Dynamical structural modeling
A collaborative design exploration
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This study is based on a generative performative modeling approach that engages architects and structural engineers in close dialogue. We focus on knowledge shared between engineers and architects to apply the Finite Element Analysis based structural design technique Evolutionary Structural Optimization (ESO) as a way to understand or corroborate the performance factors that are significant in determining architectural form. ESO is very close conceptually to the dynamical system of matter and forces of growth itself. It has parallels both mathematical and metaphorical with natural evolution and morphogenesis so it has been poignant to apply the approach to a formal architectural case study in which the generative influence of these processes is inherent.
1. Background

This research unites two groups working respectively in Spatial Information Architecture and Innovative Structures. The overall context is an investigation of the opportunities for closer collaborative design activity using compatible tools.

One of the long term research objectives of this University-based architecture; design; structural engineering research team is to investigate the generative opportunities for the technique known as Evolutionary Structural Optimization [ESO] in architectural design cycles or iterations from conceptual design onwards. An interim step in understanding how this could be done has been to apply the technique to architectural designs that already have a degree of formal determination arrived at with or without regard to structural optimization. The choice of case studies has been predicated on the detailed level of knowledge of the examples that members of the team have already gained through scholarship and project-based design research for construction.

These project-based case studies are pure research using design data available in the public domain. However, they are undertaken in parallel to work-in-progress on applied design research projects. One of the case studies investigated to date is the Passion Façade for Antoni Gaudí’s Sagrada Familia church. Using a simplified abstraction of the structure derived from the photograph of Gaudí’s original drawing, a series of optimizations were run, progressively revising the inputs and parameters to achieve an output more closely resembling the form of Gaudí’s design, in a sense, reverse
engineering the structural conditions for this FEA technique to generate particular optimized solutions. Subsequently, the process was extended beyond reproducing the conditions to produce an optimized solution resembling Gaudí’s design to investigate the formal direction a design predicated purely on structural optimization for efficient deployment of material might take initially under gravity loading and subsequently also with dynamic lateral loading.

The choice of Gaudí’s work clearly has a poetic and scientific significance beyond the existence of scholarship within the group. His understanding of “structural optimization” in natural form, development of innovative optimized structures using analogue modeling techniques and the reference to growth and morphogenesis within his work makes this an experimental site of particular significance [1],[2],[3],[4].

2. Funicular structures

Gaudí’s use of funicular structural systems to develop his architectural designs is well known [2],[4]. The original poly-funicular model for the resolution of the design for the Colonia Güell church has been reconstructed and is now displayed at the museum in the crypt of the Sagrada Familia church [5]. To describe the concept of funicular structural systems, it is worth quoting one paragraph from Shodek [6].

“A cable subjected to external loads will obviously deform in a way dependent on the magnitude and location of the external forces. The form acquired is often called the funicular shape of the cable (the term funicular is derived from the Latin word for “rope”). Only tension forces will be developed in the cable. Inverting the structural form obtained will yield a new structure that is exactly analogous to the cable structure except that compression rather than tension forces are developed. Theoretically, the shape found could be constructed of simply stacked elements that are non rigidly connected (a “compression chain”) and the resultant structure would be stable.”

There is a long history of the study of hanging structures to find the ideal form for arches in compression in construction. Hook hints at the answer to his 1670 question to the Royal Society: how to find the form of a stable arch and the forces exerted on the abutments; in a paper on watches in 1676 where he notes that the same way that a flexible thread hangs, but inverted, sustains a rigid arch [7]. The use of the term ‘catenary’ for the curve assumed by a hanging chain (uniformly distributed gravity load) is first attributed to Huygens in 1690 but already in 1669 Jungius had disproved Galileo’s claim that the curve of a chain hanging under gravity would be a parabola. (The catenary is in fact a hyperbolic cosine curve, the curve traced out by the focus of a parabola as you roll the parabola along a straight line [8].) During the nineteenth century graphical methods for determining structures were developed abstracting the polygons of lines of forces based
on the knowledge from funicular models and after 1870 this was the most widely adopted approach to structural calculation and the principal method that would have been introduced to Gaudí through his architectural structures education in the 1870s. He took up the idea of funicular modeling and, probably for the first time, extended the approach beyond verifying or achieving the structural stability of a particular design. He adopted the poly-funicular model as a means to design from the start with infinitely variable equilibrated structures [9]. The equivalent process in contemporary practice is often referred to as generative form-finding [10].
3. Evolutionary structural optimisation

Generally engineers rely on a trial and error process for establishing a structure once a design has been created by an architect, or an architect establishes a design without a resolved structural concept. Gaudí considered the structural system as an integral part of the architectural design but the structural design process is generally a reactive process of calculations based on a predetermined conceptual geometry. Even contemporary digital Finite Element Analysis is a static process, where the designer determines the deflections, stresses and forces on a structure and then uses this information to determine the appropriate member sizes (i.e. beams, columns, slabs etc.) based on the original building geometry. Generally the areas/members that are most highly stressed are used to determine the buildings’ elements sizes, and therefore there can be significant over design and redundancy in the building. It is not necessarily the most efficient design and use of resources. This can be described as a passive approach to the results of the structure.

Within the last decade a dynamic approach to the use of the Finite Element Analysis has been developed through the use of structural optimisation. This technique seeks the most efficient use of material by altering the shape and topology and geometry of the building and its various elements. Therefore there is a direct and rational connection between form and material [11]. The method of Evolutionary Structural Optimisation (ESO), developed approximately 100 years after Gaudí’s first experiments with hanging models, is a contemporary digital procedure that can produce novel forms of funicular structures.

Three of the authors (Xie, Felicetti and Tang) modified the original ESO method pioneered by Xie and Steven in the early 1990s [12]. The original ESO method consisted of removing redundant material at each iteration using von Mises stress criterion. Instead of von Mises criteria structures in the modified method can be evolved such that all remaining elements are in compression (as examples shown in this paper), or optimizing such that all remaining elements are in tension [13].

In this automated procedure each iteration consists of a finite element analysis (to determine stresses) and then removing inefficient or inappropriate elements. For example, in order to design compression only structures, elements with the highest level of tensile stresses will be removed at the end of each finite element analysis. The cycle (iteration) of finite element analysis and element removal is repeated many times until a desired geometry is produced. Typically, the number of iterations ranges from 10 to 100.
Figure 5. A block is given a uniformly distributed gravity load and assumes a catenary as it evolves.

Figure 6. This block has two non-design zones defined for the waists of the columns. The column shape is otherwise determined by iterative removal of the least stressed material.

Figure 7. The inclined nave columns of the Sagrada Familia church with branching elements at the upper level.
3.1. Case study

The ESO method creates the geometries based on a first principle mechanistic design process, rather than seeking to input the actual geometry of Gaudí's buildings into a finite element model and analyzing forces, displacements, stresses and strains. The case study focused on the inclination and general form of, firstly, the (as yet un-built) columns in the inclined colonnade in the upper section of the Passion Façade and subsequently the (built) lower columns supporting the porch with the crucifixion scene and colonnade above. The loads applied were initially vertical gravity loads only. Later studies have extended this to investigate the possible impact of dynamic lateral wind loading.
3.2. Description of process

The following is a series of trials undertaken to reverse engineer the final structural form of the Passion Facade represented by the photograph in Figure 1. Careful consideration needs to be given to the mathematical model (Finite Element Initial model) to ensure that appropriate boundary conditions, restraints, loads and design/non design regions are accurately modelled. The series of examples below are results of various trials in adjusting these conditions.

The critical conditions that were found to influence the analysis and optimization process were:

1. Non design region. The strategy in the initial model was to create a non design region along the centre line of the six main supporting columns, and also at the centre line of each of the upper level columns to the colonnade (Figure 10). It was envisioned that this would be necessary so that particular columns would not be removed by the ESO process. However it was found that creating non design regions for the columns was restrictive to the outcome. The non design regions attracted load away from directions of natural force flows and load paths, and therefore resulted in these areas being lightly stressed and removed in the ESO process. Later models turned off the non design region in the columns, giving these regions the opportunity to evolve also, thereby not hindering the natural path of evolution.

2. The base support condition. Initially base fixity was applied to each node of the finite elements at the base over the complete 4m x 4m initial design domain. Later models reduced the support zone to 1m x 1m.

3. Slots in the upper level colonnade.

4. Stiffness of the large gable lintel over the upper level colonnade (the large non design region), and the thin gable lintel to the underside of the upper level colonnade (the thin non design region.)
In Figure 10 note that the dark regions indicate areas that are “non design” regions, i.e. areas that although subject to stress analysis are not modified by the Evolutionary Structural Optimization process. The grey regions indicate “design” regions, i.e. areas that can be modified by the Evolutionary Structural Optimization process. Base support fixity is indicated by the series of triangles at the base of each column. The facade is subjected to vertical gravity loads only. It is supported by a wall, the supporting points being represented by the triangular elements to the right of the Section A-A. As this was not the aspect of interest, it was not necessary to include the supporting wall in the structural finite element model.

A: Design Analysis Model 1 – non design core to each column

This initial model included a core region to each of the columns that was a non design domain, that is, the cores could not be removed by the structural optimization process.
Note that in Figure 12 the non design core regions to columns are clearly in the view after 20 iterations. Note also how the non design column cores at the lower level are supporting the thin non design gable lintel and thus creating a redundant arch between the centre two columns, which is becoming thinner with progressive iterations. The upper level non design column cores are not co-linear with the direction of evolution of the upper level colonnade. The series of colonnade arches directly under the ridge of the upper gable lintel have disappeared, redundant due to being lightly stressed.

As it was revealed that the non design core regions of the columns were influencing the direction of evolution, it was decided that this constraint was unnecessary and that the structure should be given more freedom.

B: Design Model 2 – non design core to columns turned off

The non design cores of the columns were turned off in this model. That is, they were free to evolve with the rest of the structure. However the thick gable and the lower thin gable under the colonnade were retained as non design elements.

In Figure 13, whilst the non design column cores are no longer influencing the structural evolution, the rigid upper thick and lower thin gables are resulting in the loads being transferred to the outer columns resulting in a redundant arch. Again the non design domains were having a significant effect on the direction of evolution. Also with respect to modeling the real masonry structure, treating the gables as a continuous rigid elements is not correct. The masonry elements are, in reality, a series of interconnected blocks that can move with a degree of independence.

C: Design Model 3 – non design core to columns turned off and roller supports of the wall

In the next model the wall supports were changed (Figure 10, Section A-A). The model was now supported by the wall by rollers rather than pins, so that the wall would thus not attract much of the vertical load induced by the upper block gravity, and would have a minimum influence on the direction of evolution. The final façade model was smoothed using the
commercial software Rhino. As can be seen from Figure 14 above, the results produced by the criteria adopted in this third model produced a result with respect to the modelling of the colonnade and the 6 lower columns that was much more akin to the original image. A structurally efficient solution would have the columns to the upper colonnade sloping progressively inwards to the centre of the facade.

D: Design model 4- the base fixity area for the lower columns has been changed from square to cruciform configuration

This final gravity-only model was refined further by changing the bases of the lower columns from 1m x 1m squares to cruciform definition. It was
also run for a greater number of iterations. It passed through a point very close to the column distribution and progressively changing inclination of the columns in the colonnade in Gaudí’s drawing. Finally it arrives at a point where each of the lower columns supports a well defined group of branching upper columns, a configuration more akin to the schema for Gaudí’s design for the central nave columns. At this point the structural optimization has subsumed other expressive intentions for the passion façade in which the upper columns appear as irregular but unmistakably repeating elements in a colonnade. In the drawing the colonnade is an array of bone-like forms suggestive of both the tomb and Christ’s physical suffering in the crucifixion scene depicted in the sculpture below.
maximum wind of earthquake expectations? To the penultimate model described and shown in D above, equal lateral acceleration was applied from the north and south (or right and left of the images.) The gravity loading and support conditions and constraints otherwise remained unaltered. The result shown in Figure 17 is a structure optimized for the most efficient use of material in a compression only structure to resist both gravity and lateral loading. As shown the evolved form diverges from the gravity only case.

Whilst vertical gravity loading produces a symmetrical result mirrored about a central vertical plane, wind and earthquake forces are only applied from one lateral direction at each time period. Clearly a building/structure is subjected to wind or earthquake forces from different directions. The model is therefore subjected to multiple lateral load cases and a resultant envelope of stresses produced at each iteration. Redundant material is then deleted and the next iteration proceeds. In this case the symmetry of the model is maintained. If the load were applied from one direction only an asymmetrical form would evolve.

Whilst the structure subjected only to vertical gravity loads results in columnar elements that have a tree-like quality in supporting the upper lintel, the column elements in the laterally loaded model are angled in an A frame or cross braced arrangement in response to the lateral forces.

The ESO technique enables loadings to be applied in any direction or in combinations of directions at the same time, allowing optimisation of structures in all three dimensions simultaneously. This is a situation that it is
not possible to emulate with traditional funicular models. It is interesting to note that Gaudi’s drawing shown in Figure 1 contains elements of both the last two ESO models presented. There are lateral stability benefits from the progressive inward slope of the upper columns.

3.3. Conclusion

At the outset, the focus in this case study was on abstracting the geometrical model from original 1917 photograph of Gaudi’s drawn design for the Sagrada Familia church Passion Façade and giving as much skeletal geometric information as possible as a starting point for the optimization. Through the stages of the process this “starting point” information was progressively reduced, rather than increased, and, as the constraints were removed, the formal outcome of the optimization moved closer to Gaudi’s formal representation of the façade. This demonstrated that, not only is it vital to understand and appropriately model the constraint system and material properties (rather than try to impose a general geometrical solution), but to gain true value from ESO as a contributing generative architectural design tool, it is critical not to over-constrain the starting model. This makes perfect intuitive sense in considering the behaviour of analogous funicular models such as Gaudi’s hanging model for the Colonia Guell church but presents a challenge in digital optimization practice where all constraints are explicitly expressed rather than empirically observed as in the construction of an analogue model.

This particular exercise was a useful milestone for “calibrating” ESO applications in future projects. It also suggested that constructing simplified physical analogue models in which the material properties can be changed could be a useful parallel line of enquiry in building tacit knowledge of defining appropriate constraint systems as well as better understanding the historical precedent in this particular case [14].

Gaudi’s work applied structural optimization, through form finding funicular modeling of gravity loading. The opportunity that ESO offers through finite element analysis is complex architectural form finding under not only static gravity loading but also dynamic lateral loading. We have been able to ask the question: “what form would the optimized model assume with applied wind or earthquake loading?” This is something very difficult to represent in funicular or simple physical models. The formal output from this final exercise is so powerfully different from the architectural precedent that it opens up a raft of further questions about both the historical approaches to lateral loading in masonry buildings and the generative design possibilities that this represents.

This series of exercises has built an understanding shared across two disciplines of the opportunities and future research directions. Clearly we need to build on our understanding of how flexible early design intention models might be represented to give appropriate reign to the optimization process.
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