereotomic-like projects integrated with structural analysis (Fernando et al. 2015). By revisiting Ochsendorf's method (2010) and relearning means of erecting compressive vault structures, the Block research group at ETH Zurich was able to initiate computational modeling approaches such as Thrust Network Analysis (Block 2009) for form-finding and optimization in compression-only structures (Rippmann et al. 2011, Block et al. 2017). By applying equilibrium analysis to masonry structures and simulating their complex behaviors, they were able to find practical means of translating spatial masonry construction techniques into contemporary practice.

AGGREGATED INTERLOCKING SYSTEMS IN PRACTICE
This research documented the incorporation of digital tools into a design pedagogy for simultaneously teaching the relationships among geometry, structural forces, and architectural form. By providing an opportunity to explore the construction-informed and structurally-aware design processes of two interlocking systems - including a vaulted structure (see Figure 2) and a footbridge (see Figure 3) - in the fall semester of 2016, freshman architecture students at Texas A&M University were able to identify certain challenges to realizing these structures and developing form-finding techniques. The mortar-free bridges and vaults were held together by the boundary constraints of their discrete elements.

Here, students attempted to incorporate precision, tolerance, and craftsmanship into their coupling of the design and construction processes for several interlocking assemblies, with all of their rich structural sophistication and aesthetics. Students’ primary goal in these two projects was to take advantage of the potential of digital tools and their direct physical implications for the proposed interlocking
systems by using geometry, especially if it could be implemented in a self-supporting structure.

Both projects were non-linear and exploratory, investigated with and through computations completed within a limited timeframe. By synthesizing both physical and digital form-finding techniques and simultaneously responding to assembly challenges, the instructors hoped to lead these beginning design students to develop their own integrated digital workflow, from design to fabrication and beyond simple digital modeling. To reach beyond digital representation, these projects asked freshman students to trace the relationships among materials processing, structure, and geometry, and explore the potential of topological interlocking assemblies within an architectural framework, as well as their related design methods and fabrication possibilities.

The key to designing an interlocking system is a comprehensive understanding of geometry in three-dimensional space. Since geometry plays an important role in stabilizing an interlocking assembly, both projects helped students investigate the reciprocal relationship between geometry and structure. In these projects, the bridge and vault and their respective structural characteristics emerged from the geometry of the blocks (see Figure 4). Since each block was a configurational condition based on its geometry, the whole interlocking system was highly constrained by geometry as well. Consequently, changing the geometry of each block such that the mutual surfaces were altered affected the overall stability of the structure. Students’ prototypes were made and assembled from blocks that could be divided into several categories, depending on the forms of their interlocking joints, block interfaces, and seams, all of which were derived from the underlying geometry of their designs.

Topological interlocking assemblies have intrinsic vibration attenuation mechanisms. Thus, analyzing their principles can lead to a more profound knowledge of the behavior of seismic-resistant structures (Yong 2011) due to ground motion or soil failure. As a deep-rooted constructive solution to energy dissipation during seismic events, mortar-free interlocking structures with relative movability at their block interfaces can diffuse the energy of a dynamic load throughout. Under a dynamic load, the vault and bridge modules allowed for slight movements to dissipate significant shear forces without major failures, increasing the lateral force capacity of the entire structure. Students’ proposed vault designs were for earthquake-resistant buildings that could withstand seismic waves in Southern California.

By handling movement, interlocking vaults are able to sway but not collapse. The proposed bridge and vault designs were tolerant to local failures. Failed blocks were held in their assembled positions by their neighbors, by virtue of the appropriate joint geometry between adjacent surfaces. Conceived to be an assembly of fragmented pieces, an interlocking structure, as an exceptional material system, does not let cracks propagate throughout. In the students’ models, the chance of structural failure was decreased in cases of major damage because the structures could be broken down into smaller distinct components. By utilizing various analogue and digital tools, the projects offered a holistic understanding...
of the interplay of geometric relationships within different construction techniques and assembly configurations useful when designing forelocked systems. Both projects were initially developed with digital models.

By incorporating information, materials properties, construction knowledge, time, and space, this research clarifies how digital models can be a part of physical models. Although digital tools were used as a starting point, the final designs were developed somewhere between computational simulations and physical materials prototyping, while simultaneously transforming from one into the other. The translation of the virtual into the real provided confidence to freshman students witnessing how the possibilities of digital simulations enhance the potential of physical models.

**FIRST INVESTIGATION**

*An unreinforced interlocking footbridge*

In the first project, students were asked to envisage a relatively thin bridge system that could not be warped. Then, they examined principles key to the interlocking mechanism by applying a flat, small-span footbridge to a planar assembly. The goal was to create a structure that could traverse a small creek without requiring piers. By imposing kinematic constraints through the shape and mutual arrangement of its modules, students designed and fabricated several woven organizational patterns out of various networks of interacting elements (see Figure 5).

As mentioned above, no pillars were allowed in the middle of the students’ bridges, so loads had to be transmitted to the two ends. The closely fitted elements in the bridges were capable of resisting static and dynamic loads, thus allowing foot traffic and bicycles to cross over. The students’ bridge structures behaved as compression-active constructions to resist bending forces. The bridge elements that were assembled to mutually constrain the planar configurations were also easy to disassemble as demountable constructions. The design process for the interlocking assemblies was a systematic method of transferring loads at the interfaces between two neighboring elements that kinematically constrained each other, achieving stability through the interaction of neighboring elements and their geometrical and spatial configurations.

**SECOND INVESTIGATION**

*A self-supporting vaulted structure*

In the second project, students’ models explored the nature and potential of applying appropriate geometries to compression-only interlocking structures with non-planar shapes made out of discrete components. Here, the vault’s static components were subservient to the geometry of the joints, the respective shapes of the individual blocks, and their arrangement as a whole. The strength and equilibrium of the interlocking vaulted structures were achieved through their geometric configuration, as well as their spatial and formal characteristics.

Associating forces and form in a mortar-less structure once the interlocking blocks are completely assembled is a way of structurally defining space. As such, the second project helped students understand the dependence of an interlocking vaulted structure on its architectural spatial expression and connotations. The alliances among the structural, spatial, formal, and aesthetic qualities of the proposed vaulted architectural systems were employed to make a small structure called the Chapel of Silence. The resulting vaulted buildings required no additional side pressure or frame to hold the blocks together (see Figure 6).
Figure 6
Complex architectural geometries for three different topological interlocking modules of a vaulted structure. Freshman Design Studio, Texas A&M University, 2016.

PRINCIPLES OF THE MATERIALIZATION PROCESS

The design process for a compression-only interlocked structure uses an integral connection mechanism to deconstruct a continuous form into discrete blocks for fabrication and assembly (Griffith et al. 2006).

Since this design process meets a confluence of different formal, spatial, and structural concerns, it is cyclic and requires several feedback loops. Therefore, to coordinate the different constraints, any fabrication and assembly challenges should be brought forward early on.

Materialization involves several design and modification steps, including:

- Form-finding to determine the base surface geometry,
- Discretization of the base surface and application of a tessellated pattern,
- Generation of the geometry of the block modules based on the applicable profile geometries,
- Refinement of the structural form and tolerance design,
- Processing and replication of the fabrication of the modules, and
- Completion of the assembly sequences and required formwork.

These steps were carried out for both studio projects. The translation of the computed forms to constructible blocks was exemplified by making more than 34 complete physical scale models with interlocking assemblies built into their blocks.

Surface Geometry. In the design process, designing the appropriate surface geometry and dividing the surface into meaningful subdivided surfaces or cells can be challenging. For this research, the process began with a base surface geometry informed by structural concerns, as well as the means and methods of making. For instance, in a vaulted structure, the most important constraint that governs the stability of spatial assembly is its surface curvature radius. Therefore, the final form of the curvilinear base surfaces was geometrically defined as semi-cylindrical.
**Discretization.** The deconstruction of the base surface geometry into proper segments is a critical step in the design process. By using parametric geometric algorithms to divide the surface, it is possible to map regular, semi-regular, or non-regular surface tessellations onto the base, generating 2D or 3D grids (Rippmann et al. 2011, Weizmann et al. 2016). In both projects, the base surfaces of the bridges and vaulted structures were subdivided into almost evenly shaped cells along their initial isocurves. The ultimate goal was to achieve constructible blocks. Consequently, subdivision sizes were specified based on fabrication and assembly concerns, while retaining the designer's intent.

**Block Module Design.** The final bond of the assembly is closely related to the integral connections made between blocks. To create a continuous interlocking assembly, it was necessary to design a 3D puzzle-like configuration with particular tabs and slots. Although each cell emerged from the boundaries of the tessellated grid applied to the base’s surface, it was not possible to translate the cell boundary to a solid; a simple extrusion command, based on the cell’s normal service, can often generate the appropriate spatial geometry. However, in these structures, by using different types of surface commands and Boolean operations to subtract the desired parts of one block from another, the embedded connections could be modeled.

It can be laborious to model a block with an integral connection mechanism when it is part of a larger, complex geometry. To create the interlocking connections, the individual geometries of one or two axil sections are essential. The designers needed to progressively move closer and closer to find the desired profile geometry to drive the topological optimization process. In their projects, although the orientations of modules and geometries of the boundaries governed the material distribution of the force-locked structures, the contact faces of the mutual modules and their sections were critical to regulating the structural behavior of the assemblies. Both projects helped students to understand that the effectiveness of an interlocking system depends on the sections of its modules. In large part, the stability of these interlocking systems is not guaranteed simply by the natural compression between the blocks. However, the connections between each pair of mating joints makes it difficult to pull the entire structure apart.

In the students’ projects, the vault blocks were held in place by the appropriate geometry of the surfaces in contact with neighboring blocks, and their mutual arrangement. In the interlocking systems, it was the geometry of the joints themselves that hindered the relative movements of the block pairs. Therefore, choosing the right corresponding geometry for the joints was key to the integrity of the structure. Through the use of geometrically-informed surfaces to connect blocks (rather than sophisticated mechanical joints or chemical connections), the discrete modules were able to form bonds. Depending on the shape of any two connecting joints, the students dealt primarily with geometric principles to design large sculpted concavo-convex matching surfaces for each joint. Besides the material properties of the blocks and the constraints that could arise during fabrication or assembly, the appropriate depth of the embedded connectors of each block followed the volumetric size and weight of the entire structure, the initial curvature of the final assembly, and the number, complexity, and direction of the blocks.

**Refinement of the Structural Form and Tolerance Design.** The behavior of an interlocking structure must be predicted before assembly, rather than after completion. Although the direction and magnitude of the forces have a significant impact on the stability of the assembled structure, to satisfy a thrust network within the structure’s depth, structural concerns must be aligned with formal aspects. In this research, this step was accomplished via two substeps, resulting in an isomorphic structural condition on both large and small scales. On the large scale, to guarantee the structural integrity of the entire assembly, a global structural evaluation was considered; this maintained the structure’s equilibrium in its
final state. On the small scale, a local understanding of the forces applied during assembly was essential to satisfying intermediate equilibrium states (Ariza et al. 2017). Also during this phase, to obtain a minimum of gaps between blocks that result from tolerances in fabrication while also satisfying component strength, assembly tolerance had to be taken into account. The blocks had to have the capacity to be fitted in place during the assembly process when there were fabrication inaccuracies.

**Fabrication Process.** The fabrication workflow can vary from one project to another. In the studio projects, this process was divided into two main phases: rapid prototyping of the required modules, and the production of molds to replicate the modules through casting. Students employed cast concrete blocks to leverage the robustness, versatility, and appeal of concrete. Here, the prototypes were mainly built from uniform interlocking blocks. Although the projects took advantage of 3D printers to make individually unique blocks, it was obvious that uniformity made the mold-making processes less expensive. Since a wide array of blocks needed to be produced, most of the students’ designs used only one shape of block.

**Assembly sequences.** The process of materialization provided an additional stimulus for thinking about the interdependent relationship between the surface geometry and structural behavior of a topological interlocking system at the time of assembly. In this research, the blocks were dry fit to one another and set in place at specific moments. Registering each block to its correct location relied on the other steps being accurately satisfied. Completion of the assembly sequence did, at times, require a temporary scaffolding system.

**DISCUSSION**

In general, the main challenge with these types of structures is to maintain the structural integrity of the aggregated construction. This research allowed for a more holistic understanding of the geometric nature of interlocking structures. Through the discretization of flat or curvilinear tessellated surfaces into a limited number of cells with embedded, form-fitting connectors, the projects exhibited the potential of the structurally and mechanically responsive geometries informed by stereotomic principles.

This work reported on ways of mining the primary geometrical logic that expands topological interlocking principles towards the fabrication and assembly process. Examples of the applications discussed here were introduced through several scale prototypes. The two studio projects helped freshman students get acquainted with the challenges involved in designing and implementing this type of constructive practice by bringing into play the tangible geometric speculation of stereotomic systems. Students used hands-on experiments to unravel how the geometrical conformation of a part could guarantee the integrity of the whole. In their projects, the interlocking mutual support of the blocks in the prototypes captured the reciprocity principle upon which the topological interlocking premise was based.

The underlying organizational mechanism of interlocking systems provides the potential to streamline complex geometries. The lessons learned from these two topological interlocking projects, incorporated with digital computation and heightened by physical prototyping, reveals that their stereotomic nature could help designers investigate the potential of variations within the structure of the blocks. The fabrication of interlocking prototypes illustrates that there is a need to re-explore and improve upon the relationship between the working principles of stereotomy and their generative effect on the stability and form of a structure.

**ACKNOWLEDGEMENTS**

The authors would like to thank Jim Titus (the wood shop supervisor) for his support, and all of the students participating in the ENDS 105-500/502 studios at Texas A&M University in the fall of 2016. Special thanks to the following students that their models and drawings are presented in this paper: Chris Loofs, Jordan Marshall, Savannah Sinowitz, Hannah Parton,

REFERENCES
Fleury, F 2009 ‘Evaluation of the Perpendicular Flat Vault Inventor’s Intuitions through Large Scale Instrumented Testing’, Proceedings of the Third International Congress on Construction History, Cottbus
Glickman, M 1984 ‘The G-block system of vertically interlocking paving’, Proceedings of the 2nd International Conference on Concrete Block Paving, Netherlands
Ochsendorf, J 2010, Guastavino vaulting: the art of structural tile, Princeton Architectural Press, Princeton
Sakarovitch, J 2003 ‘Stereotomy, a multifaceted technique’, Proceedings of the First International Congress on Construction History, Madrid
Yeomans, D 1997, ‘The serlio floor and its derivations; The serlio floor and its derivations, 2, pp. 74-83

648 | eCAADe 35 - DESIGN TOOLS - PROGRAMS - Volume 1