Non-Deterministic Exploration through Parametric Design
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ABSTRACT
This paper explores non-deterministic parametric modelling as a design tool. Specifically, it addresses the application of parametric variables to the generation of a conceptual bridge design and the use of repeatable discrete components to the conceptual form. In order to control the generation of the bridge form, a set of design variables based on the concept of a law curve have been developed. These design variables are applied and tested through interactive modelling and variation, driven by manipulating the law curve. Combining this process with the application and control of a repeatable element, known as a Representative Volumetric Element (RVE), allows for the development and exploration of a design solution that could not be achieved through the use of conventional computer modelling. The competition brief for the Australian Institute of Architects (AIA) ‘Dialectical Bridge’ has been used as a case study to demonstrate the use of non-deterministic parametric modelling as a design tool. The results of the experimentation with parametric variables, the law curve and representative volumetric elements (RVE) are presented in the paper.
1. Introduction

It is often a difficult process to address a design brief with a sense of integrity towards a design concept without having to compromise due to pragmatic restrictions of conventional CAD in multi-scale design resolution. The use of parametric modelling as a design tool provides many improvements in the generation and exploration of form within a set of governing parameters. Parametric tools help to remove limitations normally imposed by the interface with traditional architectural CAD packages and opens up new avenues in multi scale design refinement. The design challenge presented by the AIA brief for a pedestrian footbridge across a river is used as a case study to test and validate a multi scale design approach using parametric technology. In this paper we explore the application of non-deterministic parametric modelling with a focus on the use of law curves as a means of model control. Within this design exercise, the goal of the study was to control and tessellate a repeatable component within a larger governing surface geometry. The aim of the design was to create a unified structure that provides immediate impact analysis and responsive interaction to constantly inform and evolve the design.

2. Deterministic vs non-deterministic parametric modelling

The forms of parametric modelling can be defined by their process and aims into two broad categories, deterministic and non-deterministic parametric modelling [1, 2, 3]. Non-deterministic parametric modelling refers to the exploration of form through a set of parameters with no definite or predetermined form. Although the initial parameter definition is known, the physical representation is not. Formulas and variable controls are used as the defining parameters to gain either a fixed form or variable representation. These can be controlled through the manipulation of raw numerical formulas or graphical representations such as law curves or graph variable bars. The work of Gehry and Associates [4] and the completion of Antoni Gaudi’s Sagrada Familia [5] are examples of this type of modelling.

Deterministic parametric modelling uses parameters and formulas to achieve a desired goal. This end goal can either be pragmatic or aesthetic but the process of creation is much more refined and specific than that of non-deterministic parametric modelling. Although this type of modelling still uses defined parameters to create a form, the distinction is in the process of design and modelling. Unlike non-deterministic modelling, deterministic modelling requires the user to start with the desired end product and work backwards to discover the formulas and parametric constraints required to achieve this goal. This type of modelling requires forethought of the end product and a program capable of multi-scale approaches such as the Python based ‘TexGen’ [6].
For architectural applications, there are degrees of intersection between both forms of modelling. Deterministic parametric modelling can be used to explore the possibilities of form generation within a set of parameters and non-deterministic parametric modelling can be used when one has an end use in mind, whether pragmatic or aesthetic. For the following bridge design the main aim was to involve as much non-deterministic form generation as possible. Through this, the completed model will help broaden an exploration of design possibilities while still adhering to a set of parametric restrictions.

3. Law curves

A law curve is a set of features which allows a level of control over its properties and relationships. This feature is represented as a law curve graph. The law curve graph consists of an x direction and a y direction, with both the x and y point values relating to properties of the parametric model curve. Essentially the graph is a means of representing and controlling elements within a stand alone model [2]. The x direction is the independent or predetermined value, meaning that the points on the x direction that make up the law curve are fixed [7]. The y direction is the dependant and derives its value from both the x direction and the shape of the law curve. In a straight law curve points are equidistant, while in a curved law curve the distances between points are variable. (Figure 1)

![Law Curve Graph](image)

Parametric modelling takes advantage of variables to allow dynamic change within a model. As a parametric model increases in complexity, the number of variables increases dramatically. The law curve can be utilised by allowing the amount of variables required (x axis) to be determined. By changing the shape of the law curve all of the variable inputs (y axis) respond simultaneously. This provides both a simplification of variables under a unified control system as well as introducing non-deterministic opportunities for form exploration within the governing parameters.

![Figure 1. The Distance between the points (y) is dependant upon the Law Curve, which in turn is dependant on the number of points in the curve (x). [2]](image)
4. Surface division and component constraints

Parametric modelling has the ability to create repeatable base components and apply them within the bounds of a governing or carrier surface. (Figure 2) This dual (multi-scale) level of design control allows a more refined process of exploration and development at both the micro and macro scale. Within this process, two main fields of research become evident,

- the meshing of a complex surface into uniform quadrilateral facets
- control and tessellation of the repeatable component

4.1 Surface meshing

One of the greatest issues that becomes apparent when dealing with complex geometry of architectural surfaces is the successful analysis and application of meshing [9]. Meshing can take a number of forms including quadrilateral, triangular and even circular facets of either a curved or planar nature. From a geometric perspective, this process of meshing is an added
level of information towards understanding and describing the surface form. In essence, it simplifies a complex form into smaller facets that can be more easily utilised. This is particularly important when describing a form that is intended to be fabricated.

In terms of tessellating a component within the constraints of a surface, this meshing process is crucial in the definition of a component. The method with which it is analysed will have dramatic results on the final model. The uniform mesh created from the analysis of a surface creates the formwork to constrain the repeatable components within its boundaries. The result is a relationship between the neighbouring components at the facet’s joining edge. If the meshing is not uniform, issues arise with warping, incorrect scaling and tessellation between the applied components.

For the purpose of the bridge design, quadrilateral planar facets have been chosen to define and control the component. It has been observed that Planar Quadrilateral (PQ) meshes are a discrete counterpart of conjugate curve networks on surfaces [10] and are argued to be preferable to architectural models defined by triangular facets or non-planar quads [11]. This form of meshing is an approximation of a shape between a given set of nodes. (Figure 3)

4.2 Representative volumetric elements (RVE’s)

The accuracy achieved in the surface meshing directly influences the precision of the input definition when applying the component. The main aim of uniform meshing is to create a stable foundation for the application and tessellation of repeatable components referred to as Representative Volumetric Elements (RVE). Representative Volumetric Elements are a representation of the smallest repeatable element within a given tessellated form [12]. One of the key differences between an RVE and a standard CAD component is the ability of an RVE to have an awareness of its place within a larger pattern as well as adhering to any set parameters and governing surface geometry. This is an important factor in the creation of both an intelligent RVE and a cohesive and tessellated product that fits within all the predetermined parametric requirements.
RVE correlation is a result of the method of definition. If quadrilateral facets are used as a surface mesh then the same singular facet becomes the basis for the RVE geometry. Due to this hierarchy of objects, the component is inherently malleable and will align itself within a given mesh facet. (Figure 4)

Unfortunately most architectural CAD packages only have a basic analysis capability for surface meshing so other means are required for more complex surface forms. Stand alone geometric analysis and Finite Elemental Analysis (FEA) packages are available, but require compatibility with the architectural CAD package and are not always useable for parametric tessellation. For this reason, the meshing performed for the bridge model was done on a basic level within the parametric program. Although this is not a high level of analysis, the relative simplicity of the surface form and the variable nature of the surface division results in a simple but effective means to successfully control the parametric model.

5. Exploring the design of a bridge

The Australian Institute of Architects proposed a design competition for their 2008 conference. It called for the design of a pedestrian footbridge to connect two sides of a river, two places and two cities. The two cities are of contrasting nature. Zeitgeist rises high and quickly, with a desire for the new and different, has a strong faith in technology to solve all problems. The second city, ‘Nostalgia’ is older, turning its back to the future and resisting progress. Zeitgeist’s side of the river is solid granite and rises directly above the river, while Nostalgia’s river bank is soft clay sloping back from the river.

Our response to the presented brief was to create bridges that intertwined in the middle of the river, providing a meeting ground for the two contrasting cities. From the soft clay bank protrudes a straight bridge, rigid and embedded in the earth, rising towards the cliff edge across the river. Reaching down from the cliff and wrapping around the straight bridge are two curved fluid bridges. The main aim of the design was to create a
point of intersection between the two cities. Due to the placement of the two cities in relation to the river, the proposed bridge site is of a distance too great for pedestrian access between the cities of Zeitgist and Nostalgia. Therefore, the aim of this footbridge is not to create a means of travel between the cities, but to create a neutral and communal destination for the residents of these two vastly different cities.

This design attempts to bridge the gap created by the river without granting access from one side to the other. Through this, the visitors from the city gain an opportunity to cross their city’s border into a neutral, communal space. The traditional function of a bridge is now changed. Instead of simply providing access from point A to point B, the journey is now about experiencing the bridge itself and the interaction between the intertwining elements. In this sense, the structure still bridges the gap through experience and passive interaction without allowing direct physical traversal. Whilst the straight bridge could be developed using conventional techniques, the generation of the curved bridges offered an opportunity for exploration through parametric CAD.

To investigate the generation of the parametric bridge, three main areas of control were explored.

- the use of a law curve to define the form
- control of the trafficable surface gradient
- surface division and component constraints

6. Defining the design parameters

Whilst the desire was for a curved, fluid bridge, the design still required certain aspect of restriction and control. This was approached by dividing the bridge into sectors over which the design parameter could be applied.

6.1 Law curve

When applying a law curve, consideration has to be given to the required amount of variables. In terms of the bridge formwork, this was dictated by a number of control sectors. The sectors were defined by the x points (the independent), which controlled a set of points and their position along a b-spline. X limits the number of points while y controls the spacing along the curve. (Figure 5) As the shape of the law curve changes, the distance between each of the point's change respectively.

In the generation of the Law Curve a point is created which is used to change the shape of the curve. The curve does not directly pass through the point but is dependant on it. This was then further explored with a law curve that was defined by two points. By introducing the second point to the law curve graph the points along the b-spline were then allowed to cross over, resulting in a more dramatic change in the distance between points. (Figure 6)
Figure 5. (Left) Straight law curve with evenly distributed points along b-spline, (Right) Law curve adjusted with the resulting change in the distance between points.

Figure 6. (Left) Straight law curve controlled by two points, (Right) Resulting point distribution along b-spline from two point law curve.
The bridge was then constructed using a set of control lines with their length based on a formula involving the law curve variable for each sector. These control lines defined the length and curvature of each sector. From these lines, points are generated to create a b-spline that defined the centre of the overall geometry. So as the law curve is adjusted the shape of the bridge changes. (Figure 7)

6.2 Trafficable surface gradient

While the law curve provides a means of non-deterministic shape generation, aspects of a pedestrian footbridge require more direct and predictable restrictions. In order to provide universal access to the bridge, a separate set of variables were created to constrain the gradient of each sector. This variable had a range imposed to it, allowing it to be adjusted within a positive or negative slope of 1 to 15. A relative vertical line (h^0) was applied to each sector of the central b-spline. This was combined with the variable sector length to create a fixed right angle triangle to control the gradient. Eqn (1).
\[ h^0 = \tan (y^0) \times X^0 \]  

where-
- \( y^0 \) – maximum gradient angle
- \( X^0 \) – sector length (variable and dependant on the law curve position)
- \( h^0 \) – relative vertical line length

This ensured that the bridge gradient was restricted to be universally accessible at all times while still allowing movement within the +max / –min values.

6.3 Governing surface definition

The central b-spline described in the previous section provides the basic shape of the bridge. The next step in the evolution of the design is to create a carrier surface based on the initial profile and then apply the RVE to this surface to create the structural form. The carrier surface was developed by using a profile replicated along the central b-spline, defining an inner and outer surface. The replication was achieved by the placement of a series of planes along the b-spline at even intervals. A second plane was then generated to ensure that the profile would remain perpendicular to the b-spline at each given intersection. (Figure 8)

From these planes a series of control lines were constructed to form the profile, with their lengths based on an equation involving the law curve variables and their distance from the start of the edge of the river bank. This allowed for the profile to scale down as it moved further away from the river bank. Within this the vertical control line for the handrail was fixed to ensure its relation to the human scale, as opposed to the general form generation of the bridge. From these control lines two surfaces were then generated, onto which a point grid was then defined. This point grid defined the quadrilateral surface meshing which, in turn, dictated the size and number of RVE’s used. This point grid was based on a UV axis with variable value that could be adjusted according to the design requirements. This method is a fairly basic form of surface analysis but was adequate for the relatively simple surface geometry.
7. Component design

The aim of the component was to create a cohesive unit that could tessellate across the meshed control surface, creating a unified structure while still adhering to its functional parameters. To accomplish this, the component was broken down into three distinctive sectors (Figure 9):
• interior skin structure (vertical element)
• exterior skin structure (horizontal element)
• blade component

Each of these components was first tested individually, to understand the uniformity of their application. This process allowed a greater level of understanding of a complex structure and resulted in a number of design changes at the component level.

The joins between the cylindrical elements was an interesting connection point and one that was left incorrectly tessellated. This decision was purely an aesthetic choice to give a subtle but definite distinction between certain elements. The first component had an end radius that was defined as a percentage of the top line of the joining facet. This method ensured that the joined radius was always the same, giving a seamless transition between RVE elements. The result of this was a very controlled and well tessellated array which, in terms of the design, didn’t quite meet aesthetic criteria.

The second RVE was referenced internally to the top facet line, giving the component a less structured appearance. By this method, the connection details have an undulating radius scale across the bridge, giving the form a better sense of rhythm.

The blade component was also accentuated to keep a degree of separation between the elements. This part is not continuous on its own, but acts as a joining piece between the inner and outer components, thus highlighting the depth in the structure. (Figure 10)
8. Rapid prototyping

The submission requirement for the competition was a scale model. (Figure 11) This presented a challenge in dealing with a complex parametric design. This was solved through the use of a rapid prototyping machine, which allowed the CAD model to be printed as a physical object. A plaster powder based 3d printer system was utilised. This type of system offers a high level of model detail, while still being reasonably cost effective due to its low levels of material wastage. The problem with this type of system is that it requires handling while the model is still unsealed and weak. This needs to be considered when determining the physical scale of the printed objects. This process was useful to better understand the physical implications of the digital design decisions. The initial prototype pushed the limitations of the rapid prototyping machine in terms of object thickness, with the model breaking during the finishing process. This resulted in the RVE having to be re-evaluated in order to provide greater stability to the model.

While the RVE required adjustment in order to produce a scaled model, the testing of a full scale version of the design could be carried through into structural analysis programs. This would ultimately determine the final requirements for the RVE design radius.

Figure 11. Completed scale model
9. Design conclusion

9.1. Micro and macro level control

The application of parametric modelling to architectural design allows designers to address the design at both the macro and micro levels of resolution. This process is highlighted in the governing surface used to define the form of the bridge and the RVE designed to tessellate the surface geometry to create the structure.

To achieve the overall form of the bridge, both single and multiple point law curves were tested. While multiple control points within the law curve create a greater chance of the bridge folding into itself, a more dynamic form is generated. With the initial form of the bridge generated, a sectional profile is produced. The sectional profile scales according to the distance from the river bank and the form of the bridge. In the current model, this scaling is arbitrary and was included purely for aesthetic reasons. Two control surfaces were produced based on the replicated profile. These offset surfaces then defined the placement of each of the RVEs. By using two control surfaces as input definitions, a more intelligent RVE could be created. This ensured that maximum control of the RVE was obtained to adhere with aesthetic choices. In future work, it is hoped that this parameter can be more heavily dictated by specific material and structural requirements. This material information can easily be included as a range of graph variables to further constrain the initial digital model. Further research is required to successfully create a feedback loop for virtually testing structural performance during the design and form exploration stages.

The model also employs the use of governing maximum / minimum restrictions that constrain the trafficable gradient of the bridge within the results of the law curve. Through this, the requirements of the brief can be addressed, while still maximising the potential for form exploration.

9.2 Paramedic vs. traditional CAD

The use of parametric modelling to develop and explore the design provided many benefits. The visual and pragmatic requirements of the bridge were simultaneously programmed and tested within a single model. This resulted in the creation of a non-deterministic form within the strict functional parameters defined by the brief. This is one of the main design strengths of parametric software as compared to traditional CAD. Rather than conceiving a design and then having to revise or rebuild the model at every change, the design process becomes cyclical through the use of variable parameters. By this method each design decision and requirement can be analysed and responded to immediately. This process is not just a means to replace a considered design methodology through random digital form
generation, but a way of increasing the visual exploration and reaction to a design concept.

10. Conclusion

The parametrically designed bridge was explored through the use of law curve based variables. The key aims of this paper were to explore form generation within fixed functional parameters and the effect this had on the bridge design. The law curve was used to define the overall form of the bridge, allowing it to be adjusted in a non-deterministic manner. This was then coupled with a strictly deterministic and performative component to create a final digital bridge. From this geometric information, a physical model could then be fabricated on a 3D printer to help understand the physical performance of the structure.

The use of parametric scripting for this design project extended the possibilities of form generation, exploration and construction. This type of digital design would not have been possible with traditional architectural CAD. The interface and construction process is still too laborious to attempt multi component tessellation within an undulating surface geometry. The well established conventions within the building industry are inexorably changing towards a new paradigm of design aided by computer generated forms. To fully realise the potential for this powerful tool, further exploration needs to be made into material and structural requirements. This will offer architects the opportunity to bridge the gap between design concept and design realisation.
References


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Figure 13. Render of completed bridge design.
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