Performing Palladio

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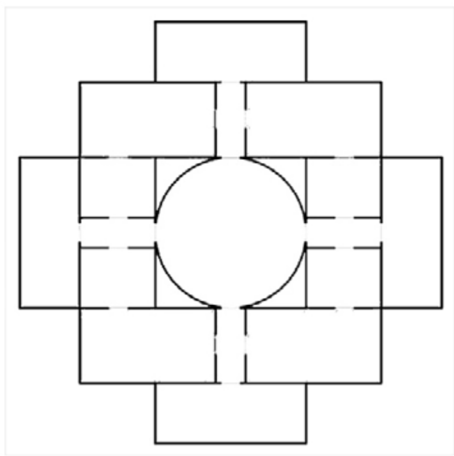
The relevance of music theory as an interpretive framework for the understanding of Palladio's work has been one of the most debated subjects in the realm of architectural theory and criticism. Typically the debate is quite abstract and it focuses on possible mappings between the ratios found in Palladio's plans and corresponding ratios used in contemporary musical temperaments. The paper here rather focuses on the actual performance of the ratios found in Palladio's work and the implications of this performance, melodic and harmonic, for the perception of the space for a situated observer/performer. To that extent the study suggests a model of mapping between space, sound and color and correlates that with polygon partition theory to simulate movement within a space. A brief account of the computer implementation with game engines technology is provided in the end. All examples to test these ideas are based on Palladio's Villa Capra.

1. INTRODUCTION

Palladian villas have long been termed harmonic. This has been discussed by many critics beginning soon after Palladio completed his designs. In his *Quattro Libri*, Palladio simply remarks that man is a microcosm that reflects the macrocosm of nature and that the rules of architecture refer to the rules of nature but Alberti, upon whose authority Palladio relied heavily, took his rules for proportion "from the musicians" and this is the general conception of what is meant by harmonic [1]. Wittkower, in 1949, argued that Palladio used proportions derived from music in his architecture on the theory that if harmonic proportions are beautiful to the ear, they will also please the eye and mind [2]. March discussed the classical proportionalities and the musical theories of Palladio's contemporary, Zarlino, in his analysis on Villa Emo [3]. The designs most closely tied with harmonic proportions come later in Palladio's oeuvre and were designed for patrons who were themselves interested in musical or architectural theory [4]. An educated man of the Renaissance era would have interpreted the musical and visual arts very differently than we do today [5]. For him, the geometrical and numerical codes contained in works of art would not always be instantly evident, but certainly present to be inspected, puzzled out, and discussed among friends and colleagues. Whether Palladio really used musical ratios in his buildings we will probably never know; it still possible though to reconstruct their sounds and their harmonies and suggest ways to inquire to what extent they might be considered harmonic or not.

This study provides a very brief history of various methods of correlating space and sound and takes advantage of the medium of color that traditionally has been used as an interface between architecture and music; color makes visible the sound which makes audible the ratios of the rooms. The study furthermore suggests a model of mapping between space, sound and color using formal systems based on theory of proportion, combinatorics and shape grammars. Here, the specific spaces that provide that prime material for the design of models of space, sound and color are all taken naturally from Palladio's work and especially from his Villa Capra, otherwise known as Villa Rotunda. The selection of this work has to do primarily with its historic significance within the whole corpus of Palladio's work and especially the significant position it enjoys within the bibliography on architecture and music as well as in contemporary architectural discourse [2], [4-7]. The paper provides four different models of patterns of movement and sight by an agent within a virtual space and concludes with the presentation of a software component based on game engines technologies that permits the interaction of an agent with the virtual space of the villa. A diagrammatic representation of the Villa Capra is shown in "Figure 1".



◀ Figure 1: Plan of Palladio's Villa Capra.

2. RATIO, COLOR, SOUND, SPACE

2.1. Arithmetic of ratios

A ratio is a relation between two numbers and *proportion* is a relation between two ratios. The least set of numbers that can establish a proportion is three. For three numbers x, y, z, and $x<y<z$, there are three possible outcomes of comparisons, one unique case of equality, $x:y = y:z$, and two cases of inequality, $x:y<y:z$ and $x:y>y:z$. For each case of inequality, there can be an infinite number of subcases with respect to the actual numbers involved in the comparison. Among these relationships, some are more significant than others; for example, for three numbers x, y, z, and $x<y<z$, if $(1/z)-(1/y)=(1/y)-(1/x)$, the inequality $x:y<y:z$ can be rewritten as an equality, namely, $(z-y)/z=(y-x)/x$. The problem has been nicely solved in antiquity by Greek mathematicians in a series of successive attempts, initially proposing two more such equalities by Archytas, the arithmetic and the harmonic mean, latter three more, possibly by Eudoxus, and finally two additional distinct sets of four inqualities by Nicomachus and Pappus respectively, with three overlapping cases among them, bringing the total number of inequalities to ten. These ten relationships of ratios plus the first initial relation of equality, the geometric mean, brought the number of comparisons to eleven and they were all treated informingly under the heading of the theory of means [8]. Among these eleven ways of comparing two ratios involving three numbers, only four survive in current discourse, the initial three, the geometric, arithmetic, and harmonic mean, and the extreme and mean ratio, otherwise currently known as the golden mean [9]. The defining relationships for all eleven means and the corresponding definitions for each of the terms with respect to the two other terms of the proportion are given in "Figure 2".

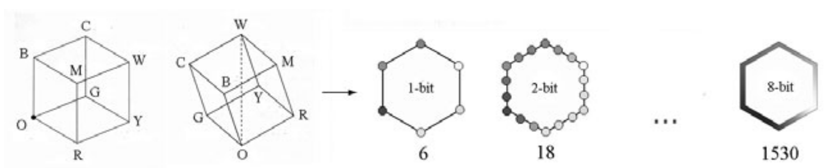
► Figure 2: The eleven means.

$P_1: \frac{z-y}{y-x}=\frac{x}{x},$	$x=2y-z$	$y=\frac{x+z}{2}$	$z=-x+2y$
$P_2: \frac{z-y}{y-x}=\frac{z}{y},$	$x=\frac{y^{22}}{z}$	$y=\sqrt{xz}$	$z=\frac{y}{x}$
$P_3: \frac{z-y}{y-x}=\frac{z}{x},$	$x=\frac{yz}{2z-y}$	$y=\frac{2xz}{x+z}$	$z=\frac{xy}{2x-y}$
$P_4: \frac{z-y}{y-x}=\frac{x}{z},$	$x=\frac{y\pm\sqrt{y^2-4(z^{22}-yz)}}{2}{}^2$	$y=\frac{x^2+z^2}{x+z}$	$z=\frac{y+\sqrt{y\ -4(x\ -xy)}}{2}$
$P_5: \frac{z-y}{y-x}=\frac{x}{y},$	$x=\frac{y\pm\sqrt{y(5y-4z)}}{2}$	$y=\frac{-x+z+\sqrt{4x^2+(-x+z)^2}}{2}$	$z=\frac{-x^2+xy+y^2}{y}$
$P_6: \frac{z-y}{y-x}=\frac{y}{z},$	$x=\frac{y^2-z(z-y)}{y}$	$y=\frac{x-z+\sqrt{4z^2+(-x+z)^2}}{2}$	$z=\frac{y+\sqrt{y^2-4(xy-y^2)}}{2}$
$P_7: \frac{z-x}{y-x}=\frac{z}{x},$	$x=z-\sqrt{z^2-yz}$	$y=\frac{2xz-x^2}{z}$	$z=\frac{x^2}{2x-y}$
$P_8: \frac{z-x}{z-y}=\frac{z}{x},$	$x=\frac{z\pm\sqrt{z(4y-3z)}}{2}$	$y=\frac{x^2-xz+z^2}{z}$	$z=\frac{x+y+\sqrt{(y-x)(y+3x)}}{2}$
$P_9: \frac{z-x}{y-x}=\frac{y}{x},$	$x=\frac{y+z-\sqrt{(y+z)^2-4y^2}}{2}$	$y=\frac{x+\sqrt{x(4z-3x)}}{x}$	$z=\frac{x^2-xy+y^2}{x}$
$P_{10}: \frac{z-x}{z-y}=\frac{y}{x},$	$x=z-y$	$y=z-x$	$z=x+y$
$P_{11}: \frac{z-x}{z-y}=\frac{z}{y},$	$x=\frac{(2y-z)z}{y}$	$y=\frac{z^2}{2z-x}$	$z=y+\sqrt{y^2-xy}$

2.2. Arithmetic of color

The history of color models is one of the most interesting stories in the history of science. The corresponding bibliography extends over a variety of diverse fields such as physics, psychology, aesthetics, philosophy, mathematics, arts, music and others [10]. The model used in this paper is based on the tri-stimulus theory of color perception that considers that any color perceived by the human eye may correspond to some mixture of red (R), green (G) and blue (B) light. The simplest color model consists of $2^3 = 8$ subsets of colors and it can be represented as a cube in a finite Cartesian space. The subsets of this space are {0, R, G, B, RG, RB, GB, RGB}, whereas red + green = yellow (RG=Y), red +blue = magenta (RB=M), green + blue = cyan (GB=C), red + green + blue = white (RGB=W) and 0, or no red, green and blue, equals to black (0). The same cube can be represented as a hexagon if it is seen through its internal diagonal; in this case the color space is represented as a hexagonal space whose central axis is the black/white axis and the pure hues without any mixture of black or white are on the periphery. The structure of the RGB color cube and its representation into a hexagon is shown in "Figure 3".

► Figure 3: The RGB color space.



The structure of the cube and the corresponding hexagon can be divided in any desired number of aliquot parts. The number k_n of possible divisions (n) of the hexagon is given by the formula (1).

$$k_n = 6\ (2n\text{-}1) \qquad (1)$$

In this study the application of the model for $n = 8$ is used to simulate the color space sRGB of computer graphics that is based on 1530 pure hues [11]. For two integers x and y, their ratio $x{:}y$ corresponds to one of these 1530 positions with an acceptable range according to the type (2).

$$\frac{\log \frac{x}{y}}{\log \sqrt[1530]{\frac{2}{1}}} = h_{1530} \qquad (2)$$

If for example the ratio $x{:}y$ is 3:2, then (2) becomes:

$$\frac{\log \frac{3}{2}}{\log \sqrt[1530]{\frac{2}{1}}} \cup 895 \qquad (3)$$

and 895 is equivalent with an RGB position (0,125, 125) producing a light blue. Any other rational number can correspond to a color in the model in this manner. The hue values of sRGB space are shown in "Table 1".

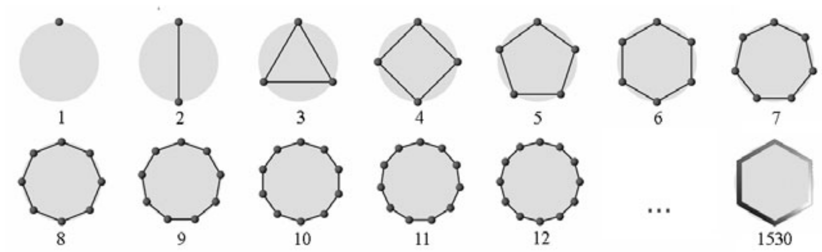
	Red	Yellow	Green	Light Blue	Blue	Pink/purple
R	255	255	0	0	0	255
G	0	255	255	255	0	0
B	0	0	0	255	255	255
	0-254	255-509	510-764	765-1019	1020-1274	1275-1529

◀ **Table 1: Hue values of sRGB space.**

2.3. Arithmetic of sound

The corresponding history of musical scales and their partition into small, discrete intervals is equally interesting, if not more. From the earliest treatises on the division of the *tetrachord* by Archytas of Tarentum with respect to arithmetic, geometric and harmonic means, to Plato's Pythagorean scale, to Zarlino's *just intonation* scale, to the *equal temperament scale* and to the twentieth century divisional experimentation, an array of different scalar divisions have been proposed [12]. Among all these ways that have been proposed on the division of the octave, the most accepted one in recent music theory is the equal tempered system which provides a division of the octave in twelve equal parts, namely in intervals of $2^{1/12}$ of the octave. This system, by making all semitones equal, sacrifices exact tuning but confers equal status in all keys and in all pitches of the octave. Equal temperament permits intervals to form a group under combination; combination of intervals in the equal tempered scale is isomorphic to the multiplication of integral powers of $2^{1/12}$, which is isomorphic to the addition of the set of integers; alternatively, combination of intervals in the equal tempered scale is homomorphic to combination of equivalent classes of intervals within an octave which is isomorphic to addition in the finite arithmetic modulo-twelve [13]. The success of the system and its ubiquitous application is due to the fact that the system has closure, that is to say, that smaller intervals do combine to produce intervals that are prescribed within the system.

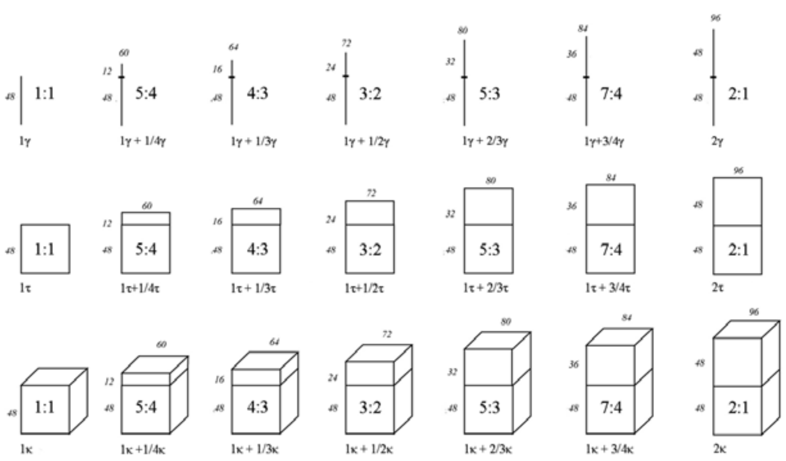
The structure that is used in this paper is based on the same idea of partition of the fundamental interval of the octave into equal parts with the key difference that the number of division of the scale has been increased to match the partition of color model into 1530 intervals. Any other case of partition of the interval of the octave would do it and in specific cases it could simulate historical models with 12, 17, 24, 48, 53, and so on [14]. Here the formula (1) was used to create a model of sound that is isomorphic with the color model sRGB that is mentioned above. A range of different partitions of the sound interval of the octave is shown in "Figure 4".



► Figure 4: *n*-partition of the interval of the octave.

2.4. Arithmetic of shape

The corresponding effort for the construction of an arithmetic of shape that describe space has produced various exciting and often conflicting models. Among all those systems that have been produced, it has also been suggested and for very good reasons indeed, that shapes representing space, function in a very different manner and they cannot comprise such algebras [15]. The model used here is a very simple application of the classical Pythagorean idea of representation of ratios in *n*-dimensional spaces for $n \leq 3$ [5]. Within this interpretative framework, relationships of lines, rectangles, and rectangular parallelepipeds may represent any ratio *x:y*. Other interpretations can be equally valid. In this paper all ratios are arranged onto a rectangular grid 1530x1530 units to create an uniform arithmetic frame for all shapes used. Any other measurement would suffice. A set of ordered ratios between an interval and its double is shown in "Figure 5".



◀ Figure 5: Pythagorean representations of ratios in *n*-dimensional spaces, for $n \leq 3$.

2.5. Parallel computations

All models above attempt to map specific ratios to systems of lines, rectangles, and rectangular parallelepipeds and corresponding frequencies of light and frequencies of sound; the goal is to provide a common framework to illustrate that given a ratio *x:y*, a rectangle in plan may in fact correspond to a specific sound and a specific color. Alternatively, the goal so far has been to design a working system to deal with explicitly with metaphorical concepts such as the sound of a space, the color of a space and the color of a sound. These three models are a good start but there are several difficulties that have not been addressed yet.

First, the actual arithmetic of ratios is differently represented in each medium. The illustration of the arithmetic of ratios with spatial compositions of rectangles can nicely be rendered following Euclid's definitions on the addition and subtraction of ratios; rectangles may be added or subtracted to provide a visual illustration of Euclid's definitions [5]. The illustration of the arithmetic of ratios in terms of frequencies of lights is not as intuitively apparent; the simultaneous appearance of multiple ratios in a two-dimensional composition is allowed only by lines and not by colors. Colors interact with one another, and they either cover one another or produce different ones. A pictorial composition of colored rectangles may well represent a specific set of relationships of ratios that correspond to specific frequencies of light but it does not permit the *emergence* of bigger ratios that can be found in the composition. The final result is a two-dimensional composition of the smallest possible rectangles whose ratios are associated with specific frequencies. Finally, the corresponding illustration of the arithmetic of ratios in terms of their frequencies of sound is equally non-intuitive; the simultaneous performance of multiple ratios in a two-dimensional composition may well denote a sum of ratios of sounds but it does not permit the discreteness of all the smaller ratios that are found in the composition. The final result is a one-dimensional composition, a unique composite sound-chord that consists of all frequencies together.

Second, the perception of each medium is very different for each case; overlapping sounds are processed by the ear differently than the mixing of light is processed by the eye; when two lights of different frequency overlap, their color values are added, but when two sounds overlap, they are distinctly heard with the addition of some overtones and beat frequencies. Similarly, the perception of space by a stationary or moving observer may differ radically with respect to the actual metric sizes involved in the dimensions of the spaces; rooms with similar ratios but different dimensions are perceived profoundly different if they are small or large.

Both set of questions lie in the heart of the inquiry here. The first set of questions regarding the perception of the arithmetic of ratios in the realms of space, color and sound is the subject matter of our model and it follows below. The second set of questions can be easily addressed here. Despite combinational differences in sound and light, we can compare the frequency of light to the frequency of sound and by shifting a light frequency many octaves down, we can bring a usable tonic frequency to an audible range such as, say, the range of a cello. Metric differences between smaller or bigger spaces can easily be addressed too: two rooms of the same ratios may have the same sound frequency, but the larger of the two can be louder in a proportional relationship to the area of the rectangle. Taking into account these last two provisions all multimodal mappings for the Villa Capra are shown in "Table 2".

► Table 2: Computation of frequencies, volume, and colors produced by room ratios in Villa Capra; tonic frequency is chosen as 160.619Hz.

Ratio	Frequency	Area	Maximum volume factor	Hue value out of 1530	sRGB (R,G,B) value
30:30 (1:1)	160.619 Hz	706.858	1.0000	0	(255,0,0)
15:11	219.026 Hz	165.000	0.2334	685	(0,255,175)
26:15	278.406 Hz	390.000	0.5517	1214	(194,0,255)
15:6 (5:2)	401.548 Hz	90.000	0.1273	2023	(17,255,0)
30:12 (5:2)	401.548 Hz	360.000	0.5093	2023	(17,255,0)

3. PARTITIONS

The mapping of ratios of rectangles to frequencies of color and sound provides a nice dataset to correlate these media but still it says nothing about the ways these media may relate to each other and in particular how they relate to an observer within this space. Various methods for conveying relationships related to perception of space exist and all are based on specific decompositions of polygons that comprise the configuration. Here we propose four models for integrating space, color and sound all with respect to a situated observer. A brief account of those follows below. The primary material for these representations is taken here from the field of computational geometry and in particular, the polygon partition theory [16]. Two polygon partitions are considered here to provide the initial framework for our model; the convex partition and the isovist partition.

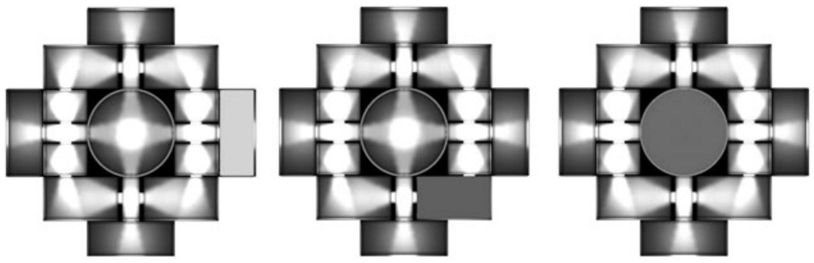
The convex partition of a polygon is the partition of the polygon in smaller polygons such that for any two points that belong to any of these smaller polygons, a line that joins them will completely belong to these spaces; if the line that joins these two points intersects with the boundary of the polygon, then the polygon is not convex and the partition is not a convex partition. The maximal convex partition of a polygon is the one that partitions the polygon in the smallest number of parts. Maximal convex dissections of architectural plans of rectangular buildings may be considered intuitively as the boundaries of space that corresponds to architectural rooms or subspaces within them [17].

The isovist partition of a polygon is a convex partition of the polygon such that the one of the two points that is used for the check of the convexity of the space remains stationary; any point in a polygon may be used to test the isovist partition of the polygon and there is a infinite number of isovist partitions for any polygon. Every isovist partitions the polygons into two areas, the visible and the non-visible area, and the relationship of the visible area versus the non-visible area characterizes the convexity of the polygon itself. The two-dimensional isovist partition of a polygon may be considered intuitively as the boundaries of space that can be seen from an observer with an omniscient 360º field of vision [18].

These two partitions provide the framework for the development of representations of space that simulate the experience of movement of a situated observer in space [17]. This movement is essentially a linear structure - a line - that unfolds in space and correlated with perspectival spaces, sounds and colors. Every partition provides a different arrangement and it permits a different performance.

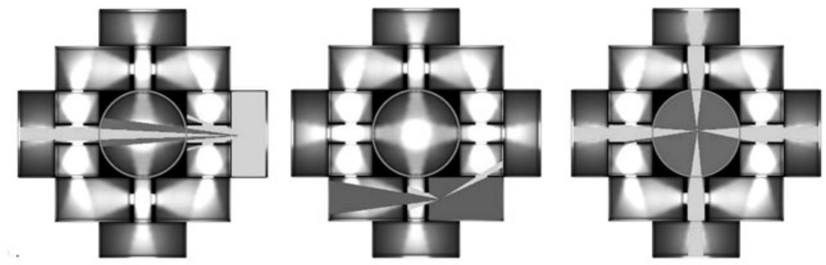
The convex partition of a plan of a rectangular building produces a set of rectangles with dimensions x and y and an area xy that corresponds to a frequency of light and sound with a ratio x:y. The movement of the agent within the space activates the discrete sounds and colors of each subspace in a similar way that the fingers of the hand strike the keys of a piano or the strings of a guitar to produce a sound. The movement of the observer within the same space does not change the sound or the color of the space; the movement of the agent in a different space with different proportions activates the corresponding frequencies of sound and light. Three different snapshots of the movement of an agent within the Villa Capra based on a field of vision of 360_ upon a convex partition are shown in "Figure 6".

► Figure 6: Convex partitions of the Villa Capra.

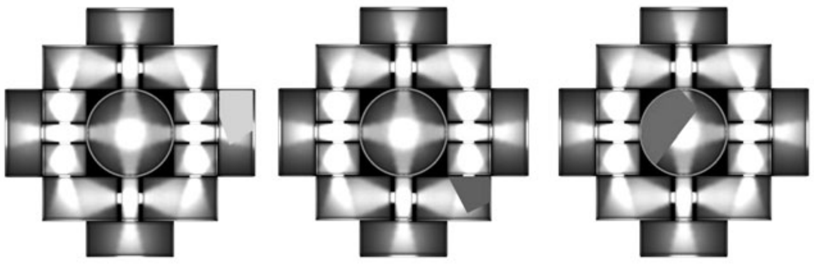


The isovist partition of a plan of a rectangular building produces a set of triangles that correspond to frequencies of sound and light based on the dimensions x and y of the convex partition of the space within which the hypotenuse lies. The volume of the sound and the area of the color depends on the ratio of the area of the triangle of the isovist to the total area of the rectangle defined by a ration *x:y*. The visual scan of the agent within the space activates the sounds and the colors of each area. The colors are distributed one next to the other with respect to what the agent sees; the sounds mix to compose a singular chord that is composed by all the sound frequencies of the isovists. Three different snapshots of the movement of an agent within the Villa Capra based on an isovist partition are shown in "Figure 7".

► Figure 7: Isovist partitions of the Villa Capra.

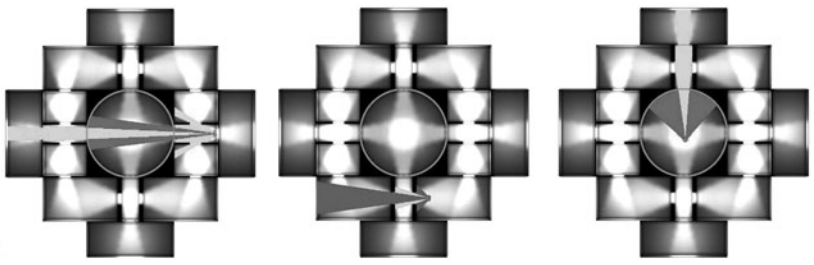


The diagrams of movement and visual scan of a space described above can simulate the movement and scanning of an agent even better if the field of vision of the agent is a subset of the omniscient 360º field of vision. In the case of the convex partition, the reduction of the cone of vision into a 90- angle creates polygons whose areas are in a direct proportional relationship to the volume of the sound frequency; the greater the distance of the agent from the boundary, the greater the area of the polygon produced by the visual field, and the greater the corresponding dynamics of the sound. The total area of the partition changes for each scan of the space and the volume of the composite sound changes in proportion to the areas of color. Three different snapshots of the movement of an agent within the Villa Capra based on a field of vision of 90- upon a convex partition are shown in "Figure 8".



◀ Figure 8: Fields of view of 90- in Villa Capra based on convex partition.

The reduction of the cone of vision in an isovist partition from a 360- to a 90- angle produces the closest model to a simulation of a moving - seeing agent into a space. The total area of the partition changes for each scan of the space and the volume of the composite sound changes in proportion to the changes of area of color. As in the previous model, the greater the distance of the agent from the boundary, the greater the area of the polygon produced by the visual field, and the greater the corresponding dynamics of the sound. Three different snapshots of the movement of an agent within the Villa Capra based on a convex partition with a field of vision of 90- are shown in "Figure 9".

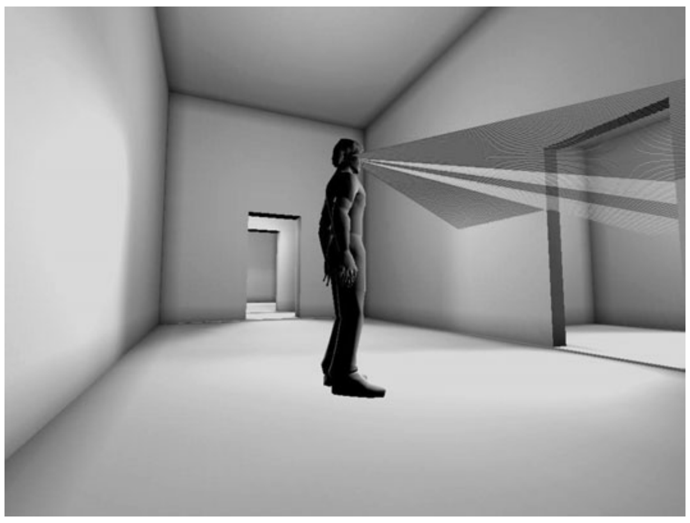


◀ Figure 9: Isovist partitions of the Villa Capra based on a 90- field of vision.

4. OPUS PALLADIO

The *Opus Palladio* is a software application based on the patterns of movements and visual scans by a situated agent within a virtual space based on the Palladian corpus. The software is entirely based on game engine technologies that are ideal for the simulation of patterns of movement and encounter within a virtual environment. All four models described above are implemented within the virtual environment of Villa Capra to simulate four different synaesthetic effects of perception of space for a situated observer. After a series of experiments we focused on the last model because it suggested a nice correlation between expansions and contractions of the visual field with changes of intensity of sound fields; in other words, the model suggested a nice way to relate what the agent sees and what the agent hears within the virtual environment.

The specific game engine technology that was used in this application is the Unreal Engine Runtime [19], and the code was scripted within the environment of the application. The design of the application was based on the ray-tracing algorithm *Trace* of Unreal Engine; this algorithm is designed to trace collisions within the virtual space. The location and specific position of the agent within the contextual environment is constantly related and checked against the boundaries of space that can be perceived by the agent. The algorithm returns a list with all the intersections of the rays projected by the moving agent with the surfaces of the surrounding space (hits), the direction of the surface at these intersections (surface normals), and the texture of the surface that has been checked for collision. This algorithm has been modified here so that it computes the frequency of sound that corresponds with each surface. The total number of rays that intersect a surface provides the volume of sound that corresponds to this surface. The total number of rays within the field of vision of the agent provides the complete volume and timbre of the sound-chord that are correlated with the surfaces that the agent sees. The tracing of the bounded space and the construction of the simultaneous frequencies are illustrated in "Figure 10".



▶ Figure 10: Automatic construction of frequencies of sound based on the ray-tracing algorithm.

5. DISCUSSION

The relationship of architecture and music has a long history that can already be traced from the early surviving ancient Greek texts; from Euripides and the harmonic proportions of the walls of Thebes, to Palladio and the enigmatic musical proportions of his buildings, to Le Corbusier and the musical scale of Modulor, a long line of thought has unfold, often vague, other times conflicting, and almost always interesting [20-22]. One central question in this line of thought is the logic of mapping in the construction of the systems that link architecture and music and other symbolic systems. Classic schemata of mapping in symbolic systems typically rely upon a duality, such as the absolute and relevant systems [23], discursive and presentational systems [24], real and nominal systems [25], autographic and allographic systems [26], and many other constructs not necessarily dual. Among all these systems the nominalist approach and especially the irrealist approach in the theory of symbols has been adopted here as the philosophical foundation of our project [26]. More specifically, this approach is considered here as the most generous and productive for the design of mappings between symbolic systems because it uses as its foundation the direction of reference between symbols: from the first to the second - denotation-, and from the second to the first - exemplification. All mappings used here are based on specific ratios and they are considered as exemplifications of these ratios in each individual medium in the sense that the corresponding proportions of spaces and frequencies of sound and light denote, signify, bring forward, a specific aspect of the relationships that condition these three media.

A range of alternative representations of Palladio's Villa Capra has been given to correlate arithmetical and geometrical aspects of form to color and sound schemata. The frequencies of sound and light that were used in these representations of the Villa Capra were equivalent to the arithmetical ratios of the dimensions of each room of the villa. Two different partitions of space were considered to suggest different methods of observation by a situated observer within the virtual environment of the villa and four different models were developed to simulate patterns of movement and sight within the virtual space. To that extent, Palladio's Villa Capra was redesigned as the main instrument to be performed and test the argument. A software application, Opus Palladio, was designed from scratch in the Unreal Engine environment to provide interactive methods of navigation and suggest different performances within the virtual space of the villa.

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