Digital Expansion of Stereotomy

A semantic classification

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This paper presents a critical analysis and reflection on stereotomy with the purpose of updating its theoretical discourse. Having risen to the apex of architecture technological possibilities in the 17th century, stereotomic construction lost its importance in favour of iron, steel and other materials and construction techniques brought by the Industrial Revolution. More recently, much owing to the possibilities offered by digital technologies, a resurgence of interest in the subject has spawned various researches which bring stereotomy back to the architectural discourse. Although technological applications and design innovations in service of stereotomy have developed in multiple interesting paths, there is a lack of a common theory on the subject which is capable of relating these multiple apparently diverging stereotomic approaches between each other and, maybe even more importantly, to the classical practice which sparked the development this discipline. The research presented in this paper shows how the digital tools were instrumental in bringing this tradition to architecture contemporaneity and how a current stereotomy is largely supported by these technologies, while keeping strong relations to its classic origin.

Keywords: stereotomy, classification, history, digital

1. FROM CLASSIC TO CONTEMPORARY STEREOTOMY
Carefully cut stone for spanning spaces is first found in Greece, and later heavily used by Romans. Complex vaults of gothic cathedrals were possible due to the masons knowledge of cutting stone in the correct angles, often shrouded in mystery. The humanist Renaissance spirit of gathering knowledge and disseminating it to empower human endeavour is all present in Philibert de l’Orme’s Le premier Tome de l’Architecture (de l’Orme 1567); this book contains in its Livre III the first comprehensive treaty in the subject of coupe des piérres, a field of knowledge still not known as Stereotomy at the time. De l’Orme was interested in bringing the art of cutting stone out of the exclusive practitioners, the masons, into the creative act of the architect.

De l’Orme’s treaty was soon to be followed by many others, revealing a keen interest in the subject by the architecture elite. The beginnings of stereotomy saw the discipline be developed and taught along side others such as geometry, optics or
statics, revealing its wide scope of relations. This allowed for multiple innovations present in the treatises on stereotomy, namely the seminal engineering statics to be developed in the treaty of de La Hire, projective geometry to be inaugurated by Desargues, or the ubiquitous descriptive geometry in the teachings of Monge.

The development of stereotomy alongside descriptive geometry was contemporary to the beginnings of the Industrial Revolution, and the new double orthogonal projection found perfect suitability for describing machines and parts for production. Iron and steel soon became cheaper and solutions based in metallic beams and trusses became more common than cut stone structures. This caused a slow but steady fading of this classic construction technique.

The maturity of the Industrial Revolution developed a more productive and precise fabrication, providing a wide range of mass produced systems. A finer control of production allowed for more accessible accurate machinery, eventually assisting the development of the computer. Its development into programmable machines accessible by professionals such as architects sparked in the 1990’s the “digital turn”, a term used by Carpo (2013) to describe the changes in architecture design brought by the computer possibilities.

This opportunity was not overlooked by researchers interested in stereotomy and in Italy first Trevisan (1996) [2] and later Fallacara (2003) made the first approaches to stereotomy using digital tools instead of the classic methods. Following many other experiments, the current development of stereotomy cannot be described without acknowledging its resurgence due to the possibilities offered by digital tools. Computational design is therefore an unavoidable defining factor in today’s explorations of stereotomy, among other agents which contribute to the development of a stereotomic project.

2. EXPANDING STEREOTOMY BY REVISIONING ITS SEMANTICS

While classic stereotomy is easily definable within closed concepts such as stone, chisel or descriptive geometry, the current range of experiments make it more difficult to enumerate what are the defining concepts of this art. This apparent distance between current stereotomic practice and the framework present in the treatises creates a void in the theoretical support which once was the backbone of the stone cutting discipline. A critical look into two systems of stereotomy analysis present in a classic treaty (Frézier 1738) and in a contemporary thesis (Fallacara 2003), shall be surveyed under current constraints and possibilities. This analysis provides information for the generation of a classification for further understanding contemporary based in three main categories: Tomotechny, Equilibrium and Voussoirs.

Tomotechny

The Tomotechny category includes all the concepts tightly related to sectioning, be it physically or geometrically. The processing of stone to create the necessary voussoirs, once relying in manual handling of the mallet and chisel, is now dependent on machinery controlled by computers, narrowing the distance between the designer and the worker. In another level, within Classic Stereotomy, the technical drawing method remained two-dimensional, while today it is possible to design a whole structure within a virtual three-dimensional virtual environment.

Equilibrium

The Equilibrium category deals with all the design factors which contribute to the statics success of the structure. The statics calculation, found in Fallacara’s (2003) triad is also present in Frezier’s (1738) treaty, although it accounts to less than five per cent of the content, showing how intuitive and rule of thumb based was this subject during classic stereotomy. Today, its engineering knowledge is far greater, and it influences directly various design options through digital tools like RhinoVault which
implement graphic statics, or Kangaroo which uses the theory of hanging chains. The subdivision of a surface, essential to the correct load transfer between parts, is also a focus of research today, using computer iterative processes to achieve optimal solutions.

**Voussoirs**

The Voussoirs category accounts for the design possibilities within the stereotomic structure constituent blocks themselves. Most of the signifiers within this category, such as geometry of the contact surface, or structural performance of the material, find direct
homologues in classic stereotomy. However, even in such a timeless concept such as constructive block, the computer has given contributions to their development, such as using different materials thanks to digitally controlled deposition in additive processes, or bespoke contact surfaces with accurate CNC control of fitting features.

Using these three categories and its terms as starting points, it is clear how a direct relationship to classic stereotomy may be formalised. In the attached image, a more developed classification based in the three main semantics is presented, hinting at the variety and richness of its sub concepts. The classification is hierarchical, providing for a logic location of stereotomy related concepts. Each of the three main categories is assigned the initial letters A, B and C. All the main terms inside each of these initial categories is assigned sequential letters, resulting in “AA, AB, AC, etc.”, and further into “AAA, AAB, etc.”

3. APPLICATION - CASE STUDIES

In this section, two case studies of stereotomic design will be subject to an analysis under this semantic classification. Besides unveiling changes and permanences from classic to current stereotomy, this classification allows for an identification of processes and properties of stereotomic design, effectively allowing for a clearer definition of stereotomy today.

3.1 Arles City Hall

General description. The Arles City Hall entrance vestibule vault is repeatedly referenced within the context of French high stereotomy for its sensitive integration within the whole project and bold ratio of span to rise. This project, ultimately realised by Jules Hardouin-Mansart in 1676, has a convoluted history. Initially planned by La Valfenière, the project was discarded and eventually demolished because a neighbouring house was acquired by the council, in order to enlarge the overall footprint. While the new project was taking shape, Mansart visited the city and was invited to comment on the project. This resulted in numerous modification proposals, most notably in the large hall in the ground floor.

Regarding this hall, consisting of a large vestibule with four central columns, Mansart is said to have expressed “qu’il avoit quelque chose de mieux à faire” (Etlin 2009). This feeling resulted in the project of the low vault with no middle support which we can see today. This vault was finalised by Jacques Peytret, a local architect (and also documented as a painter and painter-architect), who was given plans by Mansart so that he could have the stones cut in the relevant way to achieve Mansart’s vision. However, this vision was not totally realised, as Peytret was found to raise the vault without Mansart’s consent. It is also suggested by Etlin (2009) that the double vaulted corners were changed from original trompes. Notwithstanding these changes, the built vault is impressive for with swift movements connecting the columns which bring the weight to the ground, even if only figuratively. Together with its proportions of 16 meter width to a shallow 2.43 meter rise, a remarkable feature of this vault is its naked aesthetics Pérouse de Montclos (1982), which is much closer to a contemporary expression of architecture than the profusely decorated vaults of Spanish or Italian tradition. The surface of the center part of the vault looks as one continuous flowing surface, similar to concrete shells of the mid-20th century, instead of overly euclidian arches and domes of a previous renaissance period.

Semantic analysis. Following, a systematic analysis of the Arles city hall vault will be taken according to the semantic structure exposed in the previous section.
Figure 3
Bottom up perspective of the Arles vault, showing the vault in almost its entirety. 3D projection created by the author from full range panoramic view provided by [4].

Starting with the same structure of the previous chapter, the vault will be analysed starting from the Tomotechny (A) category. Most certainly the materialisation process (AA) for the production of the voussoirs in the Arles vault was achieved by carving (AAAC) stone with instruments related to the chisel, a manual and mostly sculptural way of subtracting (AAA) material from stone (CACA). Although this is probably true for most of the voussoirs, Tamborero (2003) suggests that the first rows which make the connection to the perimeter wall are corbelled and thus should have been square cut. This hint to the possibility of the usage of a saw (AAABB) for achieving planar cuts.

Regarding the first level of classification within centering (AB), its ephemerality, which could range between Temporary, Inexistent or Permanent, is clearly a Temporary Centering (ABA) in this case. Most likely built in wood truss-like support structures below (ABAA) which stones would be laid on, the literature finds evidence of a staged construction in which a main arch is first erected. This shows evidence of elected guide lines along which the vaulted construction would depart from, of which Tamborero (2003) is assertive to suggest its specific order: the periphery arches and the twin lunettes, the large arch, the small vault with the large entrance lunette and finally the large vault.

The drawing method used by Mansart to designate the vaults’ surfaces and, consequentially, the voussoir shapes is based on the trait, a series of geometrical operations which would forerun descriptive geometry. This is more over stressed by the words found in the archives (Boyer 1969) such as “(Mansart) baillera le trait a celui qui le conduira”, or “les instructions modèles et panneaux pour lesdits bâtiment et voûtes”, which clearly reference the traditional drawing methods used in stereotomy prior to Monge’s innovation. Although the drawing seen in is not by Mansart or Peytret’s hand, it is illustrative how the drawing method within this project is limited to the two dimensional realm and, although the final solution is three-dimensional, the prerogatives always depart from planar compositions which are then projected along straight or circular directrixes.

In the chapter of Equilibrium (B), the first signifier analysed is that of macro-shape (BA), further subdivided in generation method (BAA) and continuity (BAB). The vault was designed with top-down (BAAA) strategies, using established curves (such as the circular arc, or the five center basket-handle arch - anse de panier). These curves are used as generatrices isometrically translated in space, thus exhibiting a constant profile (BAAB). There is one exception regarding isometry, the central lunettes seen in red in, whose generatrix grows in order to create an interception line which promoted the reading of the V shape which seems to separate the large vault from the small vault. The continuity of the total vault surface is clearly segmented (BABA) with 17 independent surfaces which connect with each other in ridge edges.

The structural functioning (BB) of this vault is based in compression resistance (BBA); this signifier is connected to the material (CA) which will be discussed below. This kind of structural behaviour is only efficient in certain conditions, one of which is the direction of the contact surfaces (CC), whose geometry is directly dependent on the surface subdivi-
sion (BC) design. Within this signifier, it is possible to identify a clear separation of courses (rows) of stone (BCBA), predominantly running in horizontal paths. The design strategy could be that of dividing the profile curve of each surface in segments which translate into the parallel courses constant width; these courses are then subdivided in voussoirs of varying length, prioritising available stones blocks sizes. This surface subdivision in two different steps fits within the multiple level subdivision (BCB) signifier.

Another semantic within the equilibrium group is the foundation (BD), which describes the strategies used in the contact between the vault and the rest of the architecture structure. The vertical continuity of the vault surface seems to transfer the load to the columns which surround the space, providing the whole ensemble with a subtle lightness which lift the weight of the large stone ceiling. However, the columns are hardly the sole supports of the vaults, being the thick stone walls (BDAD) behind these which support the vaulted stone structure. The springer (BDB) voussoirs, although corbelled and exhibiting square angles in their extrados, have they geometry (intrados subdivision design) in continuity (BDBBA) with the rest of the structure.

The final semantic group to be discussed is that of Voussoirs (C). Within the material signifier (CA), sustainability (CAA) is analysed with standards contemporary with the vault construction. Being a pre-industrial era, the extraction (CAAA) and processing (CAAC) of raw material was not characterised by significant embodied energy, giving hints to the suitability of the construction method at the time. As expressed while analysing BB, the stone structural performance (CAB) is mainly due to its compression resistance (CABA) and significant weight (CABC). Stone itself fall into the subtractable materials (CACA) typology (CAC) for its volumetric nature upon availability, ready to be cut and carved in order to achieve the intended voussoir shape.

Regarding the geometry of the voussoirs, there are two main categories: intrados and extrados (CB), and contact surface (CC). Regarding the first, the surface geometry (CBA) is mainly characterised by its double curvature (CBAA) in the end of the large vault and small vault. The double curvature is easily identifiable by the curved intrados perimeters (rows) in , where single curvature (CBAB) is present in the lunettes, identifiable by the straight lines in the courses. The perimeters (CBB) themselves are also important regarding many materialisation methods; in this case, we may only observe the intrados perimeters, which exhibit convex (CBBAA), as well as concave curves (CBBAB). Although the extrados is not accessible, stereotomic tradition at the time suggests the extrados is analogous (CBBBA) with the intrados, even if of a rougher nature. The contact surface of these voussoirs are mainly ruled (CCAA), although the circular and linear nature of the subdivision produces also planar (CCAB) and mixed, composite contact surfaces (CCAC). The execution of this vault is very accurate, discarding the need for gap filling mortar; however, it is noticeable a thin mark of material between voussoirs, whose purpose should mainly be related to binding (CCCB) blocks together to facilitate the construction process - despite the binding purpose, every mortar is always efficient in gap filling (CCCA), even if in a micro-scale. Friction itself (CCB) is mostly related to the fine texture of the contact surface, which is hidden behind the vault intrados surface. Following the recommendations present in most stereotomy treatises, it is expected that the contact surface is left rough (CCBB) enough to cause large friction between the voussoirs and contribute to the stability of the ensemble.

3.2 Armadillo Vault

General description. The Armadillo Vault is an experimental construction built for the 15th International Architecture Exhibition - La Biennale di Venezia 2016. It’s key feature lies in the coverage of a large space with stone elements not binded by any adhesive or hardware, adding to the expressive sinuous forms which result from the performative shape. This work is a super demonstration of the knowledge accumulated within the Block Research Group (BRG),
creating the most complex stereotomic surface to date. This complexity, present in the amount of different voussoirs, free form surface, and extreme accuracy of fabrication and assembly, is counteracted by a seemingly simple and graspable curvilinear form which seeks to resolve a space within the large Venice building.

The vault is designed with the main purpose of showcasing the possibilities of contemporary stereotomic construction and material efficient equilibrium structures supported by TNA (Thrust Network Analysis), the framework developed by Philippe Block (2009) based in graphic statics which allows for a precise modelling of structure in equilibrium, be it compression or tension. The application of this design method to stereotomic constraints such as subdivision and voussoir materialization is largely developed by Matthias Rippmann (2016) in his PhD thesis.

This structure was built by the Escobedo Group, a texan masonry company which works closely with Block since at least 2009. This relationship continued with the research on the MLK Jr. Park Vault, a bold stereotomic structure which, although provided much of the pretext for Rippmann's research, still did not come to realisation. As such, the Armadillo may be understood as a sort of proof of concept for this kind of construction approach in the contemporary context, having fully proved its validity.

Contemporary freeform stereotomic construction with stone is not a first, as can be seen in the works of Fallacara or Yoon and Höweler. What is impressive about this vault are the numbers, which in the end allow for the integrated complexity of the whole. The vault is composed of 399 bespoke stone voussoirs weighing a total of 23.7 tons, whose thickness range from 5 to 12cm. These thicknesses are impressive knowing that the largest spans exceed 15m, making the thickness to curvature radius comparable to that of an egg-shell. These extreme in material economy is justified by the team by “the prescribed weight limitations on the floor of the exhibition space in the protected building”, but also important for reducing building embodied carbon and energy, together with making construction costs competitive: this can only be achieved for the compressive only nature of the construction systems, highly optimised with the TNA approach.

Semantic analyses. The Armadillo vault, being an exceptional work of stereotomy in the contemporary era of this discipline, is a very adequate example to analyse under the semantic classification.

In order to understand the shape of the Armadillo vault, the Equilibrium (B) category provides the key signifiers for this task. The macro-shape (BA) is obtained with RhinoVault, a plugin developed for the 3D computational modeller (ACBB), which is an application of TNA, based in graphic statics (BAABB). Signifiers starting with BAAB indicate that the generation method was bottom up, explaining the seemingly free form shape of the vault. This shape is
continuous (BABB), meaning there are no tangential breaks, which would hint to multiple single surfaces sharing edges. One of the reasons for such a non-euclidian shape is the design team’s purpose of reducing the shell thickness to its minimum, within the 5 to 12cm range. This means that there must be no location in the intrados or extrados set more than 6cm further than the thrust surface, much as an arch must contain its thrust line within its volume. Being the thrust surface a complex network derived from catenoids, using circle arcs would force the thickness to values deemed too large for the thin design intent.

This construction is composed of stone blocks pressed together in a dry joint, without resorting to any kind of adhesive or mechanical fixations. This is only possible due to compression only (BBA) structural functioning (BB) of this structure, which is dependent on the correct orientation of the contact faces. This orientation is given mainly by the surface subdivision (BC), which ensures that the contact surfaces (CC) of the voussoirs (C) are as perpendicular as possible to the thrust vectors. In this case, a multiple level subdivision (BCB) was chosen, mos specifically a division in courses and voussoirs (BCBA).

The foundation (BD) characteristics of this project are very particular due to the constraints related to protecting the historic flooring of the Corderie. As such, instead of inserting foundation piles, the usual outwards thrust of the springers was retained with tensioned (BDAC) cable ties between themselves. These springers are large crafted steel elements (BDBAB), whose unique geometry (BDBBB) is capable of absorbing up to 11 voussoirs.

The solutions adopted in the voussoirs (C) are key to the success of this construction. Their material (CA) is stone, a subtractable material (CACA) materialised by two 3D cut (AAAB) processes: circular saw cutting (AAABB) for the intrados rough cuts and approximation cuts for the contact faces, and milling (AAABA) with specially crafted bits for the contact faces finishing. Stone has low embodied carbon and energy mainly in its processing (CAAC) which is nearly negligible, and plays excellent structural performance (CAB) in what comes to weight (CABC) and compression strength (CABA). While the extrados surface (CBA) is planar (CBAC), the rough cuts in the intrados provide a textured finish (CBAD), one of the aesthetic signatures of this work. Contact surfaces (CC) are ruled surfaces (CCA) which feature a registry notch mainly for alignment purposes (CCDA).

Within Tomotechny (A), the signifiers technical drawing method (AC) and materialisation processes (AA) were already discussed above. Regarding centering (AB), the Armadillo vault was erected thanks to multiple temporary (ABA) extruded grids (ABAAAA). These curved wooden lattices were supported on standard scaffolding, and the voussoirs were laid on top of shims which allowed for more flexibility while assembling.

### 3.3 Comparison between classic and contemporary stereotomy

One of the possibilities offered by a semantic classification framework, besides the already intrinsic analysis of a stereotomic work, is the structured comparison of two different works, even if separated by 340 years. As such, the semantic classification will be used to understand changes and permanences from the Arles vestibule to the Armadillo vault. The graphic in allows the immediate grasping of differences and similitudes between the two analysis.

Most of the graphic is purple, revealing the common characteristics which understandably include the broader terms, such as Voussoirs, or Materialisation Processes. As the branching goes farther from the center, differences start to emerge.

Out of these we highlight as differences the tools used for Materialization - AA (chisel and mallet vs CNC sawing and milling), the Technical Drawing Method - AC (trait vs computational model), the Continuity of the vault surface - BAB (segmented vs continuous) and the Generation Method - BAA (top down vs bottom up).

On the other hand, the similitudes are shared between Subtractive Materialization Processes - AAA, similar Centering strategies - AB, same Structural
Functioning - BB and Surface subdivision in Courses and voussoirs - BCBA. The common usage of stone for the voussoirs is reflected in all the signifiers under Material - CA, whose voussoirs both present Convex - CBBAA and Analogous - CBBBA perimeters of Intra-dos and Extrados.

These two compared architectural works are set apart by 340 years, an extended time range which feels even larger due to the exponential growth of technology (Kremer 1993). Notwithstanding the expected change of paradigms, the amount of similarities between the two works is understandably rooted in their common raw material - stone - which predates even human history. The intrinsic properties of this material and its related signifiers are maintained, such as the compression structural performance, or the subtractive fabrication methods. Philibert de l’Orme is quick to exalt the qualities of stone construction, justifying the need for stereotomy as a means to build large spans with limited sized elements such as voussoirs. His arguments are still valid today as can be seen in the structural stability of the Armadillo Vault or the fire resistance of the Notre Dame of Paris vaults under the 2019 roof fire.

By distancing ourselves from stone as the main common denominator of both vaults, the changes are rooted in the possibilities provided by digital technologies. The computer is an essential part of current electronic control of machining tools, actually being responsible for the current possibility to build stereotomically today; the digital design / digital fabrication flow has taken the void place left by the disappearance of the vault designer / stone mason since the introduction of steel in construction.

Besides this relevant part taken by the computer in the resurgence of stereotomic design, there is another important role taken by digital technologies which is that of shape. Missing powerful calculation tools led classic designers to resort to known shapes, or derived surfaces from these, accounting for the abundance of circles and “basket handles” composed of up to seven arcs of circle. On the other hand, computational power led to the creation of discrete meshes and Bézier splines which are used by bottom up processes to calculate unimagined shapes which satisfy the requirements of the designer. One of these requirements might be the continuity of a vault, something trivially found in an unified mesh under a common network of forces, but not so immediate as can be seen in the triangular separation of the big and small vaults of Arles.

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<th>ARMADILLO VAULT</th>
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<td>AAAB/AAABA ABAAA ACBB</td>
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<td>VOUSSOIRS</td>
<td>CAAA/CACB CABA/CABC CABA CBAB/CBABA/CBBBA CBBBA CCA/CCAB CCBB CCCB/CCCA</td>
<td>CAAC CABA/CABC CACA CBAC/CBAD CBBBA CBBBA CCAA CCBB CCDA</td>
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Figure 6
Summary of the results of semantic analysis of both projects.
4. CONCLUSION
The surveying of the two case studies above is instrumental in identifying similitudes and contrasts between two stereotomic structures whose design and construction dates are set more than three centuries apart, with Industrial and Digital Revolutions between them. A clear set of permanences are identified, such as stone and the compression only nature of the structure, while differences arise in the generation method of the macro-shape or the tools used to dress the stone. The main differences detected in the analysis of a flow from classic to contemporary stereotomy are directly or indirectly related to technological developments made possible due to the digital possibilities. Stereotomy, being the art of construction with carefully shaped blocks which support each other, gains its built expression from its direct constraints and related meanings. These semantics become the definers of an augmented stereotomy, greatly expanded today with digital technologies. The use of a structured semantic classification of stereotomy is instrumental in reading and supporting this on-going evolution.

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