# Beyond the math visualization
## Geometrica and Stochastica

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## Abstract

Mathematically controlled imaging process provides attractive results because of its infinite scaling capabilities with some other elements that contribute to the visualization. Its global/local and precise manipulation of parameters holds potential for realizing an unpredictable horizon of imagery. When it meets the artist's taste, this method could be a strong enough system of creation, and I have been producing images using the surfaces of hyperbolic paraboloid.

On the other hand, a method absolutely free from the geometric parameter manipulation is possible with a stochastic process [1]. Like the technique of pendulum in photography, while its production rate of acceptable result is very low, its potential of generating a strong visual message is also very attractive.
It is possible to set stochastic elements at any stage of the process, and conditional probability on those elements, or the hierarchy of probability management characterizes the probability distribution.

Math space has no light. No gravity. No color on the math surfaces. And the math equation provides only the boundary in 3D or higher mathematical dimensions. The fact means that artists can keep artistic reality with their unique tastes in colors on the surface and light sources, and this is the most important element of the math based imaging. Being able to give artists' own choice of colors and that the artist may take only right ones from the results of a stochastic process guarantee the motif and aesthetics of artist could be reflected onto the work.

## 1. Introduction

Although almost all of them are not intended to express artistic motif, we can receive some beautiful visual message from mathematically produced images published in academic papers and technical documents every day. When we say "mathematically produced," they are results of two types of imaging processes. One for the visualization of mathematical objects and the other for the visualization mathematically processed. Former uses mostly geometric object with some attributes on the surfaces
and latter is not often used in the art scene but provides rather interesting results.

Here I discuss the methods of creating expressions beyond the mathematical meaning of the geometric object in 3D space and images generated by a stochastic process with actual works created by a computer graphics system for the gallery prints.

## 2. Geometrica

Geometric figures are wearing the beauty of perfection on it, but that is not from the mathematica

characteristics. When you look at beautiful math based images or sculptures, you understand that they are in fact the outcome of artist's endeavour in the artistic implementation of mathematical research [2,3,4], and the beauty derives from the harmony created in the shape and material [5]. It might be the destiny of the math artists that the creation is not effective in production because the math figures have the infinitive scalability and exist in the space without colors and lights. That means the artist have to pick out the optimum elements and the values in the infinite options.

In the context of the "generative art," the image creation by the manipulation of geometric surface is not very appropriate, but there are some strong aspects of "generating." Figure is generated by the processing
of math calculation and the manipulation of its coefficients can easily generate deformed forms. In maintaining the curvature with mathematical meaning on the surface, artist can produce varied results with the surface attributes and lighting set up.

To achieve an art work, geometric visualization has too many elements to be manipulated. Coefficients, Coordinates of the center, Colors, Values of light source, Surface attributes, and so on - they are required
in order to calculate a piece of geometric image. Such images can be easily produced by a graphics program today, but they are mostly uninteresting as art work.

The creation of geometric art might be defined as that the process of geometric data modulation controlled by the taste of the artist (Fig. 1). In other words, the effort of the artist is to look for the most effective algorithm which reflects his taste on the geometric figure, where there is no human elements. Ordinarily, the studio work comprise of using an algorithm complex designed for one time execution on the computer.

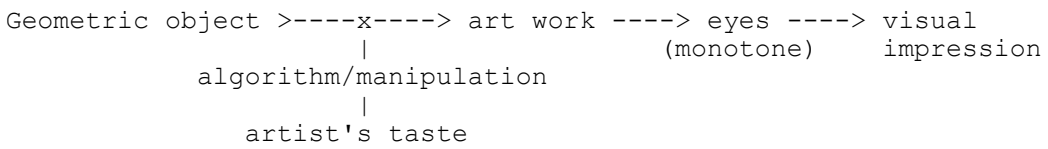


**Fig. 1**

Geometric objects that computer can produce with a simple program are mostly quadric surface, and its creative flexibility is rather limited. Since the geometry based art creation deals with geometric object without the process of modeling, the chance to add the artist's taste is limited in some processes, such as to combine geometric objects, to deform its shape, to change the surface attributes, and to tune the lighting. The produced figure keeps its geometric meaning.

To provide a strong visual impression without losing the mathematical meaning, I attempted to change the way an image is visually perceived. That means the scheme of the visual information flow is changed with an algorithm (Fig. 2). When the monotone sensibility of our sight system is controlled by an algorithm which was designed by the artist, it would produce an image that reflects more of the artist's taste.

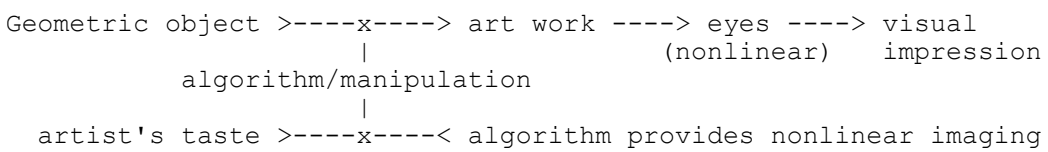


**Fig. 2**

I always start with a surface of hyperbolic paraboloid because this surface has unique characteristics of the shape and it is easy to modify the looks.

## 2.1 Legend

The series of Legend was intended to use suppressed colors, or less saturated colors, and to create figure-oriented images. The theme was to reproduce something between our visual perception and the mathematical entity, with the surface of hyperbolic paraboloid which is very sensitive to the change of the coefficients. Images were produced with some sheets of hyperbolic paraboloid [6] and each surface has its own attributes.

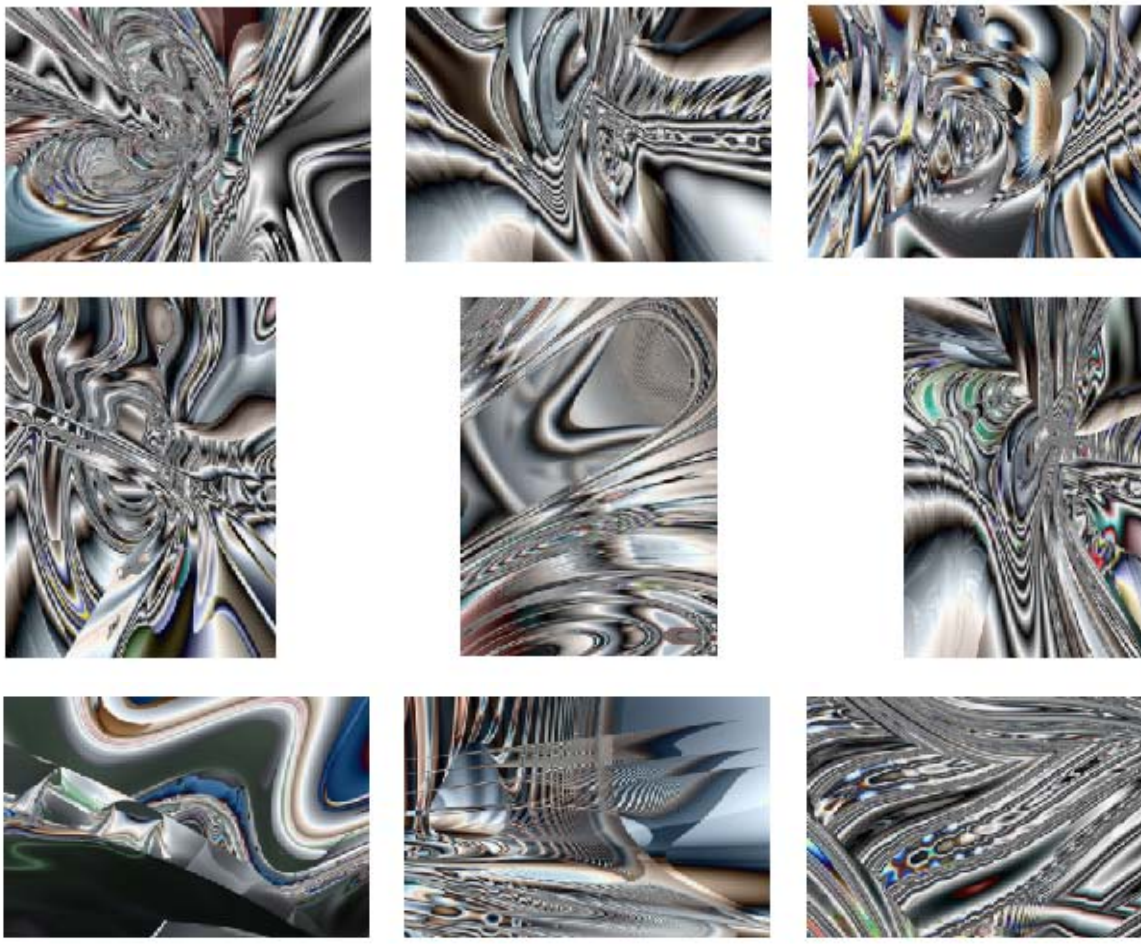

**Fig. 3** Some pieces from the series of Legend.

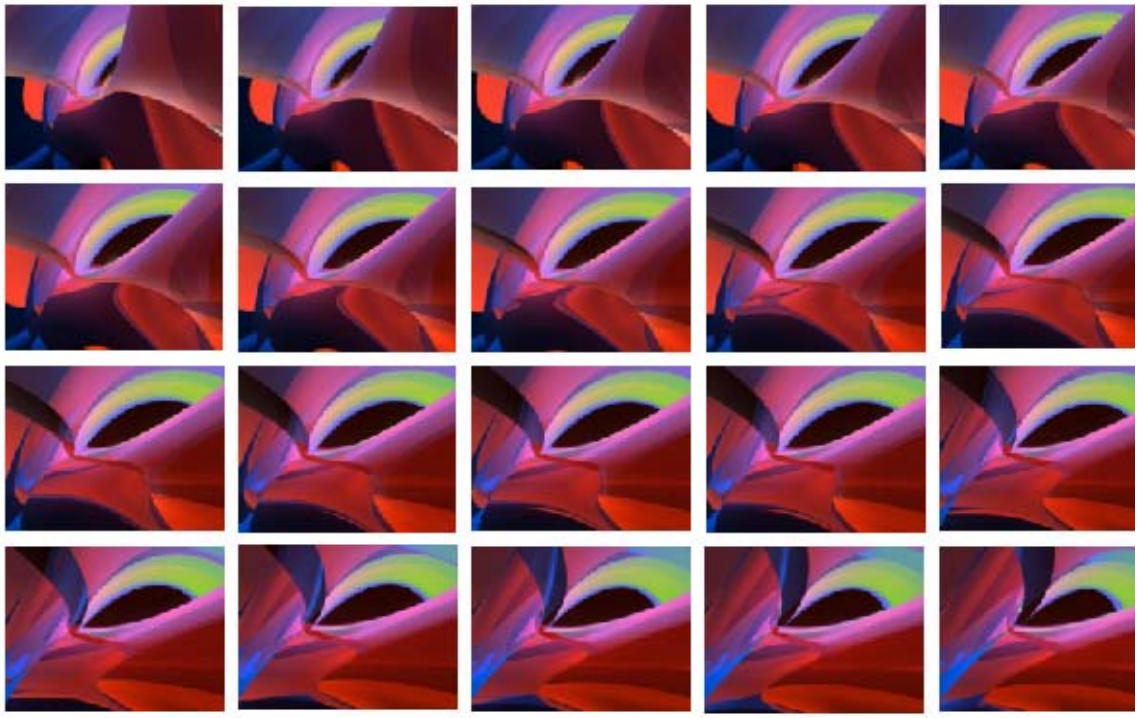

**Fig. 4** Frames from sample animation of geometrica

Sample animation was produced with image list files generated by the interpolated list file generator which used seven key frames. The program reads the key frame list files and generates frame list files in between which have interpolated data of the image such as coefficients of equation and values of coordinates, light source and surface attributes. It promises that the values keep the smooth changing from frame to frame on every elements by using blending functions, which keep the continuous second order differential coefficients. Exactly speaking, this method has some shortcomings in the smoothness but they are negligible in most cases of animation production.

## 2.2 Other works

Examples from the series of Flow and some others are in Fig. 5. Images here were processed in almost same manner as that of Legend. Only the differences are the surface attributes.

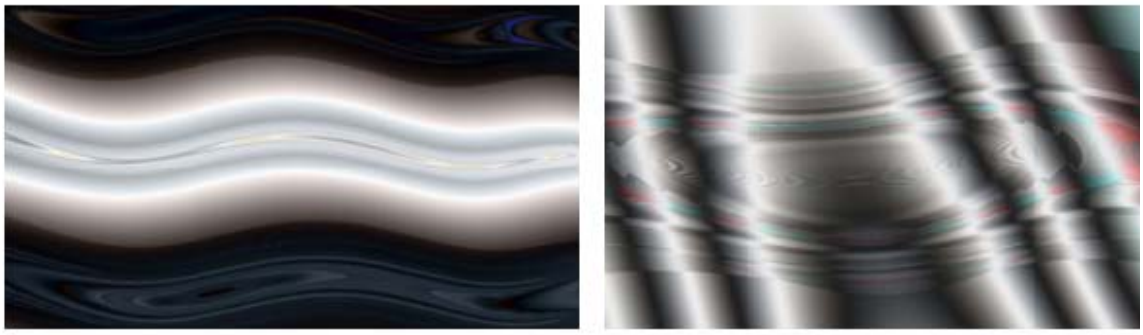

**Fig. 5** Some pieces from the series of Flow.

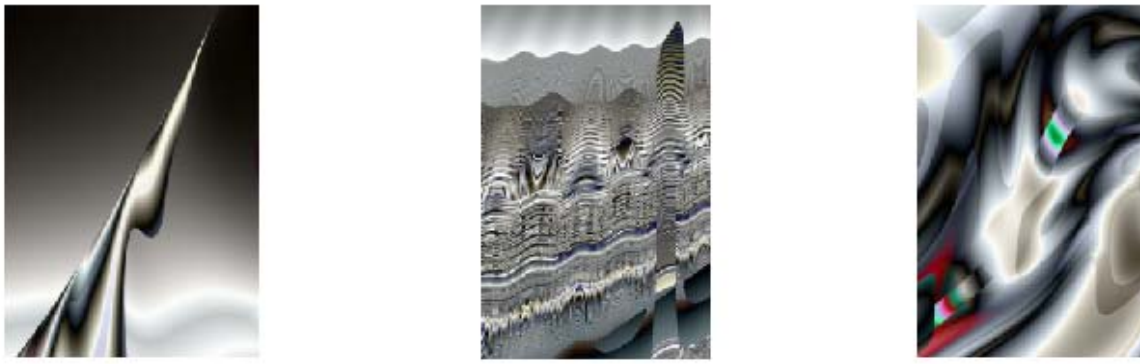

**Fig. 6a** Some other pieces of geometric images.

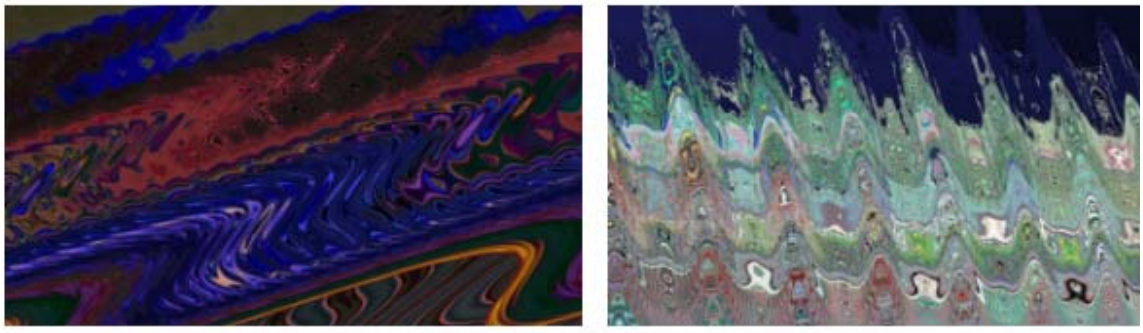

**Fig. 6b** Some other pieces of geometric images.

## 3. Stochastica

Stochastic process differs entirely from geometric imaging process. Everything goes in pre-programmed algorithmic process while we can try and modify the value of elements in the list file generating/writing
phase in the creation with geometric objects.

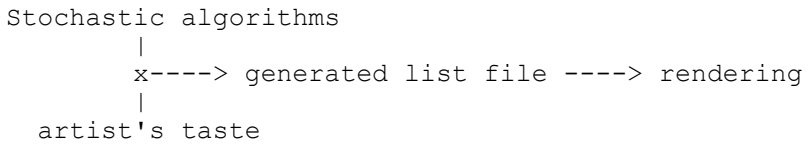


**Fig. 7**

In the series of Stochastica, the creation starts with generating light source data (i.e., the position in 3D space and the intensity of primary colors) by a process with probability elements. This phase is the place
the artist creates the algorithmic scheme to generate the list data file for rendering. The results of the ray tracing rendering of a plate which was illuminated by some tens to hundreds of light sources are the graphic work of this series. Fig. 8 shows a generated list file and its image is the left end image of Fig. 9.

```
#-----------------------------------------------------------------
# a1-2256.lst   Sat May 23 21:20:40 2000
#-----------------------------------------------------------------
# width height shadow
   6400   9024   0
#   x0  y0  z0       x1 y1 z1     viewangle     amb
    0   0  538       0  0  0         76          0.0
#type nx ny nz    px    py    pz     r           g           b
28     1 1 1      60  -290    99    0.55087311 0.47501954 0.34804038    << light #1
28     1 1 1     -16   197   143    0.23639360 0.50456657 0.39618764
28     1 1 1     -94    62   118    0.27536388 0.47026030 0.28500962
28     1 1 1    -145  -135   242    0.48221182 0.9775007 0.5819623
28     1 1 1     -58   125   118    0.63176075 0.29864681 0.37672627
  ...
28     1 1 1     -98   169    90    0.29484879 0.55379578 0.39386959
28     1 1 1    -156    93    35    0.46320018 0.8600247 0.6170587
28     1 1 1     -55    65    97    0.18293343 0.31999325 0.58633696
28     1 1 1     141  -255    41    0.21510619 0.14378468 0.17924957
28     1 1 1    -151   234   221    0.16575514 0.427784 0.19074764
28     1 1 1      98   209   209    0.63423900 0.31593906 0.24069318    << light #200
-1
# ty io s t x b  rfl rfr thr spc   r         g         b        n  a b c   x y z   x y z
rt
0  2  1 1 0 0 0 0.71 1.4  1  0.65  0.005000  0.005000  0.005000 0  0 0 1   0 0 -1  0 0 0 0
-1
# object synthesis AND  900max
0 0 -1
-1
# module synthesis OR   300max
999 0 -1
-1
```

**Fig. 8**

The reason I named Stochstica is I was intended to produce an animation with stochastic process, in the mathematical meaning, such as Markov process, Markov chain, diffusion process, Gaussian process, stationary process, martingale, branching process and a simple additive process. We can expect interesting results from the varied finite dimensional distribution of the stochastic process.

## 3.1 Crossmodulation

For an experimental collaboration with a Polish composer Igor Czerniawski, I prepared 106 A2-size prints of the image generated by a stochastic process. Most images displayed at the installation were generated by ray tracing with some hundred light sources which were generated by a stochastic program. The installation consisted of prints placed on a table and two sets of audio system playing different music simultaneously in the background. The artistic intension was to cause overflow of

intelligence in the brains of the audiences by an excess of visual and audio information simultaneously. Fig. 9 shows some of the images from the installation of*Crossmodulation*.

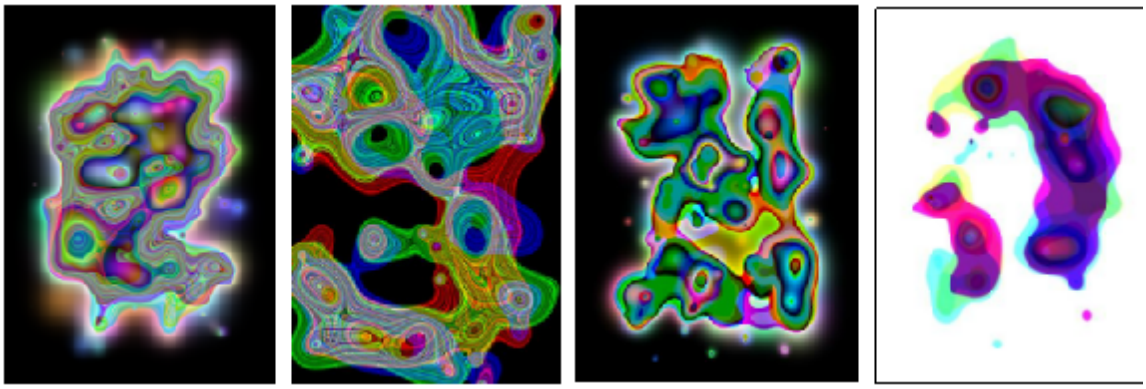

**Fig. 9**

### 3.2 Otherworks

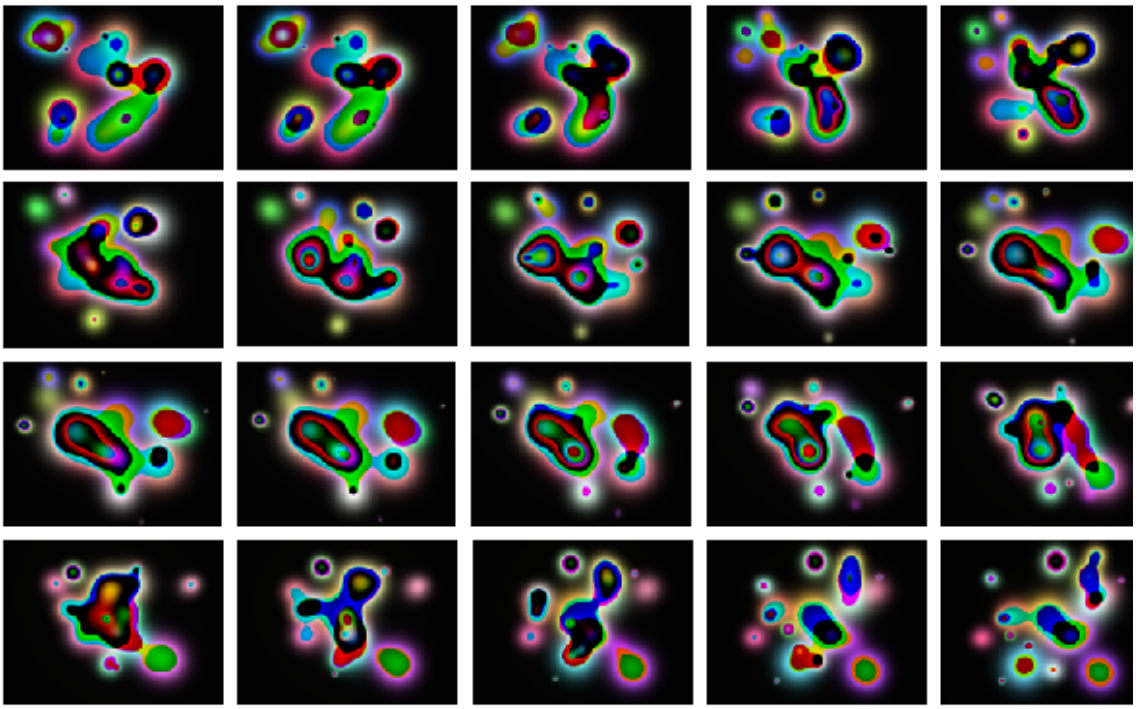

**Fig. 10** Frames from sample animation of stochastica

Sample animation was produced by my own animation tool which generates key frame light source file and its interpolated light source data are generated by blending functions. The procedure is similar to that for the geometric image animation above.

## 4. Systems

For artists using computer, the software environment is the primary subject. However, compute

graphics software aims to reproduce precise math visualization and it is not good for realizing artist's motif unless
some specific functions are added, and this is the reason why most digital artists develop their own software system [7]. Including functions to support artist's taste effectively is the primary subject for most math artists.

In my case, the first computer graphics software was written for my own designed/built machine which had no compatibilities to the commercial machines at the time in 1984, and the graphics software system have gradually grown alone from that time on. I rewrite the source codes when I try some new ideas and develop a tool if required. My environment is far from the current commercial packages, I guess, unfortunately I've never used commercial graphics software. It is mostly text based. Calculation is executed on PCs for 36(12x3) bit image data files today.

## 5. Conclusion

I discussed the artist's creative activity with two keywords of 'modulation' and 'algorithm.' I think I must point out a shadow of math based creation as well. In my experience in these activities, I have learnt that there are some complicated issues we have to overcome. For example, in the geometry based creation, the phenomenon of crossmodulation will come up when you handle a big amount of information. As like that in the radio science, it affects the quality of products and often causes troublesome results. There are so many options available that the artists easily go astray in the maze of parameters. In the algorithm driven process, artists encounters some difficulties in tuning the color or the shape because they are generated by a pre-designed algorithm and many elements interact with each other simultaneously.

No matter how you use computer, or whichever computer you use, to create an art work is not easy In spite of that, I believe artists can find a new horizon in his/her creative activities with having the experience of using geometric object and/or stochastic process in the studio. For artists who want to create mathematical art, either with geometry or with stochastic process or control, the essential element for success is the artistic serendipity.

I hope this discussion could give an answer to "that formula driven mathematically derived imagery has not enough human touch and rather sterile."

## References and Notes


[1] I used the term "stochastic process" for indicating the procedure with a probability event in this paper,
except where I discussed the stochastic process, as a math terminology, in Geometrica.

[2] Franke, H. W., *Generative Mathematics: Mathematically Described and Calculated Visual Art*, pp.101-104, ed. Michele Emmer, The Visual Mind (The MIT Press, 1993)

[3] Franke, H. W., Computergraphik-Computerkunst (Springer-Verlag,1985)

[4] Bilder nach Programm - Eine Bestandsaufnahme der graphischen Arbeiten von Herbert W. Franke (Ludwig-Maximilians-Universitat,1989)

[5] Ferguson, C, Helaman Ferguson: *mathematics in stone and bronze* (Meridien Creative Group, 1994)

[6] The surface of the hyperbolic paraboloid, which is one of the non central quadric surfaces that has not
a singular point, in an orthogonal coordinate system is expressed as:
sqr(x)/sqr(a) - sqr(y)/sqr(b) = 2cz
Because this equation includes only sqr(x) and sqr(y) for x and y, the shape is symmetric to yz

plane
and zx-plane. The intersection of the planes yz and zx provides a parabolic curve. Sliced edge at a position parallel to xy-plane shows a hyperbolic curve. The logical multiplication of the subspaces facing opposite directions which have identical coefficients but different position on the axis make a
sheet of the surface. This kind of sheets are used for primitives in most creations.
[7] Verostko, R., *Algorithmic Art and the Artist* in Computerkunst 2000, pp.21-25 (Museum der Stadt Gladbeck,2000)