Design Guidelines for Zero Waste Manufacturing of Freeform EPS Facades

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The application of curved facade designs in contemporary architectural practice has become adamant in combining the digital tools with the material properties. By expanding the focus to manufacturing as well, the topic of waste is introduced. In order to avoid the generation of waste material during fabrication, in this research a workflow is introduced which describes the design of freeform surfaces out of expanded polystyrene blocks (EPS), while producing zero waste. The main premise is that a piece cut out of an EPS block has a piece that is left inside the block, its complement. Following the premise, it is only necessary to design one half of the freeform surface over a desired facade area and the other part would align to it. After the freeform surface is generated, a tessellation process is described, prepared for robotic hotwire cutting, following the limitation of the EPS block dimension and the inclusion of the minimal insulating layer.

Keywords: freeform surface, ruled surface approximation, minimal insulating layer, complements

1. INTRODUCTION

In contemporary architectural practice, a myriad of architectural design solutions strive towards the application of freeform surfaces as building envelopes, incorporating digital design, techniques and fabrication to achieve dynamic design features (Pottman 2007). Such an approach is highly related to material choice, establishing it as an integral part between the design phase and the fabrication phase (Gramazio and Kohler, 2008). The implementation of foamed polystyrene as a material, has proven its applicability in the architecture of volume (McGee et al., 2013) going beyond its insulating purpose, and focusing on the material treatment (Brander et al., 2016) and its load bearing and aesthetical properties. However, merely focusing on the latter without implementing sustainability principles is insufficient, given the limited resources that exist and the impact that the construction industry has on the world (Juan & Habert, 2017). The research indicates that the relative sustainability of the projects is influenced, amongst many, mostly by the area of material waste management. The word “waste” normally emphasizes something around us which should be recycled, reused, reduced or even eliminated (Singh et al., 2017). While fabricating the RDM Vault (Verde et al., 2011) out of Expanded Polystyrene (EPS), the focus was placed on exploring the fabrication con-
The existence of a cut out part shown in red and its complement that is left inside the block, shown in blue; b) the depiction of the red and blue parts inclusion in the final freeform surface as complements

The term zero waste is introduced to outline an approach with aims to “eliminate” rather than “manage” waste (Curran and Williams, 2012). The term zero waste manufacturing (ZWM) is introduced, representing an approach that encourages the manufacturing of building elements without producing waste. Striving towards such an approach is represented by nesting the fabrication parts for laser cutting or CNC milling after the design phase to minimize the free space left by the fabrication process (Koo et al., 2017). Even though the waste material is minimized, it still exists. In the field of freeform concrete casting, the application of ZWM is recognized in the usage of wax casts (Oesterle et al., 2012), reconfigurable molds (Raun et al., 2011) and precast membranes (Belton, 2012), to completely eliminate the waste that custom made falsework can produce. If ZWM is introduced as a design driver early on, it becomes possible to fabricate interesting design solutions out of flat sheets of wood without contributing to waste [1,2].

The aim of this paper is to generate a workflow that incorporates ZWM as a driving factor for freeform facade design, but for volumetric fabrication i.e. for robotic hotwire cutting of EPS. Generating ZWM EPS freeform facades can contribute to the aesthetical appeal of the building, while at the same time, performing its main function as an insulating layer of the building and eliminating waste. The workflow includes:

1. generating a specific freeform surface
2. tessellating it into parts that can be:
   - cut from whole blocks of EPS without waste
   - prepared for robotic hotwire cutting
   - aligned to approximate the freeform surface as accurately as possible given the time/appearance ratio
3. fabricating the parts using robotic hotwire cutting

All the aforementioned areas are integrated in the design process early on, however, in this paper the emphasis will be on the first two.

2. GENERATING A SPECIFIC FREEFORM SURFACE

The freeform surface cannot be generated at random. Following the tessellation criteria - the fact that each part of the freeform surface is cut out of a whole EPS block without waste (Fig. 1a red part), it is safe to conclude that each cut out part has its complement that is left inside the EPS block after the cutting process (Fig. 1a blue part). Therefore, one part of the freeform surface corresponds to its complement (Fig. 1b). This fact allows for the design phase to be focused on gen-
2.1. Designing a Specific Freeform Surface on a Facade without Fenestration

The freeform surface is constructed as a network surface from two sets of guideline curves placed in mutually perpendicular planes. Before the curves are generated, the facade area to cover needs to be defined. A facade area is required to be a planar quad with a right angle between adjacent edges. Following the condition that only one half of the freeform surface needs to be generated, the next step is to determine one half of the quad in reference to one symmetry axis (symmetry axis $y$ in Fig 2a for example) and construct a box over that area, by extruding the quad half (shown in bolded lines in Fig 2a). The introduction of the box aids in explaining and determining the placement and orientation of the guideline curves. The extrusion amount of the box determines the bounding box of the freeform surface and the extremes of the guidelines curves in respective planes. After the box is generated, the box sides that are perpendicular to the facade area and parallel to the symmetry axis $y$ are chosen (Fig 2b). The side located at the perimeter of the facade area is chosen for drawing the director curve $a$ (shown in red in Fig 2b).
In order to fulfill the condition of complementary surface halves, curve $a$ is translated perpendicularly from the current box side to the other one and rotated by 180 degrees around the translation axis forming the curve $a^t$ (shown in red in Fig 2b). Afterwards, a desired number of curves, following the same boundary condition, are placed in parallel planes in between the curves $a$ and $a^t$, (shown as curve $b$ and $c$ in Fig 2c). In order to avoid a large deviation close to the boundary of the surface i.e. large curvature changes, inner curves are placed at the same distance from one another. These curves represent the first set of curves.

Lofting these curves into a surface would only satisfy the $C^0$ continuity amongst the complements. In order to generate a more smooth transition between the complements, i.e. a $C^1$ continuity, a second set of curves is introduced, whose tangent vectors at start and end points are parallel to the symmetry axis $x$. In such a manner, the surface transition is smooth at the connection between the complements. In order to generate such a curve set, two box sides which are perpendicular to the facade area and parallel to the symmetry axis $x$ are chosen (Fig 2d). A desired number of parallel planes are generated in between (shown as $\alpha_1, \alpha_2, \ldots, \alpha_{n-1}, \alpha_n$ in Fig 2e). These planes intersect the first set of curves and generate a set of points for each of the planes (shown in red Fig 2e). In order to obtain the $C^1$ continuity between the complements, for each set of points on a given surface, two points are added by translating the points located on curves $a$ and $a^t$ along the symmetry $x$ direction towards the middle of the surface area, respectively (shown in green vector on Fig 2e). Using the updated point set for each surface as control points polygon, a NURBS curve is generated (Fig 2f), forming the second set of curves. By generating a surface network from these two curve sets, a freeform surface half is generated (Fig 2g). Since the surface generation process can cause certain areas of the surface to protrude outwards from the bounding box, it is necessary to scale the surface perpendicular to the facade area if such a situation occurs.

The surface half is then translated along the symmetry $x$ axis direction and rotated by 180 degrees around the same axis to fit in the other half of the facade area (Fig 2h). However, since the facade should satisfy both aesthetical and insulating properties, it is necessary to determine how both can be incorporated in the design phase.

### 2.2. Incorporating the Insulating Layer in the Freeform Surface Design Phase

Even though the freeform surface half is generated so that it automatically has its complement, in order to increase the applicability and the functionality of such facade designs, a minimal insulating layer is introduced. The minimum insulating layer thickness, which represents the minimum amount of material between the freeform surface and the facade area is governed by the dimensions of the EPS blocks (the width and the length) available on the market. Usually, these dimensions are 100x50cm with a thickness that varies. Hence, the amplitude of the guideline curves is limited, which can be seen through a simple formula:

$$c = b - 2 \cdot i$$

where $c$ represents the curve amplitude, $b$ represents the block width and $i$ represents the insulating layer thickness. The hotwire cutting kerf width is omitted here due to the large scale of the surface.

The amplitude limitation is further emphasized through the condition that the block width value $b$ can only be a factor of the available dimension of 100cm or 50cm, meaning that the amplitude is not something that can be freely adjusted, but merely chosen, given the size of the EPS block (Table 1). As can be seen in Table 1, for a minimum insulating layer thickness of 12cm, as is the domestic standard, the freeform surface amplitude is set to either 26cm or 9.3cm, excluding the values from 100 or 25cm block dimensions as too large or too small, respectively.
Table 1
The relation between the thickness of the insulating layer on the left, shown in bold, the dimension of the EPS block on the top, shown in bold, and the available amplitude shown in regular font.

<table>
<thead>
<tr>
<th>Thickness of the Insulating Layer [cm]</th>
<th>100</th>
<th>50</th>
<th>33.3</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>50</td>
<td>33.3</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>96</td>
<td>46</td>
<td>29.3</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>92</td>
<td>42</td>
<td>25.3</td>
<td>17</td>
</tr>
<tr>
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<td>10</td>
<td>80</td>
<td>30</td>
<td>13.3</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>76</td>
<td>26</td>
<td>9.3</td>
<td>1</td>
</tr>
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</table>

In order to allow for a certain influence of the amplitude value, it is possible to preinstall an insulating layer of a particular thickness (in reference to the available EPS block thicknesses that exist), which when coupled with the parts generated in this approach, can produce the desired solution. For example a preinstalled insulating layer of 5cm requires another 7cm of minimal insulating layer, meaning that the amplitude can now be 19.3cm or 11cm, which creates more design opportunities.

Once the freeform surface has been generated, it is necessary to tessellate it according to the EPS blocks that are chosen for the tiling process.

3. THE TESSELLATION PROCESS
The usual practice when fabricating freeform surfaces is to approximate the surface to a certain degree, either through planar faces or ruled surfaces which deviate from the initial curvature, but make the fabrication process easier and time efficient. In this research, the approximation of the initial freeform surface half (Fig 3a) is done in two phases. The first phase serves to produce ruled surface ribbons, which can be cut by a straight hotwire cutting tool (Fig 3b). The main parameter here is the width of each ribbon, which is in reference to the thickness of the EPS block available on the market (10cm, 5cm, 2cm and 1cm). The second phase is necessary to further subdivide the ruled surface ribbons into patches that correlate to the size of the EPS block, mainly to the width and the length of the block (Fig 3c).

In the following sections, both phases will be explained in detail.

3.1. Tessellating the Freeform Surface into Ruled Surface Ribbons
As stated before, the process of subdividing the freeform surface into ruled surface ribbons is influenced by the thickness of the EPS block. Approximating the freeform surface with thicker blocks, for example 10cm, requires a smaller number of blocks when compared to the 5cm thickness. This in turn produces an overall shorter cutting path length, which means less fabrication time. However, the approximation of the freeform surface with wider ruled surface ribbons can lead to a certain degree of dis-
crepancy between the approximated and the initial freeform surface. Given that these two factors - the cutting path length and the approximation discrepancy are mutually conversely monotonous functions (one increasing the other decreasing), it is important to test their relation. Since the data acquired from a single freeform surface design can be inconclusive, 100 examples are generated for each EPS block thickness.

The test is carried out in the following manner. The facade area of 100cm x 260cm is chosen for generating the freeform surfaces. Such an area represents a large enough portion for acquiring the necessary information, while at the same time representing a significant portion of commonly used facade areas. By following the procedure explained in section 2.1., a myriad of diverse freeform design solutions is designed fairly quickly by changing the amplitude of the guideline curves and their shapes. The amplitude of the guideline curves is set to be from 10cm to 30cm, since values lower than 10cm offer a small amount of space for adequate surface curvature depiction, while anything more than 30cm can be too much for facade application. Such surfaces are then approximated by ruled surface ribbons of predefined thickness (as stated in section 3).

For each surface design, two data sets are extracted from these ribbons - the total cutting path length and the highest discrepancy value between the approximated and the initial surface. The total length is calculated by adding the length values of one of the ribbon’s directrices (shown in red in Fig 4a). On the other side, in order to calculate the discrepancy, a grid of points is generated on the ribbons, at a distance of 1cm in the isoparametric directions. Closest points to this grid are found on the initial surface and the distance between the two sets is measured (shown in gradient colors in Fig 4b). The peak value is extracted for the entire ribbon set.

With the total length and the discrepancy peak value determined for each of the 100 surfaces, an average value is calculated for each factor in order to use the data for graph drawing in reference to the block thickness (Fig 5).

As the graph in Fig. 5 shows, there are two trendlines. The red trendline shows a gradual decrease of the total cutting path length as the thickness of the EPS blocks increases. The data shows that 280m is an average cutting length for thinnest blocks, while almost 10 times less is necessary for the thickest blocks.
On the other hand, the blue trendline shows a significant discrepancy increase as the EPS block thickness increases, showing that the average discrepancy for 100mm thick blocks is almost 13mm. The highest peak value recorded in this test is 32mm, for the surfaces with the highest amplitude, meaning that using thickest blocks for approximating freeform surfaces with 30cm amplitudes can cause a discrepancy of over 3cm which can be significant. By observing the two trendline’s intersection, it is possible to conclude that the application of 50mm thick blocks provides the best compromise between the necessary cutting length and the approximation discrepancy. After interpreting the best block thickness choice for a specific freeform surface approximation, it is necessary to determine the size of the ribbon patches that can be cut out of whole EPS blocks.

### 3.2. Subdividing Ruled Surface Ribbons into Smaller Patches

The second tessellation phase entails that the entirety of the ruled surface ribbons (Fig 6a) should be subdivided into smaller patches so that each patch can be cut out of an entire EPS block and in compliance with the data from Table 1. For example, if a 50cm block width is used, its length is 100cm, meaning that the ribbons have to be subdivided in reference to this specific length. The second tessellation can be done in two approaches - the aligned and the staggered stack.

The first approach entails that each ribbon should be subdivided into patches so that the joints from adjacent ribbons are aligned as much as possible (Fig 6b). The aligned stack enables the connection of multiple parts in a larger panel, for example 100cm x 100cm, which makes it easier for facade application. This means that each ribbon is iter-
At the specific length is not a factor of the total height of the facade area, for example the length is 100cm, while the facade area is 260cm high, it means that only a certain part of an EPS block is cut, not the entire length, (leaving a block part of 40cm length). If the entire facade area were to be done in such a manner, a lot of EPS block parts would be left with a length smaller than the specific length thus limiting the future fabrication to using blocks of such size. In order to avoid such a scenario, the left out parts, and their specific length are further used for cutting the patches out of. Fig 6a shows the primary cut out parts in red, the secondary cut out parts in green and the tertiary cut out parts in blue. The primary parts indicate that the pieces were derived by cutting the initial specific length block into parts of equal or smaller length. The green parts are derived by cutting the primary left over parts into pieces that fit in the tessellation, while the blue parts are derived by cutting the secondary parts into even smaller pieces. Using the above described method on a facade area of 100cmx260cm, with a ribbon width of 10cm, the total amount of parts would be:

- 20x100cm primary parts
- 6x60cm primary parts
- 4x40cm secondary parts
- 4x20cm tertiary parts.

In the second approach, the focus is placed on generating staggered joints (Fig 6c), which can offer a better connection consistency when compared to the aligned joints. However, this stack makes it difficult to produce smaller, easy to navigate and assemble panels as the previous approach, taking into account the jagged panel edges that can occur or the necessity to join large pieces before facade application. The benefit is that this approach mitigates the existence of tertiary parts. The procedure is done similar to the prior approach - by iteratively dividing the ribbon in reference to the specific length. However, once the cut out part is left, the difference is carried out to the adjacent ribbon before continuing with the application of the general specific length. The entire process is repeated for the entire designated facade area. In the case of the same facade area of 100cmx260cm, with a ribbon width of 10cm, the total amount of parts would be:

- 18x100cm primary parts
- two of each - 20cm, 40cm, 60cm and 80cm primary parts
- two of each - 20cm, 40cm, 60cm and 80cm secondary parts.

As can be seen, both approaches generate the same amount of parts. The length of cutting path is the same, since the height of all ribbons adds to the same value. Given that the ribbon patches are determined...
by choosing either of the two approaches, the process of designing and tessellating a ZWM freeform facade is thus concluded.

4. CONCLUSION
In this research, the aim was placed on generating a freeform surface out of EPS blocks while producing no waste during fabrication. The important aspect was using an integrated design approach, where the surface generation and tessellation are closely related to the block size, the thickness of the insulating layer and the facade dimensions.

It is concluded that only one half of the freeform facade should be generated, since the other half is simultaneously produced as its complement. By using a bounding box as the starting point in the freeform facade generation process, the amplitude of the freeform surface is limited to a predefined value. The utilization of two sets of guideline curves with specific properties for freeform surface generation, allows for the surface to have a $C^1$ continuity transition between the complements.

The generation of various freeform surface designs on a facade area of 100cmx260cm and with an amplitude between 10cm and 30cm and approximating it with ruled surface ribbons proved to be best when done with EPS block thicknesses of 5cm. This enables the best compromise between the cutting path length and the discrepancy between the initial and the approximated surface.

Utilizing either of the two secondary subdivision tessellating approaches, the aligned and the staggered stack produces the same amount of parts. The aligned stack allows for smaller panels to be joined together before applying it onto a facade, while staggered stack enables better connectivity between the parts, but compromises that with larger sizes or jagged connection edges.

5. LIMITATIONS AND FUTURE WORK
Given the limited thicknesses of EPS blocks available on the market, 10cm, 5cm, 2cm and 1cm, the facade area is constricted to have a width of an even integer number. For example, the facade area with a width of 106cm requires a generation of its half which is 53cm. In this case, it is not possible to tessellate the freeform surface half into ribbons of the same width. Instead, the application of a combination of different block thicknesses can solve the ribbon tessellation issue here - for example ten ribbons of 5cm, one ribbon of 2cm and one ribbon of 1cm width. On the same note, secondary subdivision of the ribbons can produce left over parts of smaller specific lengths that cannot be used for the current fabrication process. If the example used in section 3.2 had a facade area of 60cmx260cm, the total ribbon height would be 1560cm, which means that 15 whole EPS blocks would be applied, while the 16th would only be cut for the 60cm length, leaving a part of 40cm for possible future fabrication.

Future work includes generating a workflow that focuses on generating a freeform surface on a facade with fenestration, fabricating such parts by using a robotic hotwire cutting process and assembling it and applying on a real facade setting.

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