New Concepts for Application of Topological Interlocking In Architecture

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The paper concerns the issue of constructing flat vaults from elements topologically interlocking inspired by the Abeille blocks. One of the new ideas that are presented is constructing the vaults in an order opposite to the one considered until now. The problem of static response on the thrust force, significant for flat vaults, is usually solved by the use of the perimeter frame, added only after arranging all the elements of the vault. The paper presents how to arrange the vault inside a previously made frame thanks to application of special components divided into parts, which are inserted at the end and play the same role as a keystone in a stone arch. The other new concept is shaping vaults based on equilateral triangles and regular hexagons, from hexagonal, romboidal and triangular elements shaped and arranged in a manner similar to the one used for shaping square vaults. The last innovative concept presented in a paper concerns shaping the perimeter frame from the components providing stiffness of the frame only due to topological interlocking. All presented ideas have been analysed purely at a geometric level.

Keywords: reciprocal structures, flat vaults, topological interlocking

INTRODUCTION

The issue of constructing flat vaults from elements shaped in such a way that due to the topological interlocking they establish a self-supporting structure is the subject of theoretical considerations and attempts of practical implementations undertaken for several hundred years. The first projects were based on the use of the idea of a stone structural arch, whose transfer to a flat structure results in the shapes of individual elements differentiated according to the position of the element in the structure. The stone vault of this type, dated to 1559-84, is still preserved in the Eskurial monastery in Spain (Addis 2007). The 280 mm thick structure was realized above a square room with dimensions of 8x8 m. Because the manufacture of non-uniform elements with precision ensuring their tight fit was difficult, researchers’ efforts have been redirected to finding the solution, which would allow to construct a flat vault from identical elements.

The first concepts of such vaults were presented in 1699 by Abeille and Truchet (Gallon 1735) and next ones in 1737 by Frezier (Frezier 1737). The vaults were designed as constructed from identical elements arranged so that each element supports two other elements and it is supported by two more elements. The idea was based on the concept of floor structures made of timber reciprocal beams, which was popu-
larised earlier as Serlio's floors, e.g. by John Wallis (Wallis 1995) and it has even been practically used in England (Insall 1972, Morton 2011) and Netherlands (Emy 1841). The differences between concepts of Abeille, Truchet and Frezier manifested in the shapes of blocks. Abeille started from the vault characterized by the fact that one of its surfaces is smooth, the other is relief. Truchet designed a vault of both smooth surfaces obtained as the result of the use of blocks with curvilinear edges. Frezier found shapes of straight line edge blocks, which were also suitable for constructing the vaults of both smooth surfaces. Two structures using this idea, both from the second half of the nineteenth century, are preserved until now in Spain: the first one in the cathedral of Lugo (Rabasa Diaz 1998) and the second in the Casa Mina de Limpia, near Madrid (Nichilo 2003). Both the vaults are very small, the external dimensions of the second one of them, which was precisely described by Ignazio Guijarro (Guijarro 2017), are 328x397 cm. Thrust balancing, which is the bigger the grater the vault, is always the most important structural challenge, which designers have to face.

Currently, increased interest in the issue is observed not only in the field of architecture, but also in material engineering. The first paper, which undertook the subject of shaping material structures from interlocked tetrahedron-shaped elements was published in 2001 (Dyskin et al. 2001). In 2003 in the field of material engineering the term topological interlocking was introduced (Dyskin et al. 2003). A limited number of theoretical papers dedicated to the issue of shaping flat structures applicable in architecture has been written in recent years (Brocato and Mondardini 2015, Weizmann et al. 2016, Weizmann et al. 2017), as well as experimental research was carried out in this area (Sakarovitch 2006, Fleury 2009). Another group of researchers focused their activities on applying the idea for shaping spatial structures (Fallacara and Stigliano 2012, Brocato and Mondardini 2012).

An important problem, in the issue of topologically interlocking flat vaults, is the tendency of elements to moving in horizontal directions, caused by the action of the thrust force. The method proposed in the literature to counteract these displacements is the use of a perimeter frame made of steel beams. Both in theoretical considerations and experiments on models made on a natural (Sakarovitch 2006, Fleury 2009) or a reduced scale (Weizmann et al. 2017), it was assumed that the frame is assembled after arranging all the elements of the vault, or at least the last element of the frame cannot be mounted until then. The experiments carried out have shown that, despite the use of the frame, the dimensions of a single vault cannot be large, due to the occurrence of excessive deflections.

THE REVERSE ORDER OF CONSTRUCTION OF FLAT VAULTS

The shapes of the elements forming the vault, designed with the intention to guarantee the topological interlocking, precludes prior preparation of the frame and filling it subsequently with only typical elements of the vault. The procedure of constructing the vault in the reverse order, i.e. first the frame, then the set of elements filling it, is possible but it requires an application instead of a given number of the typical blocks, of the ones shaped in a special way, and the assembly of the vault in a particular order. Constructing the vault in this way should be started from laying the blocks in the zone adjacent to the beams, and then be continuing towards the centre of the vault. In the middle of the vault, substitute blocks should be used, identical in form as the typical ones, but divided into pieces. The shapes of these pieces should be chosen so that it is possible to move each of them through the hole remaining between the previously arranged components, the support on the components already laid and providing the support for the element that will be introduced next. The role of these untypical blocks for the flat vault is analogous to the role of the keystone in the stone arch.

The presented procedure opens the perspective for using topological interlocking to construct floor slabs with larger spans than the dimensions of a sin-
Single vault, through the cooperation of vaults with a steel grillage, as a structure supporting the floor slab. Two types of vaults have been taken under consideration. The first one, made of blocks, which cross sections are trapezoidal and identical in two perpendicular planes, is characterised by relief surfaces from the both sides. Turning the vault upside down does not change its appearance from any side. This type of vault does not require special shaping of the edge elements if the grillage cooperating with the vault is made of pipes with a hexagonal cross-section (Figure 1). The second type of vault, made of Abeille blocks may be considered in two options, as the vault with bottom surface smooth and the upper relief (suggested by Abeille) or the inversed one, with bottom surface relief and the upper one smooth. Abeille vault requires that the blocks adjacent to the grid beams are shaped in a special way. The same applies to the vault of the first type if the beams are not hexagonal.

The role of the grillage beams is to counteract the displacement of the vault components both in vertical and horizontal directions. Therefore, the optimal way to make the grillage is to use steel I-section profiles. Filling the space between the flanges of an I-beam, which is impossible to be realized with a single panel, is possible using small-size elements. However, because the assembly of the grillage can be difficult with the use of such profiles, an alternative solution is the use of rectangular tubes with flat bars attached to their bottom surfaces, whose role would be to support the vault elements. In any case, the assembly requires the use of temporary scaffolding (Figure 2). The order of filing the grillage field by the vault elements does not depend on the type of the vault, as well as the type of the beam profile, and should be such as shown in Figure 3. Fitting the Abeille blocks does not cause friction between them (Figure 4). The elimination of friction between the blocks forming the double relief vault requires chamfering the edges of the elements (Figure 5). The consequence of the chamfering is the presence of small gaps in the vault with sizes resulting from the chamfer dimensions.

The last step in the vault assembling procedure consists in a placement of the special elements fulfilling the role of the keystone. Simulations carried out on 3D virtual models of the vaults have proved that it is impossible to put the one before last element into the space remaining after arranging all the elements, except these two last ones, so the completion of the vault requires the use of at least two elements divided into parts. The possibility of dividing the standard component into parts that can be inserted one
after the other into the available opening depends on the slope of the side faces of the component. It is assumed that the pieces that will be introduced first, marked in red in Figure 6, and numbers 1 and 3, should be of trapezoidal section (Figure 7) or in the extreme case triangular, and the angle $\theta_1$ between the plane dividing the component and the horizontal plane, should fulfill the condition (1), where $\theta_{rr}$ is the angle under which side faces of the block are inclined to this plane (Figure 7). The measure of the angle $\theta_{rr}$ which guarantees the movement of the element through the hole must meet the inequality (2) and the shorter length of the trapezoid base being the cross-section of the first part - the inequality (3).

If the elements of the keystone are assembled in accordance with the numbering according to figure 6, the part marked with number 1 will rest on standard elements, the part 2 on the same elements and on part 1, the part 3 - on one of the standard elements and on the part 2, and the piece 4 will be supported by part 3 and the blocks supporting it. Inserting the yellow parts will be easier if the measures of angles $\theta_1$ and $\theta_{rr}$ are not the same.

\[ \theta_1 \leq \theta_{rr} \]  \hspace{1cm} (1)

\[ \theta_{rr} \geq \arctan \left( \frac{3t}{s} \right) \]  \hspace{1cm} (2)

\[ c_{rr} \geq s - \frac{3t}{\tan (\theta_{rr})} \]  \hspace{1cm} (3)

The way of dividing the Abeille blocks into parts is very similar as in the case previously discussed. Since the elements of the keystone must always be inserted from the top, because their motion under the influence of gravity should be blocked by the elements previously arranged, independent geometrical analyzes are necessary for vaults with a smooth surface from the bottom, or arranged in an inverted position, i.e. with a smooth surface from the top. The basic dimensions $\theta_{rs}$ and $c_{rs}$ of the keystone for the vault with smooth surface from the bottom are expressed in formulas (4) and (5), and the dimensions $\theta_{sr}$ and $c_{sr}$ of the keystone for the inversely oriented vault in formulas (6) and (7). The condition for parameter $\theta_1$ is in each case as in formula (1), with the exception that in the place of angle $\theta_{rr}$ angles $\theta_{rs}$ or $\theta_{sr}$ are used. The shapes and dimensions of keystones for the vaults made of Abeille blocks are explained in details in Figure 8.

\[ \theta_{sr} \geq \arctan \left( \frac{2t}{s} \right) \]  \hspace{1cm} (4)

\[ c_{sr} \geq s - \frac{2t}{\tan (\theta_{sr})} \]  \hspace{1cm} (5)

\[ \theta_{rs} \geq \arctan \left( \frac{4t}{s} \right) \]  \hspace{1cm} (6)

\[ c_{rs} \geq s - \frac{4t}{\tan (\theta_{rs})} \]  \hspace{1cm} (7)

HEXAGONAL AND TRIANGULAR FLAT VAULTS

The method of constructing the flat square vaults with the use of topologically interlocking blocks can
be adopted for constructing vaults of free shapes and from elements based on a custom grid (Weizmann et al. 2017). The useful system, from the point of view of repeatability of elements and so the need for creation of only limited assortment of their spatial forms, may be designed for vaults in a shape of regular hexagons or equilateral triangles, based on the grid consisting of equilateral triangles. The method of shaping particular elements is well known (Vella and Kotnik 2016, Weizmann et al. 2016) and it amounts to establishing planes $\alpha_{ij}$ perpendicular to the grid, which cut it in chosen edges and then to rotating them $\alpha_{ij}$ by identical angles.

Such a method adopted for regular hexagons and rhombuses being a sum of six or two basic triangular meshes of the grid, as well as for a single triangle is the first step in the procedure of creation of spatial blocks designed for construction of the considered vaults. The final shape of the block is done with the use of two planes $\beta_1$ and $\beta_2$ parallel to the basic grid (Figure 9). If they have been arranged in a space at equal distances from the grid but on opposite sides, the resulted blocks are dedicated for the double relief vault. If one of the planes is the plane of the grid, and the second one is placed in a given distance from it, the blocks are similar to Abeille blocks and they allow for the construction of the vault with the one smooth surface.

Because the structure of the vault using the topological interlocking of elements requires that each element supports other elements and is being supported by other ones, it is not possible to aggregate the structure only from the triangular blocks, due to the odd number of triangle sides. In such a case, some of the blocks could base their two faces on other blocks, but there would also be those that would base only one face on the another block, while two faces would have to support other elements. This makes the stability of the structure impossible. On the other hand it is impossible to aggregate the hexagonal or triangular vault from only hexagonal blocks. The use of hexagonal blocks must be realised with the complementary use of triangular blocks or rhombic ones if possible. The triangular blocks may be applied only in such a number and orientation that each of them finds support along two side faces on the other elements, or on the peripheral frame, and only at most one side face creates support for another element. It means that they can be used also in an inversed position, but only in such a location in which the peripheral frame creates additional support (Figure 10). The stability of a set consisting of hexagonal and triangular elements will be maintained if the number of supported edges in the whole assembly exceeds the number of supporting edges, but the number of supported edges should include the edges adjacent to the sides of the vault assuming that it will be supported along its circumference.

The composition of the self-supporting structure requires a detailed analysis of all its fragments. Figure 11 shows triangular and hexagonal self-supporting structures. The universal principle allows to compose hexagonal elements and triangular structures of any size. From the same elements it is possible to assemble only such hexagonal structures whose side length is equal to 2 or 3 meshes.
PERIPHERAL FRAMES FROM TOPOLOGICALLY INTERLOCKING BLOCKS

Surrounding the vault with a perimeter frame is an indisputably effective means of counteracting forces of the thrust. In previous considerations, it was assumed to use as a frame a steel beams grillage, i.e. made of a different material and using technology other than the vault itself. Theoretically, shaping the peripheral frame from prefabricated elements made of the same material as the vault and obtaining the load-bearing capacity also due to the topological blocking of the elements can be considered. The issues discussed in this paper are focused solely on the geometric shaping of the vaults and they abstract from the mechanical properties of the material from which the vaults could potentially be made. Objections that may arise as a result of poor assessment of the suitability of concrete or stone, being the only materials used until now for constructing vaults on a natural scale, as a material for making a frame, do not have to be taken into account. It is assumed that in the near future, the development of material engineering and manufacturing techniques, including in particular 3D printing technology, can provide a completely new material and tools to implement the presented geometric concepts in real and useful structures.
The self-locking structure of the width of only one element is known for practical use in the lintel beam in the gothic gate in Alba Iulia (Motro 2009), as well as from the design of trapezoidal frame (Baverel and Popovic Larsen 2011). In both cases, the design task is to transfer gravitational loads. The idea (Figure 15) can be implemented in the structure of a flat vault for the transfer of forces operating in the plane of the vault, whose dimension in the direction perpendicular to the working force, is also equal to one element.

Prefabs included in the perimeter frame, in addition to the role resulting from the co-creation of this frame, must perform the functions of typical vault components. Hence, in addition to counteracting horizontal displacements of internal blocks, due to operation of gravitational forces they should support blocks of the pre-end band. Prefabs, which as individual elements, fulfill the aforementioned functions (Tessmann 2013), may not prevent horizontal displacement of the entire edge zone, being not stifferly linked with the rest of the structure. An effective alternative is to create a perimeter frame, which would be a self-supporting structure, capable of blocking the displacement of the edge band in a horizontal direction perpendicular to this band, by securing the sides of the frame in the manner shown in the right part of the Figure 15.

The vault shown in Figure 16 is only an example that can be adapted in structures made of elements with other shapes. Elements of this vault differ from those described earlier. They are modeled on the Abeille cube and modified in such a way that the structure of them is characterized by a non-zero thickness at each point. The modification consists in enlarging the components from the side of a smooth outer surface with a layer of constant thickness. Probably from such blocks the vault in cathedral in Lugo is constructed (Rabas Diaz 1998).

If supporting the vault is planned only by means of columns located in corners of the vault, the peripheral frame should fulfill additionally a function of a self-supporting beam. The implementation of all functions by the frame results in the exclusion of the possibility of sliding the prefabricated elements vertically from the top, due to the fact that the trapezia contained in the vertical sections, rotated one against the other, will interlock during such movement. Due to the grooves serving to block displacements in the direction parallel to the side of the frame, it is also not possible to slide the prefabricated elements from the horizontal direction. The solution is to slide them parallel to one of the inclined side faces, and give them such a shape, so that it enables the movement resulting in approach to another part of the face located in the plane of opposite slope.
Since the direction of the sliding cannot be parallel to the sides of both trapezia, the cross-sections of the prefabricated elements by vertical planes parallel to the side of the perimeter frame cannot be trapezia. The inserting of prefabricates will be feasible if the opposite sides of the cross-section are parallel straight lines, and blocking will be possible, if the upper left (right) part of the cross-section will be based on the right (left) part of its cross-section, i.e. on the right (left) part of the cross-section of an adjacent prefab, which is identical (Figure 17). Corner prefabricates should be formed in such a way that the left and right part of this section are contained in perpendicular planes, i.e. that the possibility of sliding subsequent prefabricated elements along the entire perimeter of the frame, also after changing the beam direction, is ensured.

The shape according to the description above does not allow the last component of the frame to be mounted. Its implementation can take place when, instead of one of the repetitive prefabricates, a two-element keystone is used, identical with it in terms of form, but divided into two parts (Figure 18). It is characteristic that despite the complex shapes of prefabricates forming the frame, the pattern on the upper surface of the vault is a grid of squares (Figure 19).

**CONCLUSION**

The subject of the research has been considered only in geometric aspect. The correctness of the presented results has been validated by computer simulations carried out on virtual models of particular elements and structures created from them. The next step of validation, planned by the author, is realization of material models in 3D printing technology. It is anticipated that parallel theoretical studies will be carried out on geometric shaping of vaults based on other shapes of blocks, including those invented by Truchet and Frezier. The author’s intention is to complement the concept by solving the problem of shaping triangular and hexagonal vaults also in a way allowing for the assembly inside frames previously made.

The opportunity to popularize the structural system discussed in this paper would be the invention a material that, in terms of mechanical properties, would outperform concrete and ceramic materials, but at the same time be lighter. The important problems that must be solved are: providing the edges of prefabs with resistance to damage during transport and assembly, and their walls with smoothness, because the effectiveness of topological blocking of components depends largely on the quality of the contacting surfaces.

The concept presented in the paper allows using the same vault components to construct floor slabs of various dimensions and even the load capacity, thanks to ability to cooperate with the structural grillage. The installation of floor slabs is a dry assembly that does not require the use of technological water.
and makes dismantling the structure relatively easy. For this reason, the proposed solutions are rational for applications in temporary buildings, or those for which a short turnaround time is required, e.g. used for the rapid reconstruction of infrastructure in areas affected by natural disasters.

The last but not least aspect of topologically connected vaults are aesthetic issues. The floor slabs made as single-layer, that is, without layers covering the floor or the ceiling are extremely decorative. Strengthening the aesthetic value may be obtained by the diversification of the material from which the individual components will be produced, e.g. in terms of color. There is also the theoretical opportunity to create a vault only from keystone elements, which will allow to replace any elements of the vault without the need to dismantle it, but only with the necessity of temporary local support.

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