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ENTROPIC-BASED SIMILARITY AND COMPLEXITY MEASURES OF 2D ARCHTECTURAL DRAWINGS

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Abstract. In this paper we construct an information-theoretic model of architectural drawings. This model is then used to quantify and measure the complexities and similarities of drawings. The approach is applied within a linear qualitative shape representation but can be generalized to other types of representation. The descriptive and analytic power of the proposed methods are demonstrated by studying the time evolution of the architectural plans produced by Aalto and Kahn and by comparing them to each other.

1. Introduction

Is it possible to compare drawings of different designs by a single architect or to compare drawings by different architects? Two issues are immediately raised by these simple questions. First, what makes a drawing in this context and how is it represented? Is it possible to represent drawings uniformly, i.e., in a canonical form? And the second, what sorts of measures allow such comparisons to be made? These are the basic questions we will address in this paper.

Until now this area has been dominated by qualitative subjective measures that are hard to formalize and model. The focus of this study is on finding an answer to the second question – how (within a given canonical representation) to construct an objective quantitative measure of complexity for drawings and how to measure a degree of similarity between drawings. The proposed approach is essentially representation-independent and can be applied to any representation. Because of that the first question is not considered in this paper – instead we simply use one of many published representations, which provides us with a necessary framework for this study.

Many different aspects of shapes can be compared and many measures could be used for such comparisons. Here we will consider two measures.

The first one is one of the most interesting, important and difficult measures to formalize – complexity measure. The second one is a similarity measure. Unlike feature-based models (Tversky, 1977), where similarity is defined in terms of common and different features of entities, and matching-distance (Rodrigues et al., 1999), where some metric over space is used, our model of similarity relies on measuring "distance" between shapes represented in a more abstract form as probability distributions in feature space. When constructing such measures, we will use a set of tools provided by classic information theory (Shannon entropy) and by its modern offshoots (approximate entropy and Ziv-Lempel complexity). Each of these tools provides an integral value over the whole drawing or set of drawings being measured of its information content, within the chosen canonical representation. The intuitive idea behind this approach is that if we represent drawings using the same canonical representation then the more information that is necessary to describe the particular drawing (or ensemble of drawings) the higher its complexity is. Further, the closer to each other are the distributions of features in two shapes, the higher is the degree of similarity between them. The necessary conditions for this conjecture to be true is that the canonical representation must be able to carry information related to the drawings' similarity as intuitively asserted by the human observer and the model of a drawing (which underpins each of the above entropic measures) must be a "proper" one. At the very least, the constructed means of measuring similarity should have a discriminatory power, i.e., should be able to judge the similar (from the human intuitive view point) drawings as similar, the non-similar as non-similar and the more complex as more complex.

Although the proposed approach is essentially representation-independent, only the overall quality of its results (that is, the combined underlying representation and the measures) can be evaluated. We will perform such evaluations for the measures we construct by applying them to the extant body of architectural plans of two twentieth century architects Alvar Aalto and Louis Kahn, represented using the Q-code schema of Gero and Park (1997). This will demonstrate the discriminatory power of these entropic measures and will also derive some interesting insights about the work of the architects examined.

Many shape complexity measures and shape metrics have been developed in various domains including cognitive psychology, computer science, computational esthetics, etc. It was long ago realized that the majority of them are based (explicitly or implicitly) on information theoretical notions (see, for example, Leeuwenberg, 1968). We strongly agree with this assertion and would argue that all of the known measures of shape complexity can be modeled using the approach proposed in this paper. For example, the well-known notion of figural goodness can be modeled as a particular realization of this approach (Hochberg and McAlister, 1953) as the redundancy index computed from the particular shape coding. Similarly, Leeuwenberg's measure that is based on ab shape's minimal code (Leeuwenberg, 1968) can be interpreted as a particular case of entropy computed by using the particular set of code transformations.

2. Qualitative Shape Representation

Architectural drawings in the form of plans, along with elevations are one standard form of representing an architectural design. There appears to be no standard way of representing such drawings even when using CAD systems. This lack of a canonical representation becomes clear when examining a CAD drawing. By looking at the drawing it is not possible for the viewer to know how the drawing was constructed. Even an object whose image is that of a square exhibits this: is it a single polyline, four line segments, an L-shape reflected around its endpoints, and so on. Figure 1 indicates this with output from a CAD system.. Without a canonical representation it is not possible to compare drawings.

(a) (b)

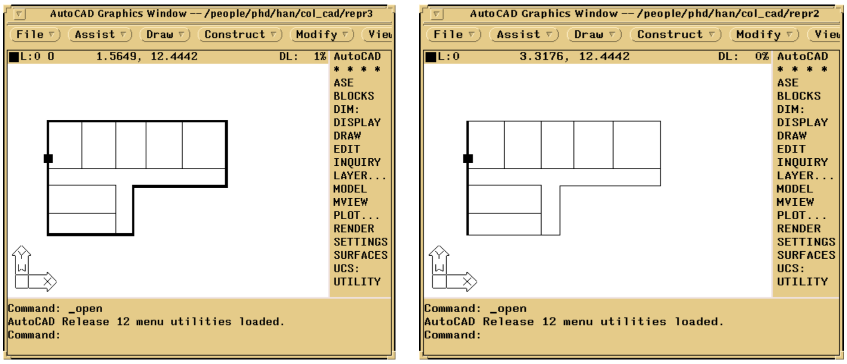

Figure 1. Different representations of the same drawing in a CAD system, Selecting the same point in does not result in the same set of lines being selected. The selected lines in (a) and (b) are a function of the representation (Jun and Gero, 1997).

Numerous representations are available for the description of architectural drawings (Mantyla, 1988). They can be divided into two broad groups – the numerically-based (quantitative) and the symbolically-based (qualitative). It has been recognized that quantitative representations are inappropriate representation for spatial reasoning (Kuiper, 1978) and that qualitative representations provide a better framework for developing computer-aided tools for spatial reasoning (Egenhofer and Shariff, 1998).

In principle, the proposed approach – measuring the shape complexity by measuring its information context with the chosen representation and measuring the shape similarity by measuring the "distance" between their underlying statistical properties – is equally applicable across both those groups of representations, but we describe it here within the framework of a qualitative representation.

We will use the published qualitative shape representation scheme called Q-codes (Gero and Park, 1997) as our canonical representation. Q-codes allow the outline of a shape to be encoded qualitatively in terms of the relative changes of directions at landmarks such as corners and the relative changes of lengths of adjacent sides. This converts the drawing into a circular string of Q-codes, Figure 2. This creates the common frame of reference for different drawings.

The Q-code coding process follows formal guidelines developed in qualitative reasoning (de Kleer and Brown, 1984) and includes two stages:

(a) The set of landmarks is placed on the outline of a drawing. Landmarks are defined as points that are considered as distinguished by the coding procedure. Here we define them as points where the direction of the outline has a discontinuity on a coarse "macroscopic" scale. For example, there are 19 landmarks ("corners") on the Aalto's drawing that is shown in Figure 2.

(b) In a counterclockwise order each landmark is coded as {A+}, {A0} or {A-} correspondingly, depending on which of the intervals $\{(0,\pi)$, $[0,0]$ or $[\pi,2\pi)\}$ the angle between two segments adjacent to this landmark belongs to; and each segment is coded as {L+} if its length is longer than the length of the previous segment, {L0} if they have the same length and {L-} if it is shorter than the previous one. The resulting symbol string is understood as a circular structure, that is, its last symbol is followed by the first one. An example of the Q-code coding is shown in Figure 2. Note that the granularity of the coding can be increased by reducing the sizes of the corresponding intervals.

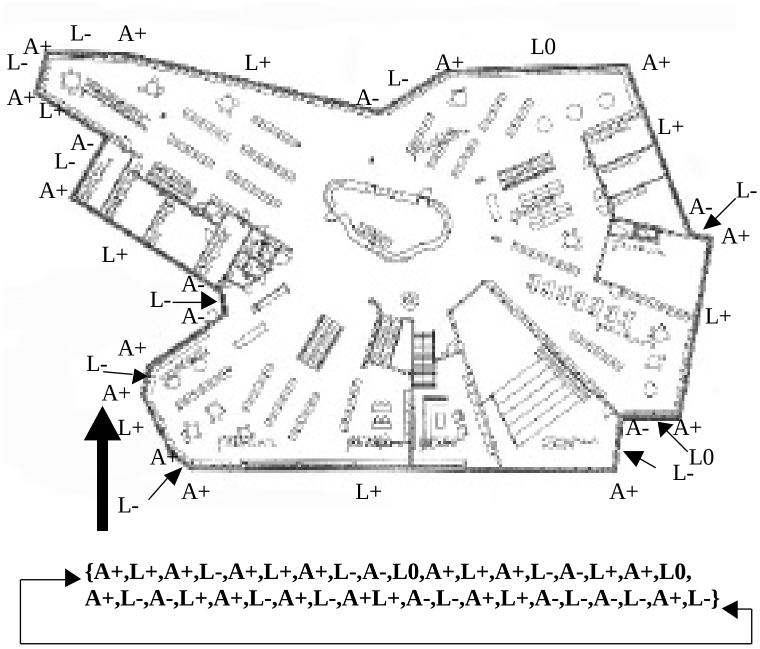

Figure 2. Q-code representation of a building plan by Aalto. The angle in each corner is coded as {A+} if it is < π as {A–} if it is > π and as {A0} if it is equal to π. The coding is carried out in a counterclockwise fashion, beginning with the corner to which the bold arrow is pointed. The length of each border segment is coded as {L+} if it is longer than the previous one and {A-} if it is shorter and {A0} if they have the same lengths. The resulting string is understood as a circular structure, where the its two end points are connected.

These Q-codes are composed in a similar way to any natural language such as English. A sequence of Q-codes forms a word that may represent a shape pattern of significance such as a face of an object. A sentence, as a sequence of words, may describe an entire object.

The major advantage of Q-code representation over other similar qualitative representations is its “architectural” and “design-oriented” character, namely, its ability to explicitly represent some drawings’ features that are based on concepts of space in architecture and design (symmetries, Alexander’s pattern language (Alexander et al., 1968), etc). The second major strength of Q-code representation is its ability to represent architectural sketches, used at the early stages of designing when objects do not yet have a fixed geometry; only an indicative one based on their topology. A detailed discussion of Q-code representation can be found in Gero and Park (1997). From our viewpoint this representation provides us with a good initial test bed because:

(a) it can be mapped onto a feature space that intuitively seems to be related to the concept of shape complexity and similarity, and
(b) it has a linear nature, which simplifies the algorithmic and computational realization of the proposed approach.

If the Q-code representation is employed then the problems of estimating complexity of drawings and measuring the similarity between them are reduced to the problems of estimating complexity and similarity for symbol strings and corpuses made of symbol strings, for which a large number of methods is readily available.

3. Applying Entropic Measures to Q-Code Strings

Once the drawings are encoded in this canonical form, we can work with the objects that represent these drawings – Q-code strings (or we can call them Q-code sentences) and Q-code texts that consist of Q-code sentences – and measure the amount of information they contain. A wide range of algorithms is available for this purpose. The drawing’s complexity is then defined by this amount of information. In order to calculate this amount of information a statistical model of this Q-code text is developed for each distinct class of such texts. The similarity between two classes can then be defined as degree of similarity between their statistical models.

3.1. SIMILARITY MEASUREMENTS FOR DRAWINGS

3.1.1 Shannon entropy and statistical language modeling

Shannon entropy is a central notion in information theory. It is based on a simple statistical model of data, which assumes that they are generated by an ergodic Markov source, symbol after symbol (in the simplest case and feature after feature in the more general case). This generation is executed stochastically on the basis of the history of which symbols have been generated immediately before the current one. The features here could be Q-code words (continuous string of Q-codes) or any semantic or structural regularity in the Q-coded data. So for the 1-st order model and the alphabet of Q-codes $Q=\{q_1,q_2,\ldots,q_n\}$ ({A+,A-,A0,L+,L-,L0} for the Q-code in Figure 2) the model’s estimate of the probability of generating q_i after q_j, $Prob_M(q_i \mid q_j)$, is assumed to be constant. These conditional probabilities are estimated from a sample of Q-codes text. The cross-entropy is then calculated as:

$$En = -\sum_{i,j} P_S(q_i, q_j)\log\mathrm{Prob}_{\mathrm{M}}(q_i, q_j) \quad (1)$$

where $P_S(q_i, q_j)$ is the empirical probability of the symbol q_i following q_j in this sentence. Entropy is an ensemble-based measure that can only be calculated for an ensemble of similar (generated by the same source) sequences.

Thus, if we have a group of similar shapes (for example each drawn by the same architect) that are converted into the Q-code text, we can estimate these conditional probabilities and then calculate entropy for each individual shape (sequence) from this group. Those that have higher entropy values, will be declared as more complex than the ones with the lower values of entropy. But this judgement will only be valid if the Markov model that underpins this construction is valid.

Assume we have two different groups of shapes (for example drawn by two different architects) that were coded as two Q-code texts. The comparison between these two texts can be carried out by constructing a Markov model for each group as described above and then by computing cross-entropy for each text with respect to another

$$En = -\sum_{i,j} P_T(q_i, q_j)\log\mathrm{Prob}_{\mathrm{M}}(q_i, q_j) \quad (2)$$

where $P_T(q_i, q_j)$ is the empirical probability of the symbol q_i following q_j in the other text (so the model comes from the text and the empirical probabilities from another source). In practice when such analyses are carried out one normally uses perplexity, PP, instead of cross-entropy. Perplexity, *PP*, is defined as

$$PP = 2^{En}. \quad (3)$$

The higher is the perplexity the further apart are the two generators from each other and less similar are the two Q-code texts being compared. The perplexity can be viewed as a “distance” between the model and the sample. Thus, when perplexity of one corpus of Q-code text is computed with respect to the model constructed for the other corpus, the “distance” between this model and the underlying empirical model of the second corpus is obtained.

Note that problem of comparing two texts (sets of symbolic sequences) drawn from a closed alphabet is the central problem in statistical language modelling (Clarkson and Rosenfeld, 1997) and all these techniques come from thiat field. This technique accounts for an N-gram history (conditional probabilities that depend on N-long Q-code words), data sparsity of the samples, and other various technical issues.

3.1.2. Comparison of Aalto and Kahn bodies of work

Consider the architectural plans (actually only the outlines of plans) produced by two distinguished 20-th century architects: Alvar Aalto and

Louis Kahn. Aalto practiced primarily in Finland and Kahn primarily in the US, although designed buildings for disparate parts of the world. Each plan was coded as a Q-code sentence, which was stored in a database as a single record jointly with the year when the building was designed. Each Q-code sentence includes 4 parts. The first two are based on relative length coding (with two granularities) and the other two are based on the relative angle coding (again with two different granularities). For example, the plan from Figure 2 was coded as {A+,L+,A+,L-,.A+, L+,A+,L-,A-,L0,A+,L+,A+,L-,A-,L+,A+,L0,A+,L-,A-,L+,A+,L-,A+,L-,A+,L+,A-,L-,A+,L+,A-,L-,A+,L+,L-}. Then, the resulting Q-code texts were separated into groups that belong to the different "decades" in which they were designed. These decades not necessarily cover 10 year-long intervals but rather such variable-length intervals that hold approximately the same number of plans. For Aalto these decades are the 1920s, 1930s, 1940s, 1950s and 1960s, and for Khan they are 1940s, 1950s, 1960-64, 1965-69 and 1970s. We will also use the term "overall" (overall sample and overall model) to denote the complete set of coded plans developed by each of these architects over all decades. The overall sample for Aalto contains 132 plans and for Khan it has 218 plans.

The Markov model (of the 4-th order) was developed for each of these architects and for each decade they worked. The "overall" models were also constructed for both Aalto and Kahn using the corresponding complete database of works as training set. The computations were carried out using the CMU-Cambridge statistical language modeling toolkit (Clarkson and Rosenfeld, 1997). Since it is well known that the quality of the model depends strongly on the size of the training sample we normalized the sample from each decade using the bootstrap re-sampling procedure (Efron and Tibshirami, 1993). Here a set of artificial samples was produced for each decade by re-sampling with replacement of the original sample for this decade. The results were then averaged over this set of samples. This forced all the training samples to have the same effective size. These models were evaluated (the perplexities were calculated) with respect to the original samples.

As we described in the previous section, at the very least we expect to find that the proposed technique is able to differentiate between Aalto's and Khan's works and between their work from different time intervals. This is based on the assumption that there have been changes in the complexity of drawings each produced during their lifetime. In another words the corresponding models must be statistically different. Our analysis could also allow us to gain insight into the changes that occurred in their works, designed during various periods of their design lives.

Table 1 shows the distribution of classes of building plans used in this analysis. It can be seen that there is a reasonably good correspondence between them except for office and industrial buildings.

TABLE 1. The distribution of classes of buildings for Aalto and Kahn.

	Cultural	Health	Housing	Offices	Religious	Industrial
Aalto	28.9	4.6	34.4	9.9	19.1	3.1
Kahn	21.2	3.9	39.3	23.8	11.8	0.0

The perplexities of the statistical models, developed for the different time intervals for Aalto with respect to the plans from different decades are shown in Table 2. The perplexities for Khan's decades vs. samples are shown in Table 3. The perplexities of the "overall" models of Aalto and Khan with respect to all of their plans are shown in Table 4.

TABLE 2. The perplexities of Markov models developed from Aalto's architectural plans from different decades of his work. The columns denote data sets and the rows denote models developed for the corresponding decades.

	1920s	1930s	1940s	1950s	1960s	overall
1920s	2.19	7.48	5.01	8.98	10.40	7.62
1930s	5.28	2.30	4.64	5.71	7.11	5.53
1940s	4.14	5.68	2.34	4.69	5.71	4.64
1950s	3.49	4.61	3.44	2.58	3.84	3.46
1960s	3.63	4.13	3.49	3.46	2.72	3.27
overall	2.65	2.74	2.61	2.78	2.86	2.83

TABLE 3. The perplexities of Markov models developed from Khan's architectural plans from different decades of his work. The columns denote data sets and the rows denote models developed for the corresponding decades.

	1940s	1950s	1960-64	1965-69	1970s	overall
1940s	2.41	4.20	5.45	4.51	4.10	4.25
1950s	4.13	2.47	4.40	3.88	3.53	3.41
1960-64	3.91	3.49	2.69	3.59	3.38	3.30
1965-69	4.24	3.79	4.92	2.50	3.48	3.84
1970s	7.19	6.68	7.56	5.68	2.40	6.29
overall	3.00	2.52	2.98	2.79	2.66	2.75

TABLE 4. The perplexities of Markov models developed for Aalto's and Kahn's architectural plans. The columns denote data sets and rows denote model developed using the shapes from corresponding architect as a training sample.

	Kahn	Aalto
Kahn	2.75	3.49
Aalto	5.15	2.83

3.1.3. Results from comparing Aalto and Kahn bodies of work

From these tables one can see that the perplexities vary significantly which confirms the discriminatory power of the approach. The values in the tables indicate the utility of a data model as a predictor. Thus, in the tables the diagonal numbers refer to the model's perplexity for the data set in that row. In Tables 2 and 3 this refers to decades or the overall set of data, whilst in Table 4 it refers to each architects overall data set. Then each other column in a row refers to the use of that model as a predictor of the decade of that column. The higher the perplexity the poorer is the model as a predictor. Examining the results for Aalto in Table 2 we can see that the model of the plans of designs produced in the 1920s is a poor predictor of later plans. It has a perplexity of 7.62 for plans of all the designs. These results imply that Aalto changed the complexity of the plans of his designs over the decades of his practice.

We can see that as the decades progressed the model for each decade becomes an increasingly better predictor of future and preceding decades as well as for the overall corpus of Aalto's work. Interestingly, the model of the overall body of work is by far the best "predictor" of each previous decade of his work. We can see a parallel but slightly dissimilar result for Khan's work. Based on these results Khan's plans exhibit a smaller change over time than do Aalto's except that in the 1970s there was a dramatic change. Again, the model for the overall set of data is the best "predictor" of each decade of his previous work.

Table 4 shows the model developed for the overall body of each architect's work, which in Tables 3 and 4 are the best predictors, used as the predictor of the work of the other architect. This is a measure of the degree of similarity of the complexity of each architect's plan drawings. The model developed for Khan's drawings is a better predictor of Aalto's drawings complexity than is the model developed for Aalto as a predictor of Khan's drawings complexity. Surprisingly, the model for Khan's drawings complexity is a better predictor of Aalto's complexity than the early work of Aalto himself.

3.2 COMPLEXITY MEASURES FOR DRAWINGS

While readily computed, the Shannon entropy is applied to the process generating the Q-code text rather than to the resulting Q-code text itself. Thus, its usefulness is limited since different processes can generate the same Q-code text. A practical complexity measure should be readily computable, meaningful even for very small sentences, and defined by the Q-code text itself rather than unknown process that created it. Two measures with those properties are the Lempel-Ziv compression or complexity and approximate entropy.

3.2.1 Lempel-Ziv compression

The Lempel-Ziv measure LZ (Ziv and Lempel, 1978) is essentially the number of cumulatively distinct Q-words in the Q-code shape description. It determines how far this description can be compressed. The heuristic idea here is that the most complex drawings are those ones whose description cannot be compressed. Unlike the Shannon entropy the LZ can be computed for individual shapes and can be used to compare shapes that belong to different groups (for example, drawn by different architects). This is important as it has the potential to show how individual drawings can be used to track changes over time.

Because a random Q-code sequence will have a characteristic number of distinct Q-words we can calculate the maximal value of LZ for a given length of the sequence and size of alphabet. This provides a natural scale for LZ and allows the use of an absolute (relative to these two limits) value of LZ for an individual shape.

The results for Aalto are shown in Figure 3 and for Khan in Figure 4. Each of these figures contain three graphs that show the empirical dependence on decades of the maximal yearly LZ, the average yearly LZ and the difference between the maximal and minimal yearly LZ and their linear regressions. The results in these graphs form one basis for a time-based comparison within an individual architect's drawings.

Figure 3 indicates that the average complexity of Aalto's plan drawings increased with the length of his architectural practice, although the maximum complexity did not increase as fast. The bottom graph in Figure 3 indicates the change in the range of complexities produced in each decade. As can be seen there was virtually no change in this range.

We can see a different behaviour over time in Figure 4 depicting the same information about Khan. The average complexity of Khan's plan drawings was unchanged over his lifetime and the maximum complexity dropped over time. However, the range of complexities produced during any decade increased.

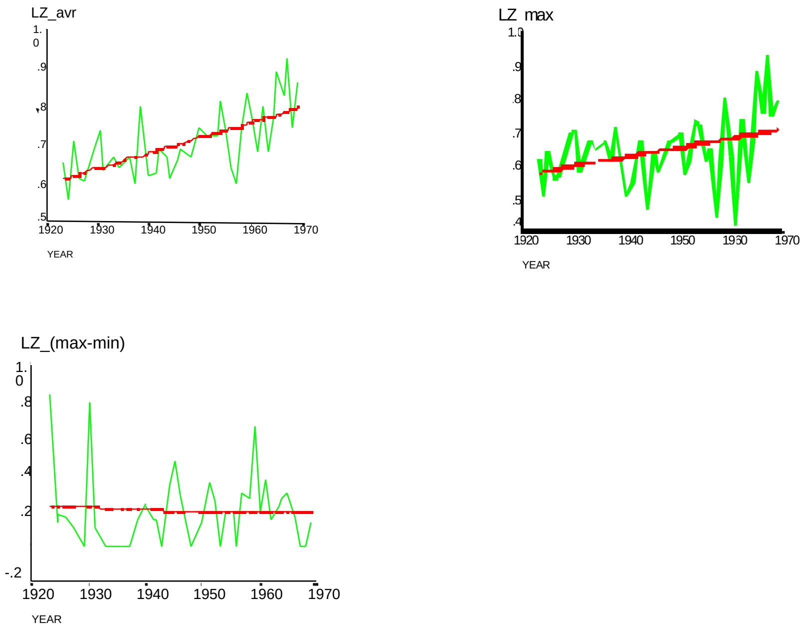

Figure 3. The dependence of the maximal yearly LZ, average yearly LZ and the gap between the maximal and the minimal yearly LZ on time for Aalto.

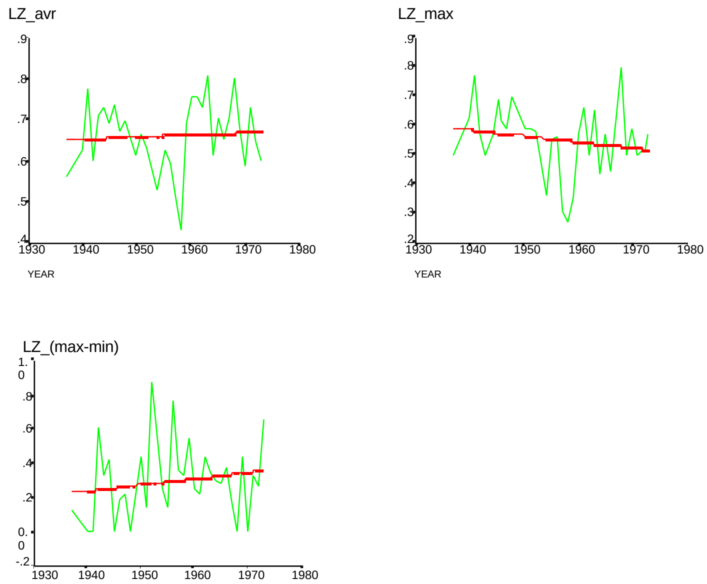

Figure 4. The dependence of the maximal yearly LZ, average yearly LZ and the gap between the maximal and the minimal yearly LZ on time for Khan.

3.2.2 Approximate entropy

The focus of approximate entropy is the frequency of all possible overlapping m-letter Q-code words across the entire sequence. The purpose of this measure is to compare the frequency of overlapping words of two consecutive/adjacent lengths (*m* and *m+1*). The approximate entropy (Pincus, 1991) is then defined as

$$ApEn(m) = \Phi(m) - \Phi(m+1) \qquad (4)$$

where $\Phi(m)$ is the empirical entropy of the m-letter Q-code words in the string

$$\Phi(m) = \frac{1}{n+1-m} \sum_{i=1}^{n+1-m} C_i^m \qquad (5)$$

C_i^m is the relative frequency of occurrences of the i-th Q-word out of n+1-m possible. ApEn characterizes the log-likelihood that two parts (of a Q-sentence) of size m+1 are the same given that they each contain the same parts of size m.

Figure 5 shows the results of measuring approximate entropy in Aalto’s plans for 3-letter words. The graphs show the average, maximum and range over the decades of Aalto’s design life. These results parallel those obtained using the Limpel-Zev complexity for average and maximum. For the range of complexities the use of approximate entropy results in a constant increase over Aalto’s design life. One interpretation of this is that Aalto’s designs as indicated by drawings of his building plans, became richer as he continued to design over many decades. Richer in the sense that they became more complex.

Figure 6 shows the results of measuring approximate entropy in Khan’s plans for 3-letter words. The graphs show the average, maximum and range over the decades of Khan’s design life. These results are close to those obtained using the Limpel-Zev complexity for average and maximum. Unlike Aalto, Khan’s plans of his designs did not become richer of his design life.

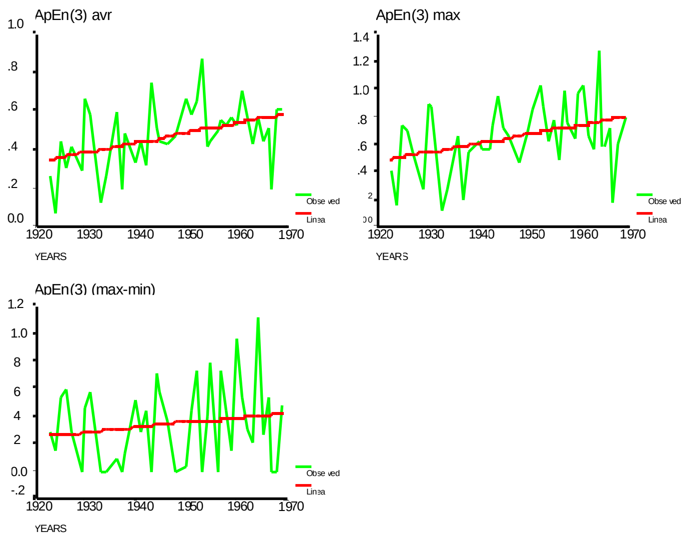

Figure 5. The dependence of the maximal yearly ApEn(3), average yearly ApEn(3) and the gap between the maximal and the minimal yearly ApEn(3) on time for Aalto for the 2-nd granularity of length based Q-code.

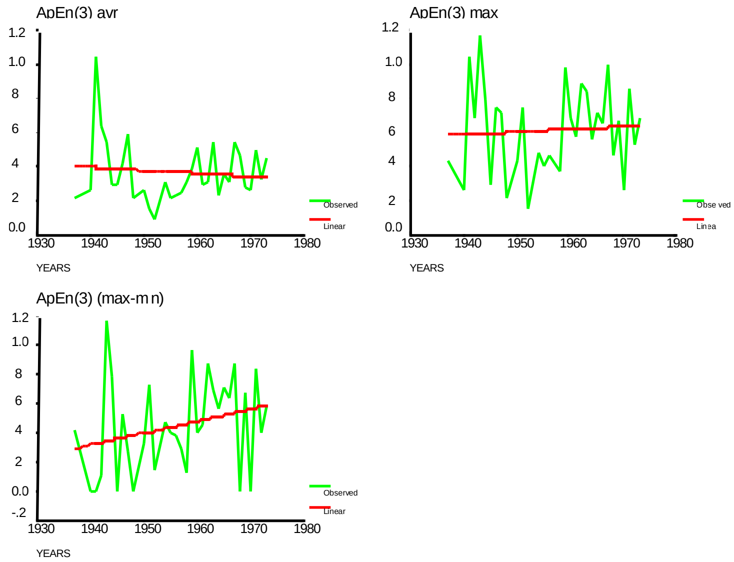

Figure 6. The dependence of the maximal yearly ApEn(3), (for 3 letter long words) average yearly ApEn(3) and the gap between the maximal and the minimal yearly ApEn(3) on time for Khan for the 2-nd granularity of length based Q-code.

4. Conclusion

This paper has described the development and implementation of similarity and complexity measures of 2D architectural drawings. These measures can be used to begin to formalise the measurement of the "style" of a visual designer in a formal manner. Further, the measures can be used to examine the change in complexity of drawings of designs of an individual designer over time. They can also be used to compare the works of different architects based on their complexities.

What we have proposed could also be viewed as the use of a quantitative objective method for developing what might be the basis of a "digital signature" of a visual designer, that is, a descriptor that allows authentication of designed shapes generated by a particular designer, architect, school of architects, style of architecture, etc. This method is based on building a statistical model of a "typical" encoded design object and on using this model to construct an information theoretic functional with the desired descriptive properties. The tools, which are based on this method, can be used not only for the analysis of existing designs but also for generative purposes as a evaluator in conjunction with a generative system, which provides this system with a feedback and directs it into the subspace of design space where the designs similar to the targeted one are located. The proposed approach can be generalized to 3-D shapes, using 3-D Q-code (Gero and Damski, 1999) but that requires methods for the measurement of the entropy to semantic graphs to be developed.

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