Running, Running, Stop

Applying graph theory to pathfinding analysis to improve circulation efficiency in vertical high schools

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Understanding how people circulate in a space is crucial to effective spatial organisation. However, while many studies focus on wayfinding within environments such as train stations, there are few discussing multi-storey circulation and pathfinding within educational institutions. Vertical high schools prefer multiple stairways over elevators, and seek to devote the maximum floor area possible to learning spaces. These considerations impact a student's ability to quickly traverse multiple floors. In this research, a computational tool is designed to simulate and assess circulation efficiency within a vertical high school with the goal of lightening stairwell congestion and decreasing classroom transit times. Using action research methodology, the research problem was formulated in cooperation with the industry partner, TERROIR, and solved in an iterative manner. As a method to assess spatial circulation performance, principles of graph theory were applied to a simplified navigation mesh derived from the high school's floor plan and timetable using Grasshopper. The ability to understand the influence of a designer's decisions on circulation efficiency factors such as travel time and congestion is vital to creating practical and enduring designs that avoid expensive, post-construction building alterations to fix emergent problems—a core contribution of this research. While the simulation designed in this research engages with a vertical high school context, it could be appropriated for other building types, such as offices or shopping centres.

Keywords: *Graph Theory, Congestion, Internal Circulation, Education, Spatial Organisation.*

INTRODUCTION

Functional circulation in vertical high schools is vital for a healthy and balanced learning space. When a space is poorly designed, overpopulated, or neglected, the people within can find themselves crowded and stressed, and this continues to negatively impact their behaviour while they remain in the area (Baldea & Dumitrescu 2013, p.176). This is compounded by reprimands students may receive due to tardiness, or the impact of lunch hours lost to travelling. To future-proof designs against such scenarios and ensure the stable mental wellbeing of the student body, the testing of circulation within vertical high schools is paramount. The issue of circulation in vertical high schools differs from other spatial typologies, such as train stations and plazas, due to not only a school's avoidance of elevators and preference towards one or multiple stairways, but the large numbers of people needing to traverse the space simultaneously with conflicting pathways. However, this presents a new issue regarding student density around stairways. The egress of these stairs create congestion traps during class transitions, as students crowd the landings in an attempt to get to their next class or outside of the school. This has the causal effect of forcing students to be late to class, or to devote a significant proportion of break times to traversing the school. A clear answer to this problem is to test for congestion through the means of circulation simulation and make changes to the floor plan or timetable to fix issues that arise. The industry partner, TERROIR, provided the form and floor plan of an example vertical high school, which was used as the foundation for the circulation simulation. From the hallways and stairways, a navigation graph was constructed from the principles of graph theory. After, students were iteratively sent between classes along this navigation graph, with their travel speed responding to the level of nearby congestion. The results of this process were analysed to reveal highly congested areas, and students who were unable to arrive to class on time. This method of circulation simulation and analysis exposes the influence of spatial organization and timetable planning on the efficiency of student flow. Designers can use this evidence-based data to create long-lasting designs responsive to their context and justify the relevance of their design decisions to either their client, or other designers working in collaboration. Thus, the vertical high schools may maintain high circulation performance levels long after construction, and expensive building alterations are avoided.

RESEARCH AIMS AND QUESTIONS

The research aim is to give both qualitative and quantitative circulation information to vertical high school designers, that informs them of congestion-prone intersections and unrealistic timetable decisions together with the goal of improving design by catalysing decisions that streamline people flow and reduce wait times during class transitions.

Consequently, we ask the following question: How can graph theory be applied to pathfinding analysis and circulation simulation to improve the pathfinding efficiency in vertical high schools?

BACKGROUND RESEARCH

In the traditional architectural realm, circulation analysis is often manually conducted through two-dimensional means, such as the diagrammatic simplification of a floor plan, graph theory, or by predicting natural circulatory flow routes within a space (Emmons 2005, p.51; Steadman 1983, pp.61-65). However, Kalay and Yan (2004) assert that the positive impact of these methods is limited due to their reliance on architectural precedents and case studies whose context does not align with the relevant project. They continue by stating this is common as the context of a project will never exactly match another, and that conflict would primarily occur through difference of location, or difference in relevant architecture regulatory requirements and norms (Kalay & Yan 2004, p.372). When considering circulation in the computational realm, agent-based simulation, or automated pathfinding, may allow one to overcome these issues of context using case-by-case computational analysis that is responsive to issues of obstacles or congestion. Moreover, applying graph theory principles to circulation simulations and pathfinding can significantly aid the computational analysis process and influence related conclusions.

Wayfinding and the Pathfinding Problem

In most spaces, pathfinding is secondary to wayfinding when determining how a user navigates an area, but in a school, wayfinding is predominantly inconsequential as both the students and teachers are expert users who have months or years of experience navigating the space. Thus, unlike in circulation simulations for plazas or train stations where agents must be responsive to signage within their cone of vision (Mackenzie et al. 2014, pp.436-437), the problem of circulation in a school becomes one not of wayfinding but instead one of pathfinding efficiency when moving from classroom to classroom, and the impact of congestion. The solution to the pathfinding problem often takes the form of shortest path tests from Point A to Point B in a grid or interconnected arrangement of vertices while navigating around one or multiple obstacles. It was determined the A* Algorithm was most suitable for this project in comparison to others such as the Dijkstra or Greedy Best-Search-First due to its accuracy and efficiency (Mehta 2015, p.1-2).

Obstacles and Congestion in Pathfinding

Arash Naderpour (2018) calculated the congestion in a theatre lobby space by counting the number of agent pathfinding routes that passed through each $9m^2$ cell in a square grid. His project goal was to inform the architect rather than alleviate high pedestrian flow in these areas, and neither a simulation or animation of people colliding along their pedestrian routes was completed (Naderpour 2018, p.56-64). Nevertheless, the separation of congestion analysis into fixed areas, whether in a grid, or as segments of hallways and points of egress, is seen in several studies of pathfinding congestion analysis. In their study 'Multi-agent pathfinding in the crowded environment with obstacles', Hudziak et al. (2017) focussed on agents pathfinding around obstacles within a navigation mesh using both A* and Dijkstra in separate project iterations, and similar to Naderpour they separated their analysis of congestion into rectangular areas, with each space outputting a value representing the density of agents during the simulation. Crucially, this agent density variable would cause an agent to slow down should they enter the space while it was nearing capacity (Hudziak et al. 2017, pp.1566-1567). agent density variable would cause an agent to slow down should they enter the space while it was nearing capacity (Hudziak et al. 2017, pp.1566-1567). However, they needed to ensure that no density value would cause an agent to become stationary, as this may stall the simulation. To combat this problem, they ensured that an agent density above a certain threshold would cause the associated area to become a temporary obstacle (Hudziak et al. 2017, p.1568). Furthermore, when planning routes using pathfinding algorithms, Hudziak et al. found that crowded areas resulted in slower travel times but had an unchanging shortest path distance. They concluded that calculating from path distances alone produced incorrect results for congestion analysis, and instead focussed on shortest time to account for cases where alternate routes around congestion were favourable (Hudziak et al. 2017, pp.1561-1562).

Application of Graph Theory to Congested Pathfinding

In the context of this research, graph theory is the simplification of areas or rooms based on their topological relationships, and it may take one of several forms. Convex space analysis graphs represent the spatial relationships between the entries and the paths between rooms in their floor plan configuration, and are directly applicable to the issue of pathfinding (Dawes & Ostwald 2013, pp.1-3). These convex space graphs are used to analyse accessibility and connections (Dawes & Ostwald 2013, p.1), and are not frequently utilised in congestion analysis. Nevertheless, their abstraction of space may be used to define the fixed areas for circulation and congestion analysis under the aforementioned congestion pathfinding method. Moreover, there is precedent to applying graph theory to circulation analysis in the study of Kim et al. (2014) where they used a simplification of circulation routes to determine the point in space farthest from fire egress and ensure that the related building regulations were satisfied (Kim et al., 2014, p.1-2). To conclude, for vertical high schools, circulation is a fundamental issue as a blockage on one level will propagate across multiple floors, and the tendency for blockages to form is dependent on the size of the school's spine, the provided egress, and the school's timetable (Fouad & Sailer 2019, pp.1, 11-12). Using the A* algorithm, graph theory, and congestion analysis, satisfactory quantitative conclusions around any blockage origin may be uncovered, thus positively impacting pathfinding efficiency within the institution.

METHODOLOGY

This research project was conducted under the overarching methodology of action design research, where the researcher undergoes their design practice and then evaluates that practice to identify its strengths and weaknesses. These are addressed to improve one's practice for consequent iterations of the project, or later projects. In relation to this iterative manner of practice, Jean McNiff describes action design research *"as a powerful form of educational research"* (McNiff 2013, p.23), as the designer is consistently reflecting on and critiquing their practice, researching the outcome, and then sharing their processing of learning with others.

McNiff also separates the theory of action design research into two distinctive areas. The 'action', referring to the practice, the processes performed when developing a prototype or product iteration, and the 'research', referring to the data gathering processes after the prototype's completion and what evidence, reflections and conclusions may be gathered from this data (McNiff 2013, p.25).

This research project centred around the circulatory analysis of a vertical high school is to determine whether a student travelling from one class to another can do so within a given time limit. To achieve this, a pathfinding simulation that generates and analyses circulation data is needed. During the initial stages of the project it appeared the Grasshopper plug-in *PedSim,* would be the best method to generate circulatory data, however it was soon apparent there were functionality issues that made it unsuited for the vertical high school project. Most prominent among these was the inability for *PedSim* to function across multiple levels, as it only operates on the XY Plane. This would introduce inaccuracies into the script and, regardless, was impossible due to the long and thin shape of the floor plan, with a centralised stairway ensuring level collision when on a single plane.

Furthermore, due to the nature of high schools *PedSim* may not be required for the project. In a vertical high school, students are well-versed in their surroundings and do not require signage to move from classroom to classroom. One of the main benefits to the *PedSim* plug-in is the real-time response of agents to signage and stimuli within their field of vision, or the agents' wayfinding response, and this is not needed when the objective is to attain pathfinding data. Therefore, after its usefulness was deemed insufficient, *Pedsim* was abandoned in favour of a pathfinding-based simulation.

After reaching this decision and progress had been made towards the circulation simulation, the student-agents initially travelled from classrooms to recreation spaces, and travelled along a far more complex navigation mesh derived from the face edges of a Delaunay Mesh. This navigation mesh was later simplified to a curve network, and students instead travelled from classroom to classroom to enable timetabling testing and analysis. Analysis and ongoing evaluation of project iterations is evidence of action research in practice, and its effectual improvement of the final simulation's functionality and relevance to the context.

CASE STUDY

The case study investigates the efficiency of circulation in transitional spaces using vertical high school geometry provided by the industry partner, TERROIR. This is accomplished through the development of circulation simulation and analysis tools within the Grasshopper (GH) program. Grasshopper was chosen due to its flexibility in both base components and plug-ins, and because of the industry partner's preference of using the Rhino for building modelling, of which Grasshopper is a companion program. While the given context example is a vertical high school, and the script is designed with high school timetable utility in mind, this script may easily be adapted beyond the high school and into stair-reliant educational institutions, residential complexes, or commercial spaces as required.

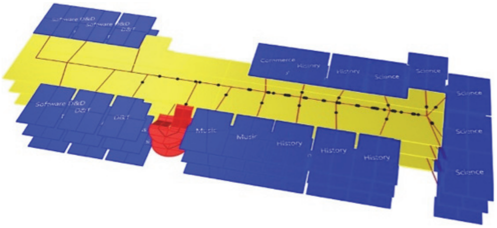

Figure 1
Circulation simulation in progress

Script Inputs

There are two types of inputs used in this Grasshopper script. Geometry, which is used for the creation of a navigation mesh and the definition of start and end locations, and slider inputs which are used for the loop and the time-based 'arrival analyses'. Inputs and values in the script are:

Geometry Inputs. Hallways, stairs, start location and end location, these are inputted as surfaces and are grouped according to their level.

Level Count (3). Defines the number of levels in the building, includes the ground floor.

Human mm/s (1400). The average speed of a human in millimetres per second. It is used in comparison to the 'Movement in Millimetre per Loop' input to determine how many seconds occur per loop and how many loops it takes to reach the maximum analysis time.

Movement in Millimetres per loop (350). The base distance travelled, in mm per loop iteration.

Seconds for Transition (150). The maximum analysis time. This value indicates the maximum time, in seconds, available to a student to reach their next class, however the simulation will continue beyond this point. Students that have not reached their next class by this time are highlighted by the simulation arrival analysis.

Students per Classroom (30). The number of students in every classroom, all classrooms currently have the same number of students.

Loop Iterations between Students (10). The number of loops waited between the first student leaving the classroom, and the second student.

Divide Curve Parameter for Navigation Mesh Division (23). The number of times the hallway centre curve is split. This is used for congestion analysis, where it is necessary to determine the number of students occupying a section of the corridor.

Origin Timetable Period (2). Designates the period that has just ended, and the origin classrooms for all students.

Destination Timetable Period (3). Designates the period that will soon begin, and the destination classrooms for all students.

From these slider inputs, the following is calculated:

(1) *Seconds per Loop* (0.25)

(2) Maximum *Loops to Destinatio*n Classroom (600)

'Seconds per Loop' represents how many seconds occur every loop iteration, and the *'Loops to Reach Destination Classroom'* is the number of loops required to reach 2.5 minutes.

Geometry Simplification & Classification

The vertical high school geometry (Figure 2) provided by TERROIR is a Rhino model representative of a possible floor plan arrangement, and its primary purpose is to prove the functionality of the Grasshopper script. In addition to the geometry, several vital pieces of data were given by the industry partner to ensure accurate simulation and analysis outcomes. These include the target storey count of 6, the total student count of 2160, the students per class count of 30, and the analysis timeframe of 5 minutes. To reduce prototype complexity, the storey count and the total student count have both been halved, to 3 and 1080 respectively. Similarly, and for the same reason, the 5-minute analysis timeframe has been halved to 2.5 minutes. When simplifying the school floor plan, it was split into three different areas: classrooms in blue, hallways in yellow, and stairways in red (Figure 3).

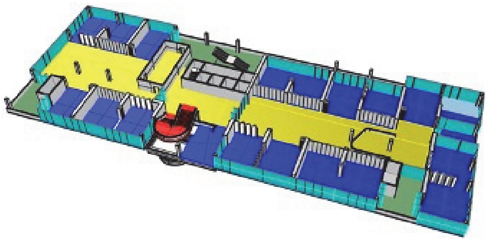

Figure 2
School geometry with hallway, classroom and stair surfaces overlaid – provided by TERROIR.

These were defined by creating surfaces from the rooms in the floor plan, not including walls. Both stairways and hallways were used for the creation of the navigation mesh, while classrooms were used for the origin and destination locations.

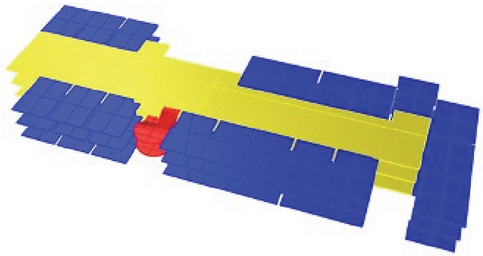

Figure 3
Simplified floor plan

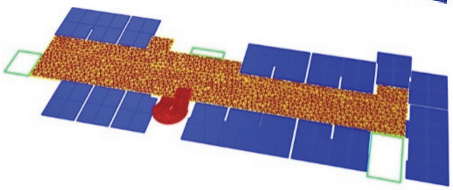

Figure 4
Delaunay navigation mesh

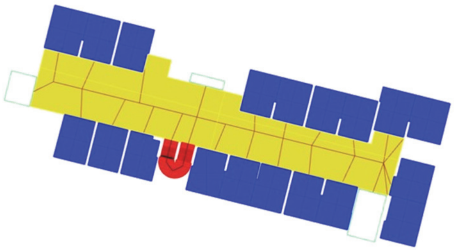

Figure 5
Simplified navigation graph

Graph Theory & Navigation Mesh

The problem of the navigation mesh was considered from a graph theory perspective, specifically from convex space analysis graphs. The primary reason for this, aside from the benefits in its usage as demonstrated in previous examples and research, was to simplify the later analysis of the circulation simulation. Navigation meshes based on Triangulated Mesh Edges, or the Delaunay Mesh algorithm, create needlessly complex hallway paths (Figure 4) that are also inefficient in their direction; to find the most direct path, smoothing these curves before or during navigation would be required. However, the simplification of this navigation mesh based on convex space graphs reveals the pathways shared by students between classrooms (Figure 5). Analysing the simplified path network is far easier to accomplish and makes the visual display of students occupying and congesting a space clearer to the user. Shared path curves were generated based on the centre line of hallway and stairway, which are divided a given amount, a default of 23 times, to assist with the later generation of shortest paths and simulation analysis. The stairway and the classroom entrances are then connected to the hallway centre line to complete the curve network. This was achieved by separating classrooms and hallways by their storey and connecting the classroom entrances to their closest end point on the hallway centre curve.

Applying the School Timetable

As master timetables for high schools are not publicly available, a school timetable was created in Microsoft Excel based on existing student timetables and necessary high school subjects, such as English, Maths, etc. Each of the six-year groups were separated into six separate streams, resulting in a total of 36 classes occurring at across the school. MS Excel was chosen due to its normalised use for high school timetables, and resulting in a total of 36 classes occurring at across the school. MS Excel was chosen due to its normalised use for high school timetables, and the low learning curve required for designers to make or apply changes to the timetable input without the need to understand Grasshopper. This timetable is imported into Grasshopper, and the origin period (P1) and destination timetable period (P2) are chosen from two input sliders at the beginning of the script. Along with the timetable, another Excel-imported input is the amount of each classroom type that exists in the school building. These are imported as two columns (list of classrooms / list of the amount of rooms allocated to each class). The classrooms are grouped across the 3 storeys and are distributed in a way that multiple classrooms of the same type are positioned on the same floor. From these lists of origins and destinations, and the placement of classrooms, the Shortest Walk GH component calculates the shortest path between the origin and destination along the navigation mesh. Only paths that are included in classroom transitions for the selected period are calculated, for example curves for movement from P3 to P4 won't be calculated if that is not the input given. After the Shortest Walk GH calculation, the outputted shortest paths are redefined, so their curve domain is equal to their length. Due to the combining and splitting of curves, this is not the initial case, but vital for calculating curve positions by parameter in the circulation simulation.

Circulation Simulation & Simulation Congestion Response

The circulation simulation operates as a loop within Grasshopper, a flow-based program. This loop calculates and saves two values — the *curve parameter* that represents the position of a student along their shortest path, and the *shortest path curve*. These are sorted in corresponding lists that update with every loop iteration. When creating the loop there were several inputs that had to be considered. Key to the generation of new curves and parameters were the number of students per classroom and the number of loops between people leaving a classroom. These needed to be accounted for to prevent an infinite number of students, and to create a realistic gap between departees. This was accomplished within the loop by calculating two equations to be true:

$$x < (y * z)$$
$$\text{and}$$
$$\text{floor}(x / z) = x / z$$

Where x is the loop count, y is the number of students per classroom, z is the number of loops between departing students, and *floor* indicates rounding to the lower value. The default amount a person moves every loop is set by 'Movement in Millimetres per Loop' input, with value of 350 mm. Yet, depending on the number of nearby students while the loop is running, as the degree of congestion the student is dealing with, this value will drop. One of the issues encountered when developing this section of the script was the simulation stalling due to a high number of people who were continuously slowing down as the congested traffic jam worsened.

To prevent this, a minimum travel distance was set to 100 mm, and now students will slow down only to this threshold, and people will continue to flow. The ability to enable or disable the congestion response is available in-script and allows the user to choose whether to have the students slowdown in highly congested areas or always move 350 mm with every loop iteration.

Simulation Analysis

The simulation is analysed and visually represented through three different methods.

(1) Congestion analysis, where the hallway surfaces on each level are divided along their cross-sections, and stairway surfaces are divided by their risings and landings. The number of people positioned on each of these surfaces is then displayed above the surfaces' centroid and updated with every loop iteration. Furthermore, as this number is increased the colour of the cross-

sectioned surface shifts from blue, to green, to yellow, to red (Figure 6).

Figure 6
Congestion analysis

This is defined as the minimum and maximum number of students that occur in the simulation. The congestion analysis is the primary method to convey the locations of bottlenecks to the designer, as to decrease their number in later project stages through design changes or timetable changes.
(2) Arrival analysis, where all origin surfaces are coloured blue at the loop start and change to red when all members from the origin's class have reached their destination. This provides a visual representation of a student's journey beyond their presence in the hallway, and in the case where a student is still travelling after reaching the time limit, the problem can be easily and immediately identified by the unchanged, blue classrooms (Figure 7).

(3) Year group analysis, where each student is coloured and labelled according to their year group. Colours can be assigned or randomised, and all students of the same year will be labelled the same. This allows the designer to understand the flow of year groups within the school during class transitions and can highlight parts of the school that attract specific year groups, i.e., if there are separate general learning spaces for each year group, this will be apparent from clusters of similarly coloured and labelled students.

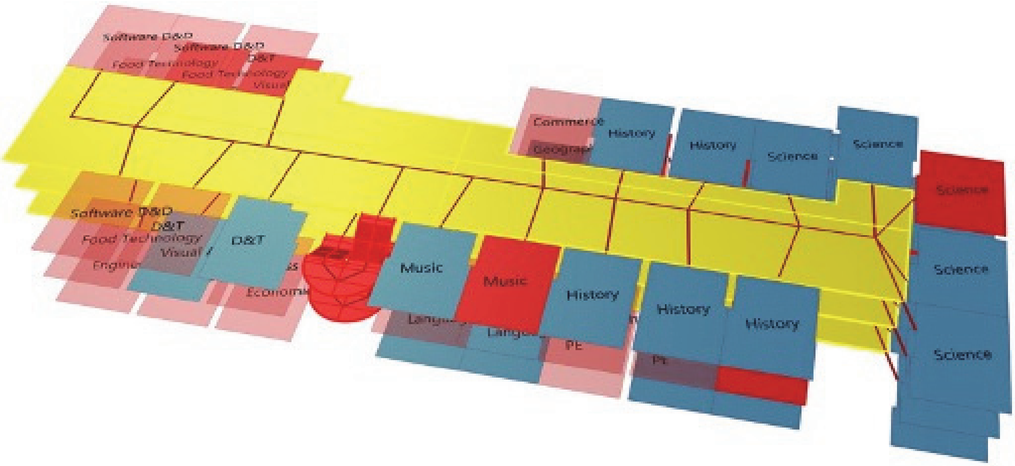

Figure 7
Arrival analysis

DISCUSSION

Through the circulation simulation and analysis, vertical high school designers are empowered to develop enhanced design solutions that reduce congestion and improve timetabling arrangements. The strength of the script lies in its flexibility, as it's able to be applied to other typologies such as shopping centres. In this scenario, the hallway, stairway, origin and destination inputs would need to change, but most inputs, for example the average human speed, would remain the same. Similarly, the timetable accepts generalised inputs, such as 'English' or 'Maths', where a student-agent is placed in the first classroom available, as well as specific inputs, such as 'English 1' or 'English 2', where each classroom is labelled with an associated number and

student-agents are allocated to specific rooms. Therefore, if the script is to be adapted to specific classroom names it will continue to function, or if there are multiple rooms with the same name and purpose there will be no issues that arise. The same concept would apply to other typologies.
However, while the base aims of the simulation were achieved, there are several areas where this project may be improved.

The script has been developed only to a point where all 30 people in every class leave and arrive from the same two classrooms. Meanwhile, a more realistic school set-up would have 30 students leave a classroom and split up into multiple classes for their next period. Additionally, this would require the individual high school timetables for a minimum of 1080 people, which wasn't within the available resources. Furthermore, the solution to simulation stalling under 'Simulation Congestion Response' did not account for all scenarios, as there would be real-world cases where students arrive at a near stand-still, and the 100 mm minimum movement prevents these scenarios from occurring. A set-up like the solution found in Hudziak et al. (2017, p.1568) would solve this issue, where temporary obstacles to navigate around appear when the congestion value in an area passes a given threshold. However, this does not work when there is a single path between classrooms. To expand the capacity of the script in this direction would require an overhaul of the navigation mesh and shortest path generation, as the referenced obstacle set-up relies on multiple pathways and a higher-level of complexity than is generated in the graph-theory-based navigation mesh used here. However, other limiters would arise with the complication of the navigation mesh, such as high-processing requirements, and long loop calculation times. The ability to rapidly process congestion data, create a simulation, and maintain the script's flexibility when applied to new building forms is significantly aided by basing the design on graph theory principles. Thus, while obstacle-based congestion would negate script stalling issues, the benefits are outweighed by the negatives of the extended set-up and processing times.

CONCLUSION

This research has explored the development of a computational workflow that aims to use circulation simulation and pathfinding analysis to empower the designers of vertical high schools to make evidence-based choices from generated data. Moreover, it has investigated how the use of graph theory principles can address the limitations of existing analysis methods for pathfinding and time-to-destination calculations. While this project focussed on analysis reliant on manual changes to inputs and geometry, with more time automatic input and optimisation methods could be added, where the timetabling process and classroom allocation is automated from the geometric and slider inputs. This is an addition to the script that would benefit designers and architects who are checking or justifying design feasibility, and is achievable if using a Revit model combined with Rhino.Inside Revit. Over recent decades, as the development of vertical high schools has gained popularity, the need for data-gathering and analysis catering to education has increased in tandem. Already architects and designers are required to support their design choices with related data, and this research enables the fulfilment of these requirements for vertical high schools.

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