A Parametric Approach to the Design of Vaulted Tensegrity Networks

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Significant new research in tensegrity theory and technology encourages tensegrity’s implementation in architecture. A recently developed technology makes possible the rapid modular assembly of deployable tensegrity units, and the construction of alternate curved configurations by re-using the same modules [1]. Although a form exploration method for tensegrity structures already exists [2], estimating the structure’s new geometry remains a challenge due to difficulties designers encounter in understanding and following the method’s geometric construction process. Besides, the method doesn’t address the geometry of vaulted configurations. This paper presents algorithms that link together the geometric parameters that determine the shape of tensegrity vaults by addressing different design-construction scenarios, and a software code that generates parametric models of tensegrity vaulted structures. The application of the algorithms to the morphological study of a tensegrity vaulted dome, which constituted the main feature of an entry to a recent international architectural competition, is also presented.
1. Tensegrity in building design

Initially a materialization of an aesthetic quest first conceived by artist Snelson [3], the concept of “tensegrity” (tensional integrity, as coined by Buckminster Fuller [4]) was later used to denote a novel class of structures with unique morphological characteristics and mechanical properties, deployability being one of them. Tensegrity structures are composed of bars and cables. The bars, that act as compression members, are connected only to cables, and not to each other, while the cables take only tension and form a continuous closed system. Internal tension is what holds struts and cables together and allows tensegrity structures to reach equilibrium at geometric configurations where bars look like floating in a network of cables. Stable configurations and related mechanical and structural properties have been extensively covered in bibliography [4] [5] [6] [7] [8].

Tensegrities, also defined as “kinematically indeterminate and re-configurable systems in which all parts coexist in a dynamic equilibrium” [9], are characterized by an optimum relation of geometric, architectural and structural expression.

The morphological features and variations of tensegrity networks are also extensively covered in literature [10] [5] [9]. Double layer tensegrity networks that occur from the assembly of tensegrity units of prismatic or pyramidal shape, are shown to be the most appropriate for building applications. Their form depends on the configuration of the composing units, mostly of triangular or square base, and the method by which units are attached to each other to form a network. In order to maintain bar independence throughout the structure several patterns for connecting the units have been proposed [11] [6] (Figures 1 and 2).

Although researchers from various disciplines have been experimenting with tensegrity for over a decade, the implementation of tensegrity in the built environment has been very slow. Geometric and conceptual complexity that characterizes them is among the reasons that hindered the

![Figure 1. Tensegrity units of a) triangular base, b) square base](image)
use of tensegrity structures in construction [6]. An important feature of tensegrity structures is that they are potentially deployable. Based on this, a method was recently developed for deploying tensegrity structures, which makes possible the rapid, on site, modular assembly of deployable tensegrity units, and the construction of alternate configurations by re-using the same modules [1]. This is possible because individual modules are not connected at the nodes but by cable overlap, which allows for varying the pattern of cable overlap between adjacent units and as a result several configurations can be achieved. Among the possible configurations enabled by this new technology, structures of vaulted shape are the most probable to draw the attention of the construction industry.

2. Challenges in the design of tensegrity structures

When tensegrity is considered for building design, one of the most challenging issues is the generation of alternate spatial configurations of a tensegrity network. This is particularly important, when the same tensegrity units are to be used in more than one patterns of spatial arrangement. Indeed, as already reported in relevant literature, the geometric complexity of tensegrity structures, which is inherent to their structural and mechanical basis, makes their study difficult, and is responsible to a large extent for their limited application in building design [6].

Although patterns and general rules for the assembly of double layer tensegrity networks already exist, the geometry that occurs when these patterns are applied is very hard to determine and to graphically generate. Unlike regular space trusses, tensegrity networks are composed of tensegrity units which are of antiprismatic shape with bars lying on asymptotic axes, and therefore do not fall within the range of regular polyhedra. In addition, in order to ensure cable continuity and bar independence throughout the structure, as seen in Figure 3 that illustrates an assembly of four square-based units, the units do not connect end to end, but, by a partial overlap of their upper and lower base cables. Taking the
above into account, the configuration of tensegrity networks cannot be addressed as a regular space packing problem. A main issue therefore was to determine the geometric basis of the problem; and to come up with a method that takes into account the interdependence of the various parameters and constraints of the problem.

A geometric approach for the investigation of the spatial configuration of tensegrity structures of spherical shape, composed of tensegrity units of square base, has already been developed [2]. Specifically, fundamental geometric principles that rule the generation of form in tensegrity structures have been identified and demonstrated, and a method for the development of 3D digital models of spherical tensegrity networks, based on the integration of Descriptive Geometry procedures with Computer Aided Design tools, has been formulated. The method allows for the visualization of the effect of each of the interrelated parameters, such as proportions and size of units, overlap of upper and lower unit cable bases of adjacent units, and structure height and curvature, which determine the shape of a tensegrity network, and allows designers to resolve and visualize the complex geometry of such structures. A feature of the method and a key issue in determining the configuration of a network is the construction of geometric loci which allows for studies on the effect of each one of the interrelated parameters in the design of a tensegrity network.

Though the above method is of critical importance since it sets the geometric principles for the generation of any tensegrity network, it also presents limitations because of difficulties the designers may encounter in understanding and pursuing the step by step geometric construction process proposed by the method, and the time involved, particularly when a number of different configurations need to be generated during the design.
exploration process. Extensive experimentation with the effect of various parameters, such as the overlap between the upper or lower bases, renders the method rather inefficient. In addition, the method at its current stage of development can be directly applied to spherical configurations only. To address vaulted configurations, additional geometric relationships and rules need to be derived and a graphical representation procedure.

To address these issues a new approach to the design of vaulted configurations that encompasses the geometric basis of the problem into an analytical parametric expression has been developed and discussed in the following sections.

3. Vaulted configurations: geometry generation

Vaulted structures are understood in pure geometric terms as structures of cylindrical shape. In this paper they are also referred to as “single curvature” networks to distinguish them from spherical networks, which are configurations of double curvature. For the generation of their geometry the following objectives had to be met:

I) Define the geometric principles and general rules that apply to vaulted tensegrity networks.

II) Develop mathematical expressions to address parameter interdependence and constraints for various construction/scenarios.

III) Develop a software code to generate parametric models of tensegrity vaulted structures to facilitate the design process of projects incorporating such structures.

3.1. Geometric principles and relationships

Earlier research work has shown the following general geometric principle that applies to all regular tensegrity configurations with a curvature that are composed of identical square based units: When two square based tensegrity units with overlapping upper bases (upper and lower bases typically are of different dimensions) are placed with their axes parallel to each other, in order to determine their lower bases overlap, each unit needs to rotate around the common cord of the intersecting circles that circumscribe the upper bases of the two units, until their axes meet at a point, which is the center of curvature of the structure. As units start rotating, they maintain only one point in common, which is shown to be the middle point of the overlapping section of their upper bases (Figure 4) [12].

In order to determine the spatial geometry of vaulted tensegrity networks composed of tensegrity units of square base, additional geometric conditions need to be met. These are: a) the axes of overlapping units along the axis of the cylinder remain parallel to each other and perpendicular to the direction of the cylinder’s axis, and b) the axes of adjacent overlapping units along the circumference intersect at a point that lies on the axis of the cylinder.
Based on the geometric principles and rules that pertain to vaulted configurations, and considering unit-re-usability, which is in accordance to the approach taken in this study, additional geometric relationships that apply when unit dimensions are given, are as follows: a) the upper base-overlap and the lower base-overlap of adjacent units along the axis of the cylinder are fixed, b) the upper base-overlap and the lower base-overlap of adjacent units along the circumference of the vault can be varied so that different curvatures can be achieved. According to this, when a value for the upper overlap is given, the value for the lower overlap can be derived, and inversely, once the lower overlap is given, the upper can be derived.

3.2. Algorithm development

The geometric conditions and principles that determine the form of vaulted tensegrity networks were implemented in an algorithm that allows for the generation of the geometry of vaulted tensegrity networks from (a) the dimensions of composing units, and (b) unit-overlap conditions.

The algorithm integrates a step by step geometric design process. Specifically an initial step in the process consists of shifting the centers of adjacent units with overlapping upper bases along the axis of the cylinder, until their lower bases meet. Then, the required rotation angle between adjacent units along the circumference is calculated, so that both upper and lower bases meet (Figure 5); at this stage the point where the axes of adjacent units along the circumference meet, after rotation, is also determined. This provides the radius and the curvature of the structure. Subsequently, all tensegrity units located across the axis of the cylinder are rotated at the calculated rotation angle. A 3D rotation matrix is used to rotate the tensegrity units, and then the 3D coordinates of all nodes are calculated. A diagram indicating the steps in the algorithm development process is shown in Figure 6, and a description of the analytic geometry expressions used in each one of the steps of the algorithm follows.
3.2.1 Shifting of units along the axis of the cylinder

The scheme in Figure 5a shows two rows of tensegrity units with overlapping upper bases along the axis of the cylinder. When the dimensions of tensegrity units are given, the upper and lower base overlap along the axis of the cylinder can be calculated by following the trigonometric function indicated in the scheme of Figure 7. Specifically, if \( d_1 \) is half of overlap size, \( d_2 \) is half of top base, and \( r \) is half the length between the centers of adjacent units, the unknown \( d_2 \), calculated from the triangle \( o, q_2, q_3 \), is:

\[
d_2 = -2d_1 + \sqrt{4d_1^2 - 8(d_1^2 - r^2)}
\]
3.2.2 Rotation of units along the circumference of the cylinder

The angle of rotation between adjacent units along the circumference is calculated by determining the intersection point between vectors $u$ and $v$ as indicated in the scheme of Figure 8.

In order to express this process mathematically, a general analytical geometry procedure for determining the intersection point of two lines in a 3D space is utilized. More specifically, for determining the angle by which each unit needs to be rotated with respect to the axis of the cylinder, the process described below has been followed:

We first need to determine the intersection of the lower base with a line that is perpendicular to the axis of the cylinder and passes through the mid point of the upper base overlap. Defining zero, (0), as the global origin, as indicated in the scheme in Figure 8, and $P1$ and $P2$ as the lower base vertices on the $x$-$y$ plane measured from $0$, and $P3$, $P4$, as unit value points on the two directions of the $y$ axis, let line $L_1 = P_1 + u(P_2 - P_1)$ and line $L_2 = P_1 + w(P_3 - P_1)$ and substitute $P_2 - P_1$ with $D_1$ and $P_4 - P_3$ with $D_2$.

Since the dot product of lines $L_1$ and $L_2$ with their perpendicular vector, $(P_3 \times D_2)$, is zero, (0), the following expression is derived:

$$ (P_3 \times D_2)(P_1 + uD_1) = (P_3 \times D_2)(P_1 + wD_2) = 0 $$

Accordingly, the values of scalars $u$ and $v$ are derived as follows:

$$ u = \frac{(P_1 \times D_1) \cdot P_3}{(P_1 \times D_1) \cdot D_1} \quad v = -\frac{(P_1 \times D_1) \cdot P_3}{(P_1 \times D_1) \cdot D_2} \quad (1) $$

After finding intersection point $Q_1$, $Q_2$ can be also found by following the same process. The rotation angle of adjacent units, $\varphi$, is calculated from the dot products of vectors $u$ and $v$ that originate from 0 and end at intersection points $Q_1$ and $Q_2$.

$$ \varphi = \cos \left( \frac{u \cdot v + u \cdot v}{\|u\| \|v\|} \right) \quad (2) $$

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3.2.3 Geometric conditions and constraints for various design/ construction scenarios

The boundaries of possible solutions to a parametric vaulted structure geometry that consists of units of given dimensions are defined by calculating the unit overlap values that generate maximum and minimum structure curvatures. Specifically, as shown in Figure 9, maximum curvature is achieved when the overlap of adjacent units along the cylinder’s axis becomes zero, (0), and minimum curvature is achieved when the overlap of adjacent units along the cylinder’s axis is equal to the overlap of adjacent units along cylinder’s circumference. In this instance the overlap on both upper and lower cable bases, is the same and the resulting surface is a spherical structure of zero curvature.

A constraint regarding the proportions of the selected unit that results from the geometry in Figure 7 and is integrated into the algorithm is that the unit’s lower base should be greater than the $1/\sqrt{2}$ of the length of the upper base, otherwise the overlap along the cylinder axis can not be generated.
3.2.4 Form generation and graphical representation

Based on the algorithms, two codes to calculate and graphically generate the parametric form of vaulted tensegrity structures, that address two design/construction scenarios have been developed.

The first code applies to instances when the tensegrity unit is given and the configuration of the structure that results from a given unit size, proportions, and assembly pattern needs to be determined. The code enables the user to input known values of unit dimensions. Subsequently, he can modify the overlap length of adjacent units along the circumference, and the number of units in both directions, to create a new geometry. The output provides the parameters required to generate the geometry of the structure. These are: a) radius of the vault, b) structure’s angle of opening.

Based on the algorithm, once the value for the upper overlap is given, the value for the lower overlap is derived, and inversely, once the lower overlap is given, the upper can be derived. In general, by increasing the overlap of adjacent units along the circumference, the curvature decreases. The upper base-overlap and the lower base-overlap of adjacent units along the axis of the cylinder, as already mentioned occur from the dimensions of the re-usable unit and cannot be changed by the designer.

The second one applies to cases where in addition to unit dimensions, the curvature of the structure is given, and the unit overlap pattern for the given curvature needs to be determined. The code enables the user to input known values of unit dimensions inputting unit dimensions, and the desired structure’s curvature, and then lets the program calculate the corresponding overlap values between adjacent units along the circumference. The output provides numerical values for a) radius of the structure, c) angle of rotation between adjacent units and d) overlap of lower and upper bases. After the number of units in each direction is decided, the geometry of the entire structure is generated. For this particular scenario, the algorithm includes constraint equations, that determine the max unit dimensions values that will permit a given structure radius.

Additional general observations with regard to the proportions of the selected re-usable unit that apply when overlap values for adjacent units along the circumference of the cylinder are given, and result from the application of the codes are: a) by increasing unit upper/lower base ratio, the curvature increases, b) when the height of units remains fixed, a higher upper/lower base ratio gives a lower curvature, and respective the lower the upper/lower base ratio the higher the curvature. Similarly, when units upper/lower base ratio is fixed, the greater the unit height, the lower the curvature, and inversely, the lower the unit height, the higher the curvature.

The codes were developed in a MATLAB software environment that provides a graphical interface and enables the visualization of parametric models of vaulted tensegrity structures. Figure 10 shows models of vaulted networks with varying curvature and number of units.
Additional general observations with regard to the proportions of the selected re-usable unit that apply when overlap values for adjacent units along the circumference of the cylinder are given, and result from the application of the codes are: a) by increasing unit upper/ lower base ratio, the curvature increases, b) when the height of units remains fixed, a higher upper/ lower base ratio gives a lower curvature, and respective the lower the upper/ lower base ratio the higher the curvature. Similarly, when units upper/ lower base ratio is fixed, the greater the unit height, the lower the curvature, and inversely, the lower the unit height, the higher the curvature.

4. Application to a design entry for an ephemeral structure for the 2004 Athens Olympics

The concept of tensegrity was central to the design development of a semi-open exhibition space, which was submitted as an entry to the international architectural competition titled “Ephemeral Structure for the City of Athens”. The competition, which took place in the fall of 2002, was part of the Cultural Olympiad for the 2004 Athens Olympics.

The ephemeral structures, according to the competition program, were to be mounted and remain in a site only for a limited amount of time. They were then to be disassembled and transported to another cityscape. A contemporary cityscape is defined in the program as “a site for experimentation, a site of practice, and a site of thought” [13]. The ephemeral structures were thus expected to promote innovation in meeting practical needs, as well as to respond to theoretical issues raised by their temporary nature. According to the category-specific competition program, the main requirement for the semi-open exhibition space was to design a structure in which the level of spatial enclosure varies in accordance to the specific needs of the exhibitions or installations to be hosted. The total area of the semi-open exhibition space, in its closed form, was supposed not exceed 50 m².

Figure 10. Models of vaulted networks with varying curvature and number of units
4.1. Features of the proposed design

The main feature of the proposed exhibition space is a vaulted dome composed of interconnected detachable and deployable tensegrity units. The metaphors and associations of tensegrity to the spirit of the Olympics and the ephemeral nature of the proposed structures are numerous. One explicit association is the parallel of the continuum of tension/compression in tensegrity systems to the bone/muscle interaction in the human body in motion.

According to the competition, the ephemeral structures are also to be seen as parasites. In this regard, the complex network of cables and nodes, inherent to the architectural expression of the tensegrity concept, generates multiple grid lines, which can be used in determining the relationship of the structure to its hosting urban context (Figure 11). The proposed structure is not intended to reside in the urban infrastructure, but to intrude into the city's network of motion lines, align with one or more of them, and provoke a flow through it. The same system of grids would permit parasite structures, belonging to the same city network, to relate to each other with the use of sensors mounted on the nodes of the structure. As the structure migrated from one cityscape to another, the number of grid lines would keep increasing which will allow the public to trace previous stages in its life.

Figure 11: "Semi-Open Exhibit Space": Entry for the international competition on the design of "Ephemeral Structure for the City of Athens."
At a different level of interpretation, the proposed tensegrity structure performs as a hard shell, or exoskeleton, that supports a system of reconfigurable partitions/doors that extend the structure to the direction of flow through it, as well as a system of display panels, raised over the ground and attached to the tensegrity roof bar nodes. The nodes that support the panels support and penetrate a translucent and stretchable fabric covering membrane that reveals from the interior the view of the floating bars against the sky-dome. As the ephemeral structure moves to a different urban site, the tensegrity vault, as well as the space defining partitions, can change configurations to adjust to different functional or contextual requirements.

4.2. Tensegrity vault: form exploration

For the design of the tensegrity vault for the proposed ephemeral structure, various factors had to be taken into account including technical considerations such as constructability, efficiency of deployment and structural performance. Each of the main parameters in the design of a tensegrity network (number of units, proportions of unit, size of units and overlap) as indicated earlier, is linked to the others, and any change in one parameter affects not only the form of the structure but also the above mentioned considerations. As an example, since the structure requires the on site assembly of modules, reducing the total number of units is, in principle, a desired feature. However, reducing the size of units, and maintaining their proportions, will significantly increase the depth of the structure. From a purely geometric point of view, to maintain the same depth of structure, when larger units are used, requires reducing unit height, and thus changing unit proportions, and possibly changing upper and lower overlaps. However, lowering unit height and keeping the same size of lower bases, will lead to units with bars at a very low angle which will affect structural performance.
Figure 13 shows two configurations that were originally considered for the design of the ephemeral structures. Both configurations are identical in terms of their basic geometric features: overall width, depth of structure and curvature. Their main difference is that the first one consists of four units and the second of six. From a constructability point of view the first one is easier to assemble, due to the reduced number of units. On the other hand, in the second configuration bars are shorter, less prone to buckling, and at higher angle than the ones in the first configuration, which may result to improved structural performance. It becomes obvious at this point, that additional analysis is required for determining the optimal design.

In addition to the above, since in the proposed solution space defining elements, such as roof membrane and display panels, are all attached to the nodes on the interior layer of the structure, any change in the system of nodes, will affect space features and spatial definition (Figure 14).

Although an optimized approach that takes into account all of the above aspects in the geometric design of the network was not the objective of this study, the parametric code which was mentioned in the introduction...
greatly facilitated form exploration and experimentation with various design parameters. After a number of alternate configurations were explored in the MATLAB environment, two different configurations of the same square footage, composed of 5 by 7 units and 4 by 6 units, respectively, were selected for development in a CAD environment. From the MATLAB model, angular measurements, measurements of radii and distances, as well as node coordinates were obtained. These values were used for the generation of the selected network in the CAD environment. Figures 15a and 15b show consecutive stages in the development of the final solution. The approach followed in the design of the ephemeral structure has also indicated that a further automation of the process, which will allow for any model generated in MATLAB to be automatically recreated in a CAD environment, is also needed.

![Figure 15. Stages in the development of the final solution](image)
5. Conclusions

The promising applications of tensegrity structures in the deployable building industry provided the broader framework and context for this study. Resolving difficulties in obtaining the geometry of vaulted configurations, that architects encounter particularly when they consider the use of a new approach to the design and construction of tensegrity structures that allows for unit re-usability and structure re-configuration, was the main motivation.

The paper presents a parametric method for generating tensegrity networks that enables the experimentation with many design variables. Specifically geometric design algorithms were developed to determine the form of vaulted tensegrity networks. A mathematical code that integrates these geometric algorithms and simulates the design process performed by an architect when different design/construction scenarios are considered, has also been developed. The code allows for the generation of alternate configurations of 3D models of the structure in a graphical environment.

The method has successfully addressed the challenges faced during the design stage of a tensegrity vaulted structure that was submitted as an entry to the “ephemeral structures for the city of Athens” competition. The approach followed in the design of the ephemeral structure has indicated that a further automation of the process is needed so that the models generated by the code will be immediately imported, or automatically recreated in a Computer Aided Design environment.

In general, the parametric method has been shown to resolve issues of geometric complexity that are inherent to the configuration and representation of vaulted tensegrity structures. However, the models of structures developed in this manner are of initial geometry only, and do not address displacements that occur when material weight, cable prestress, and loading conditions are taken into account. An improved method that integrates structural analysis into the geometric design code to generate models of pre-stressed geometry and applied loads, is under development.

Note: This paper is a development of a 2003 ACADIA conference publication titled “A Digital Parametric Approach to the Design of a Tensegrity Vaulted Dome for an Ephemeral Structure for the 2004 Olympics” by the same authors.

References


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