LINESPACE: AN AUTOMATED GENERATION OF THE 19 SPACES WITH 1 AXIS OF GROWTH

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Abstract

The structures of 3-dimensional spaces with one axis of growth are discussed in detail. The mathematical structure of all 19 possible patterns is briefly discussed and a computational framework to generate these spaces is presented in the end.

1. Introduction

Recent architecture discourse has been systematically exploring complexity in design in new and unprecedented ways. In architectural design complexity typically arises in the interplay of the design of components, the ways these components are assembled, and the degrees that both types of descriptions deviate or not from established conventions and paradigms (Mitchell, 2005). The possible interconnections between these models provide a nice structure to explore complexity. For example, standard and uniform shapes and standard and uniform spatial relations are associated with the least degree of complexity in design and quite often with mass produced designs. Standard shapes and non-standard spatial relations as well as non-standard shapes and standard spatial relations point to different ways of tackling complexity and usually are associated with overall complex arrangements with some discerned degree of order. At the end of the spectrum non-standard shapes and non-standard spatial relations are typically reserved for highly expressive architecture designs composed by non-repetitive non-uniform shapes and components. Most of contemporary designs can be cast in any of these four classes. “Figure 1” shows a part of a fabricated surface comprised by non-uniform components assembled in a uniform way for the current exhibition of Monica Ponce de Leon’s class on emergent technologies at the College of Architecture at Georgia Institute of Technology.

Among these four classes of designs the first three exemplify various degrees of controlled repetition;

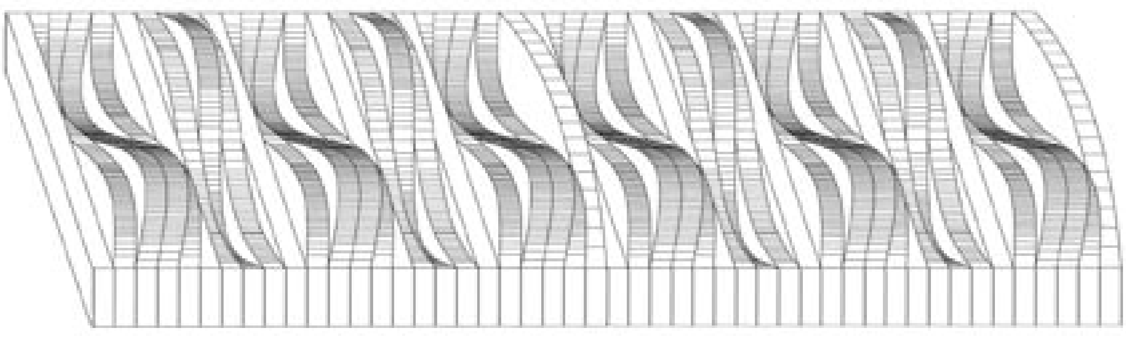

Figure 1: Fabricated surface with an axis of growth (Tristan Al-Haddad and Monica Ponce de Leon, 2005).

repetition of parts and joints, repetition of parts, and repetition of joints. The first three classes involve some explicit degrees of repetition and can be nicely studied using traditional tools of algebra such as group theory (March, 1974). It would not be farfetched to claim that a substantial part of the fourth class of designs that rely on non-standard parts and non-standard relations can be dealt within the same framework under parametric rules. Among the first three classes of configurations the class of 3-dimensional designs with 1 axis of growth is ubiquitous in architectural design and appears in walls, columns, modular arrangements, tectonic configurations, row housing and tall buildings.

This paper looks closely at this specific class of 3-dimensional designs that have 1 axis of growth and presents a computational framework that generates designs based on all basic algebraic structures that capture the symmetries of these designs. The classification scheme adopted in this paper describes designs in terms of geometric generators and defining relations (Shubnikov and Koptsik, 1974). All these spatial structures can be derived from the interaction between 6 spatial generators. These generators are: a translation (*a*) along an axis, a rotation (*n*) of order *n* about an axis, a screw rotation (n_j) with an elementary translation *a/j*, a reflection (*m*) along a plane *m* coincident or perpendicular to an axis of translation, a rotor reflection ($\widetilde{2n}$) along a mirror-rotation axis of order $\widetilde{2n}$, and a glide reflection axis ($\widetilde{a}$) with an elementary translation *a/2*. The signs between the generators denote the spatial relationships between them. The two-point sign (:) between 2 generators indicates that they are perpendicular to one another; the one-point sign (.) indicates that these generators are parallel to one another. These symbols are enough to describe the symmetry structure of any spatial configuration with 1 axis of growth. A pictorial representation of all 6 generators is given in “Figure 2”. A dashed label denotes the initial position of a label before any generators apply to it. Black and white labels denote the positions of the labels after the transformation; black labels denote the position of the label towards the viewer and white towards the back. Numbers denote the internal repetitions of a label along the translational axis within the duration of a complete translation of the pattern. All axes of transformations are assumed to be perpendicular to the axis of viewing.

Figure 2: A pictorial example of all 6 generators in 3-dimensional space a) Translation b) Rotation c) Screw Rotation d) Reflection e) Rotor reflection f) Glide reflection

2. The 19 configurations with 1 axis of growth

The complete list of 3-dimensional structures with a single axis of growth consists of 19 classes. Several nice mathematical treatments can be found in the literature (see for example, Toth 1964, Shubnikov and Koptsik 1974). Here a very brief description is provided and the emphasis is given in the diagrammatic illustration for each class for an *n*-fold rotation, $n \leq 6$.

2.1. n.a

The pattern *n.a* is generated by successive translations of shapes with symmetry *n* along their primary axis of rotation at a distance *a*. A substitution of the symmetry axis *n* by the screw axis n_j produces 3 additional types n_j, for ($j < n/2$), ($j = n/2$), ($j > n/2$). A diagrammatic representation of these 4 types of patterns, for $n \leq 6$, is given in “Figure 3”.

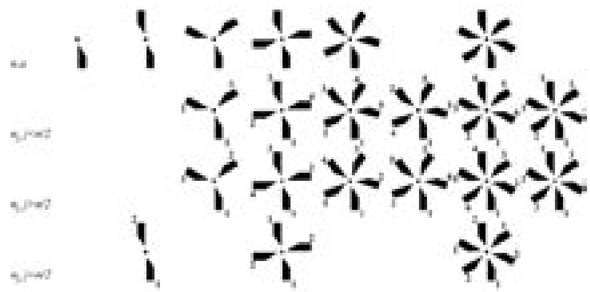

Figure 3: The 4 types of linear spaces based on the class n.a.

2.2. 2ñ.a

The pattern $2\tilde{n}.a$ is produced by successive translations of shapes with symmetry $2\tilde{n}$ along their rotor reflection axis at a distance *a*. A diagrammatic representation of this single type, for *n*≤3, is given in “Figure 4”.

Figure 4: The single type of a linear space based on the class 2ñ.a.

2.3. n:m.a

The pattern *n:m.a* is produced by successive translations of shapes with symmetry *n:m* along their primary axis of rotation at a distance *a*. A substitution of the symmetry axis *n* by the screw axis n_j produces the pattern n_j:*m* for (*j* = *n/*2) or alternatively $2n_n$:*m*. A diagrammatic representation of these 2 types of patterns, for *n*≤6, is given in “Figure 5”.

Figure 5: The 2 types of linear spaces based on the class n:m.a.

2.4. n:2.a

The pattern *n:2.a* is produced by successive translations of shapes with symmetry *n:2* along their primary axis of rotation at a distance *a*. A substitution of the symmetry axis *n* by the screw axis n_j produces 3 types of patterns. A diagrammatic representation of these 4 types of patterns, for *n*≤6, is given in “Figure 6”.

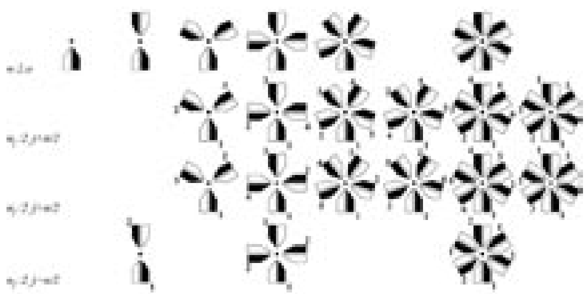

Figure 6: The 4 types of linear spaces based on the class n:2.a.

2.5. n.m.a

The pattern *n.m.a* is produced by successive translations of shapes with symmetry *n.m* along their primary axis of rotation at a distance *a*. A substitution of the symmetry axis *n* by the screw axis n_j produces the pattern n_j.*m* for (*j* = *n/*2) or alternatively $2n_n$.*m*. A substitution of the mirror plane *m* by a glide reflection $\tilde{a}$ produces the pattern *n.*$\tilde{a}$. A diagrammatic representation of these 3 types of patterns, for *n*≤6, is given in “Figure 7”.

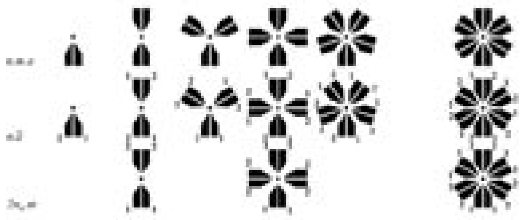

Figure 7: The 3 types of linear spaces based on the class n.m.a.

2.6. 2ñ.m.a

The pattern $2\tilde{n}.m.a$ is produced by successive translations of shapes with symmetry $2\tilde{n}.m$ along their axis of rotor reflection at a distance *a*. A substitution of the mirror plane *m* by a glide reflection $\tilde{a}$ produces the pattern $2\tilde{n}.\tilde{a}$. A diagrammatic representation of these 2 types of patterns, for *n*≤3 is given in “Figure 8”.

Figure 8: The 2 types of linear spaces based on the class $2\tilde{n}.m.a$.

2.7. n.m:m.a

The pattern *n.m:m.a* is produced by successive translations of shapes with symmetry *n.m:m* along the axis of rotation at a distance *a*. A substitution of the symmetry axis *n* by the screw axis n_j produces the pattern n_j.*m:m* for (*j* = *n/*2) or alternatively $2n_n$.*m:m*. A substitution of the mirror plane *m* perpendicular to the axis of rotation n by a glide reflection $\tilde{a}$ produces the pattern $n.\tilde{a}:m$. A diagrammatic representation of these 3 types of the patterns, for $n \leq 6$, is given in "Figure 9".

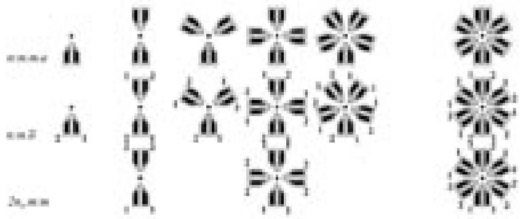

Figure 9: The 3 types of linear spaces based on the class n.m:m.a.

3. A computational framework for the generation of the 19 linear spaces

The mathematical complexity of the underlying structures of these 19 3-dimensional linear spaces and the potential complexity of the 3-dimensional shapes and their spatial relations translated along an axis, clearly suggest the need for a computational framework that can compute the spaces that exhibit these properties. For that reason we designed a program to allow users to design and visualize these 3-dimensional growth patterns. This program, called *LineSpace*, was built on top of the Persistence of Vision Raytracer (POV-Ray), a ray-tracing program that has a built-in three-dimensional coordinate system and includes several function calls, such as *rotate*() and *translate*(), as well as more advanced spatial manipulations through matrix transformations.

LineSpace utilizes a library of function calls to extend the built-in geometric system of POV-Ray, and allows the user to construct any of the previously enumerated linear patterns by calling on these functions. Although each function operates on a single object, complex shapes can be defined by the union, intersection or difference of two or more objects using POV-Ray's constructive solid geometry (CSG). These original objects can be simple geometric primitives, as well as more complicated shapes, such as those defined by a triangular mesh. CSG operations can be nested, and can thus allow for extremely complicated initial objects. The functions, which are referenced in a standard input file, pass a bundle of new objects to the next function called until all of the desired transformations have been completed; the resulting scene is then rendered by the POV-Ray Engine.

LineSpace utilizes a graphical user interface written in Visual Basic 6 to take input from the user before creating the nested-function text file required by POV-Ray. The program contains a basic and an advanced user-interface, which are associated with different approaches to the design process. The basic interface caters to users who are concerned with visualizing a particular linear transformation as applied to a given object. The basic interface allows users to modify certain options (the shape and color of the initial object, the axis of growth, the number of rotations, n, etc.), and then apply one or more of the 19 linear space transformations to the object. The advanced mode caters to users who are more concerned with the process of composition. In this mode, users are allowed to design their original object, and then have direct access to the function library included in *LineSpace*. By manipulating which functions are called, and the order in which they are applied, users create 3-dimensional axial designs that are described by one of the 19 patterns described in this paper. "Figure 10" shows all 19 possible spaces based on a simple spatial relation between a prism and an oblong generated within the software.

4. Discussion

A major motivation underlying this work is the exploration of the relationship between languages and configurations. Languages of design are generated by many different kinds of rule-based systems, including shape grammars, cellular automata, L-systems and others (see for example, Knight and Stiny 2001); configurations are similarly explored to discern what is structurally possible in a design context (Economou 1999). Both provide a complementary insight in design explorations and their differences really lie in their functionality in design inquiry rather than anything else. The work here privileged the second mode of design and explored a specific configurational problem, the class of 3-

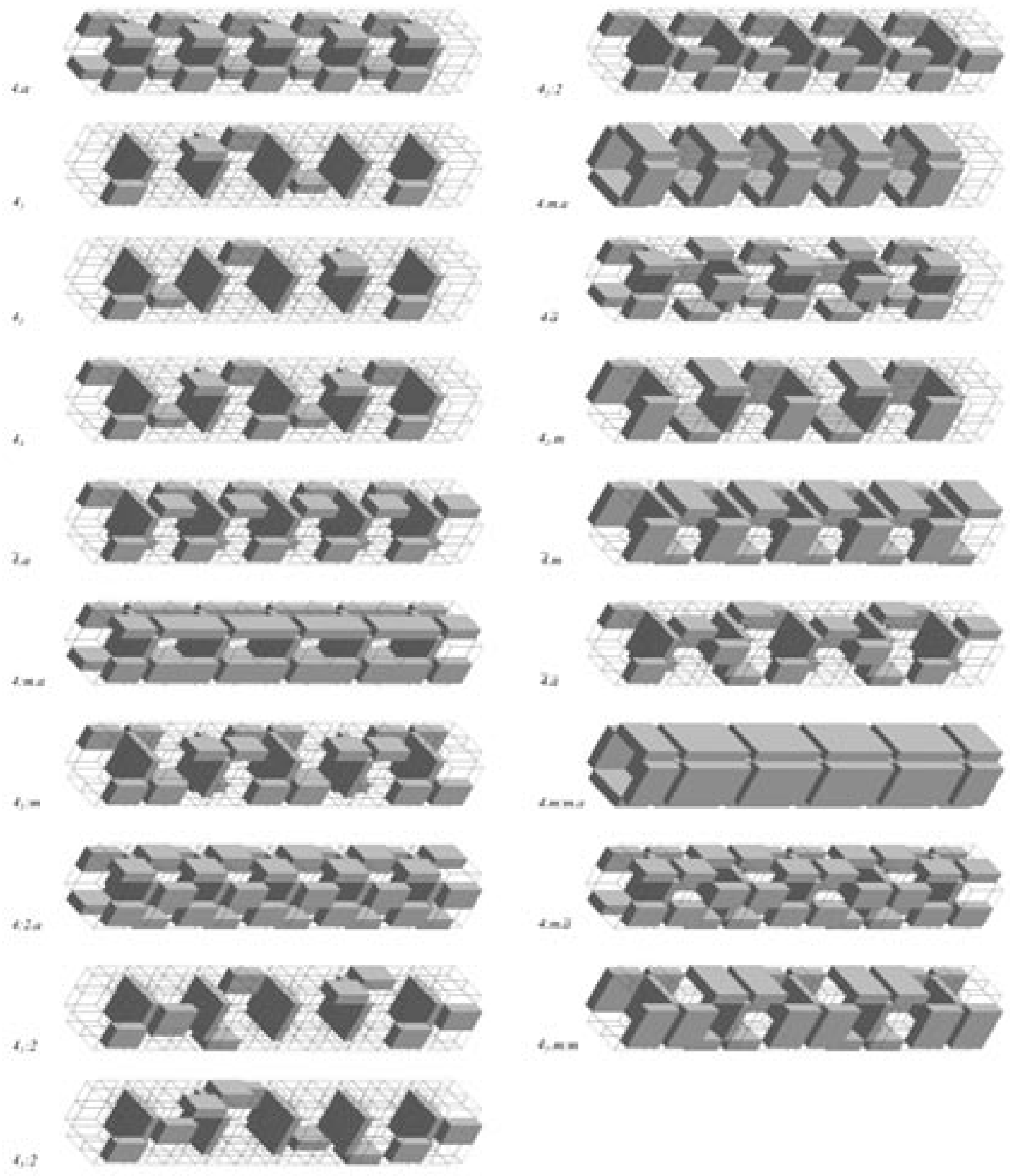

Figure 10: The 19 possible spaces based on a simple spatial relation between a prism and an oblong.

dimensional spaces characterized by a single axis. The 19 types of group structures that capture the properties of these spaces were briefly described and illustrated to show pictorially the structures of these spaces. Future work will deal with the creation of a plug-in within a solid modeler to be used in the analysis of existing architectural works, and in the synthesis and fabrication of new designs informed by these 19 diagrams.

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