SIGRADI2018
TECHNOPOLITICAS
xxii congresso da sociedade
iberoamericana de gráfica digital
22th conference of the
iberoamerican society
of digital graphics
07|08|09|novembro|2018
iau usp | são carlos | sp br

Digitally Fabricating Expandable Steel Structures Using Kirigami Patterns

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Abstract

This article presents a computational approach to generating architectural forms for large spanning structures based on a "paper-cutting" technique. In this traditional artform, a flat sheet is cut and scored in such a way that a small application of force prompts it to expand into a three-dimensional structure. To make these types of expandable structures feasible at an architectural scale, four challenges had to be met during the research. The first was to map the kinetic properties of a paper-cut model, investigating formative parameters such as the width and frequency of cuts to determine how they affect the resulting structure. The second challenge was to computationally simulate the paper-cut structure in an accurate fashion. We accomplished this task using finite element analysis in the *Ansys* software platform. The third challenge was to create a prediction model that could precisely forecast the characteristics of a paper-cutting pattern. We made significant strides in this demanding task by using a data-mining approach and regression analysis through 400 simulations of various cutting patterns. The final challenge was to verify the efficiency and accuracy of our prediction model, which we accomplished through a series of physical prototypes. Our resulting computational paper-cutting system can be used to estimate optimal cutting patterns and to predict the resulting structural characteristics, thereby providing greater rigor to what has previously been an ad-hoc and experimental design approach.

Keywords: Transformable Paper-cut; Design method; Prediction Model; Regression analysis; Physical prototype.

INTRODUCTION

In recent years architectural designers have become interested in the traditional art of kirigami, otherwise known as "paper-cutting." Similar to the better-known art of origami, which involves only folding paper, kirigami allows for both folding and cutting of a thin sheet to create intricate three-dimensional designs. In the architectural sense, kirigami can be seen as a model for generating large-scale deployable structures that can transform their shape without any complicated joints or sophisticated electromechanical means. This is a significant shift from the current paradigm of large-scale adaptive designs, which usually require precision robotics and heavy electromechanical, pneumatic, or hydraulic components. Adaptive designs based on a "paper-cutting" approach have the potential to greatly reduce the power requirements, cost, and complexity of these structures, producing lightweight alternatives that can quickly transform from a thin 2D sheet to a large 3D structure with minimal force requirements.

One of the reasons that origami and kirigami have attracted the attention of designers and scientists is because these transformable patterns replicate many natural processes in which structures regularly expand and collapse according to environmental variables. A broad range of such mechanical behaviors has been observed, in both biological structures and inorganic materials (Babaee et al., 2013; Bauer et al., 2014; Bertoldi et al., 2010; Bückmann et al., 2014; Kane & Lubensky, 2014; Kang et al., 2014; Grima et al., 2004; Mandelbrot, 1983; Overvelde, 2012; Schenk, 2013; Taylor et al., 2014). One example of a natural system that uses structural expansion to produce different shapes is a stem cell. An embryonic, pluripotent stem cell can differentiate into any type of cell in the body (Slack, 2008). By adjusting its shape and then recursively dividing, the stem cell develops its structural potential and undergoes drastic transformations. Previous researchers have used this model to conceptualize the development of flexible electronics, which are scored at the molecular level so that they can take on a range of different potential structural shapes and macroscopic properties (Cho et al., 2014; Fan, 2014).

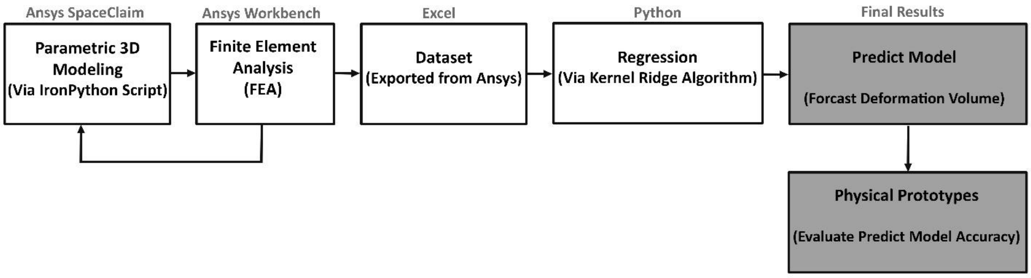

Figure 1: Software components used in the computer simulation and regression analysis. Source: authors.

In general, however, the application of conventional origami and kirigami approaches in design and engineering has been limited by the complexity of these designs' structural properties, as well as by material constraints on the amount of reversible bending and folding that construction materials can undergo. Researchers are now beginning to reduce these limitations. Strain engineering has become increasingly important in materials science due to the move toward kinetic and transformative designs (Kim et al., 2013; Lee et al., 2014; Sekitani et al., 2008). Some researchers have attempted to improve traditional materials to eliminate the nanoscale and microscale defects that serve as stress concentrators and lead to microcracking and eventual failure (Cranford et al., 2012; Khayer et al., 2013). Others have worked to develop entirely new materials such as elastic nanocomposites, which can undergo tremendous strains without permanent deformation or failure (Lee, et al., 2014; Kim & Kotov, 2014; Shyu et al., 2015). While this materials development has not yet reached the point to where folding structures can be widely and inexpensively deployed, it is beginning to open new possibilities for architectural designers.

In previous research architects and engineers have already demonstrated the potential of structures based on paper-cutting and paper-folding methods, on a wide variety of scales. For example, physicists at Cornell University cut a microscopic sheet of graphene with the goal of building nanostructures that can be used in flexible and stretchable electronics. Other researchers have used this same approach in efforts to develop advanced materials for breathable yet protective garments (Blees et al., 2015). In architecture, thin sheets of materials such as PDMS have been cut in pre-defined patterns to allow rigid rotation, bending, and twisting, thus creating open pore spaces in response to environmental factors such as sunlight (Baseta et al., 2017). So far, however, this model in architecture has been mostly limited to shells and flexible curtains. Its potential uses in large-scale transformable structures have not yet been extensively developed.

Previous work with paper-cutting approaches in architecture has mostly been carried out on an ad-hoc basis, following a time-intensive process of experimentation to develop unique design solutions. Paper-cut systems tend to have strongly nonlinear behavior, and there is a lack of formal, mathematical knowledge about their structural properties. The characteristics and optimization of these kinds of transformative systems have not yet been rigorously studied. The purpose of the current research was to contribute to this knowledge, and to help develop a more systematic paper-cutting design approach, by collecting data on paper-cut transformations and integrating that data into a computer software model.

Our work focused on predicting the deformation of a paper-cut design; in other words, the amount of "stretch" that a particular cutting pattern will produce. As a first step toward creating a more holistic paper-cut analysis system, we focused exclusively on circular cutting patterns (described in more detail below). Future work may address other types of cutting geometry. To be able to predict the results of a particular cutting pattern, we used a simulation and regression-analysis approach. The end result of the project was a custom software tool that can rapidly calculate the deformation volume of a circular cutting design based on a few simple inputs.

METHODS

The goal of the research was to develop a software design tool to calculate the amount of deformation produced by various circular paper-cutting designs, and therefore assist in creating structurally efficient, flexible, and spaces using the paper-cutting approach. To accomplish this goal, a regression-analysis method was used. First, the behavior of circular paper-cutting models was analyzed to determine the relevant transformation parameters. Second, a computer simulation was constructed and used to generate a very large data set on the behavior of different circular cutting possibilities. Finally, the data collected from the simulation was statistically analyzed to create a prediction model (Figure 1). Each of these steps is explained in detail in the following sections.

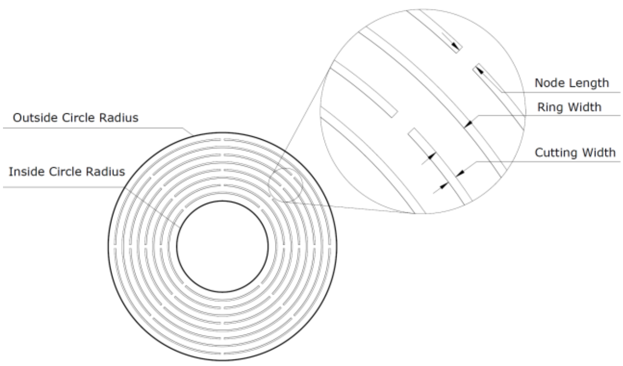

Figure 2: Some of the basic parameters of a circular paper-cut model. Source: authors.

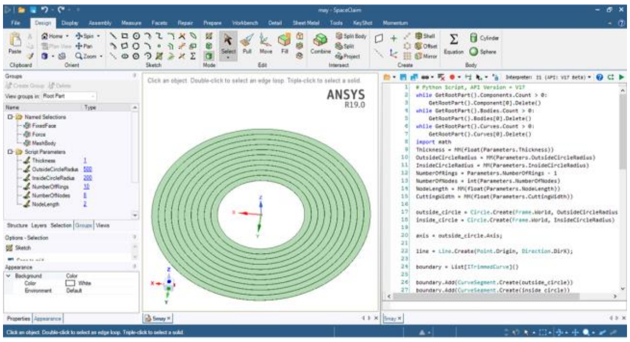

Figure 3: Creating the flat paper-cut model in ANSYS SpaceClaim using an IronPython script. The input variables are shown in the left-hand panel, as "script parameters."

MORPHOLOGICAL EXPLORATION OF KIRIGAMI ARCHITECTURE

For this initial study we decided to focus on one type of kirigami pattern – a circular cut. This form has the benefits of a step-by-step opening ability, geometrical predictableness, and relatively straightforward force and expansion calculations, while also providing important adjustable variables in the cutting pattern. Physical prototyping was used at the beginning of the project to investigate the possibilities and limitations of various circular paper-cut models. These initial experiments allowed us to understand the kinetic possibilities and geometric properties of successful designs, and to identify relevant parameters for later simulation (Strickland, 2011). As shown in Figures 2, we extracted the basic principles of circular paper-cutting designs and quantified them as variables:

Number of rings

Ring width

Number of nodes in each ring

Node length

Sheet material

Sheet thickness

Outside circle radius

Inside circle radius

The "rings" in a circular paper-cut design are the sections of solid material. Each solid ring is separated from the other rings by making a thin circular cut. The "nodes" in the design refer to the locations where two rings are joined together; in other words the nodes are breaks in the circular cuts. Experimentally adjusting the pattern of rings and nodes in several hand-made models gave us a general, tactile impression of how these variables affect the resulting structure's adaptability and form (Figure 2).

From this process we gained several important insights about the constraints and potentials of the models' geometry. For example, we discovered that in a circular paper-cutting pattern, cuts far from the center play a significant role in the model's expansion, whereas cuts near to the center hardly affect the expansion range at all. Iteratively exploring the possibilities of the paper-cutting model made it possible to clarify what at first seemed to be a very complicated geometry. Soon we began to quantify some of the effects that were created by adjustments in the variables.

COMPUTER VISUALIZATION AND SIMULATION

In the second phase of the research we moved from physical experiments to computer simulation. The simulation of an architectural model is routinely used in the second phase of the design processes to generate predictive data and to fine-tune a design solution (Ellis et al., 2006). For our purposes, we wanted to develop a simulation that would allows us to quickly collect deformation data for a huge number of circular paper-cut models. Eventually this approach could also be used to visualize the structures' behavior, investigate various material features, apply loads on the model, and set support areas for analysis.

Our first effort at simulation involved importing the 2D paper-cut model from Grasshopper/Rhino into the popular SolidWorks software platform. Unfortunately, a lack of fluid compatibility between the design program and the simulation program made this process infeasible for our needs. For every new simulation, we had to make changes to the 2D model in Grasshopper/Rhino and then re-import the data back to SolidWorks, which became a very time-consuming process. We also soon realized that other alternatives offered significantly better 3D meshing and simulation tools for our purposes. We therefore decided to recreate the 2D model in a different program that would be capable of natively simulating its expansion to a 3D structure. We were able to use the ANSYS software suite for this purpose. ANSYS is primarily a finite-element-analysis software tool, with the capacity to provide an accurate analysis of the structural behavior of a transformable architectural model.

We began with the "SpaceClaim Direct Modeler" tool, which is available as part of the ANSYS Workbench. Scripting in the SpaceClaim environment uses an open-source programming language called IronPython. Using the built-in script editor in SpaceClaim, we created a program to generate circular paper-cut geometries with options for all of the relevant variables (Hueg-Huang, 2017). The script was designed so that it accepts simple user inputs for (a) the number of rings, (b) the number of nodes, (c) the node length, (d) the sheet thickness, (e) the outside circle radius, and (f) the inside circle radius. We did not need to include an input for "ring width" in the model, because the combined values for inner radius, outer radius, and number of rings will define the ring width. The "sheet material" variable was fixed throughout all simulations to one particular type of structural steel, as discussed below. Once the relevant information is entered into the ANSYS program, the script produces the flat paper-cut model (Figure 3).

After the script created the basic structural geometry for a particular set of input values, the ANSYS Workbench was used to apply finite element analysis. This is a mathematical technique that is used to prepare the model for simulating 3D forces. In finite element analysis the overall structure is broken down into a large number of substructures ("finite elements") so that the effect of forces on the individual parts of the structure can by analyzed. There are various methods available in ANSYS workbench for this process, and we experimented with several of them. The most effective method seemed to be "automatic with adaptive size function" (Figure 4).

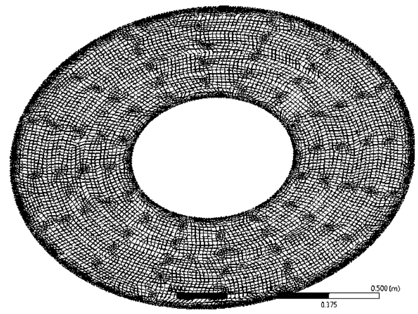

Figure 4: Generating a 3D mesh for the paper-cut model using finite element analysis in the ANSYS Workbench.

Finally, the simulation material was selected. We chose nonlinear structural steel, with yield strength of 2.5*10^8 Pascals, a tangent modulus of 1.45*10^9 Pascals, and a large deformation condition. To ensure that the simulation of structural transformations was carried out as accurately as possible, we selected the ANSYS option for nonlinear analysis. Using a linear analysis of the structural transformations would be less resource-intensive, but that approach assumes that the stiffness of the structural material does not change throughout its transformation. For our paper-cut model the stiffness will change as a result of its large deformation (a phenomenon that we noted consistently in our initial physical prototypes), so we needed to use the nonlinear alternative. It is also worth mentioning that ANSYS provides two sub-options for nonlinear materials, called "bilinear" and "multilinear" (Rosen, 2018). The multilinear option is more resource-intensive but generally more accurate. For our model, we found that using the multilinear option significantly slowed the processing speed with only a minimal increase in accuracy. Considering this, the bilinear option was chosen.

At this point we were ready to begin applying forces and transforming the model. In setting the boundary conditions we considered the disc structure to be resting flat on the ground, under the influence of normal gravitational forces. A dynamic upward force was then applied to the inner edge of the structure. The outer edge was considered to be fixed in place. We applied an increasing upward force to 300N over a one-second time period to expand the structure, and then gradually removed this force over another one-second time period to allow the structure to collapse back to a flat position (Figures 5 and 6).

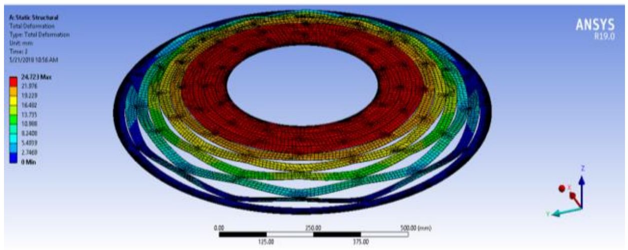

Figure 5: An upward dynamic force up to 300N was applied over one second to the inner ring of the paper-cut structure and then removed during another one-second time period.

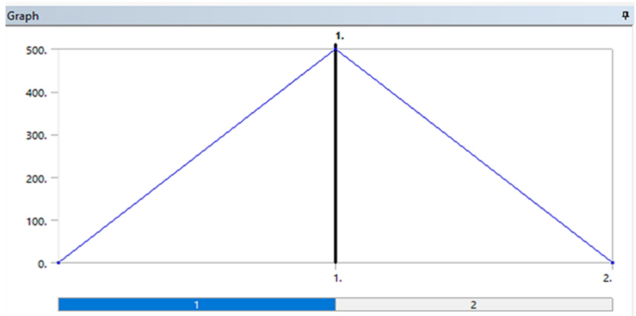

Figure 6: A partially expanded paper-cut simulation. Upward force is being applied to the inner ring of the model to create the expansion. The colors in this image represent the extent of displacement.

The ANSYS software provided a variety of data about the structures' behavior and transformation in response to the applied force. Displacement values were obtained for three regions on the structure (inner ring, middle ring, and outer ring) and the stress load was calculated for these same structural locations (Figures 7 and 8). At the end of the process, after the structures had been expanded and then allowed to collapse, a small degree of permanent deformation was noted, as indicated in the difference between the initial condition and the final displacement value (Figures 7 and 8).

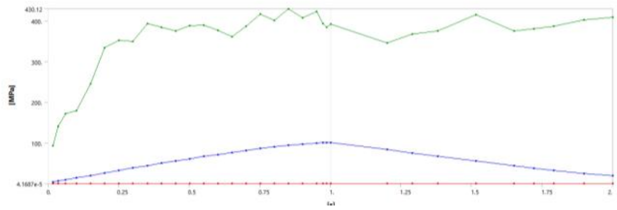

Figure 7: A graph of a sample structure's displacement over time. The green line indicates the inner ring, and the blue line indicates a middle ring. The red line indicates the outer ring, which was fixed in place.

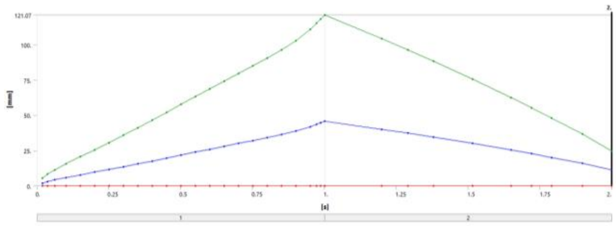

Figure 8: A graph of a sample structure's stress load over time. The green line indicates the inner ring, and the blue line indicates a middle ring. The red line indicates the outer ring, which was fixed in place.

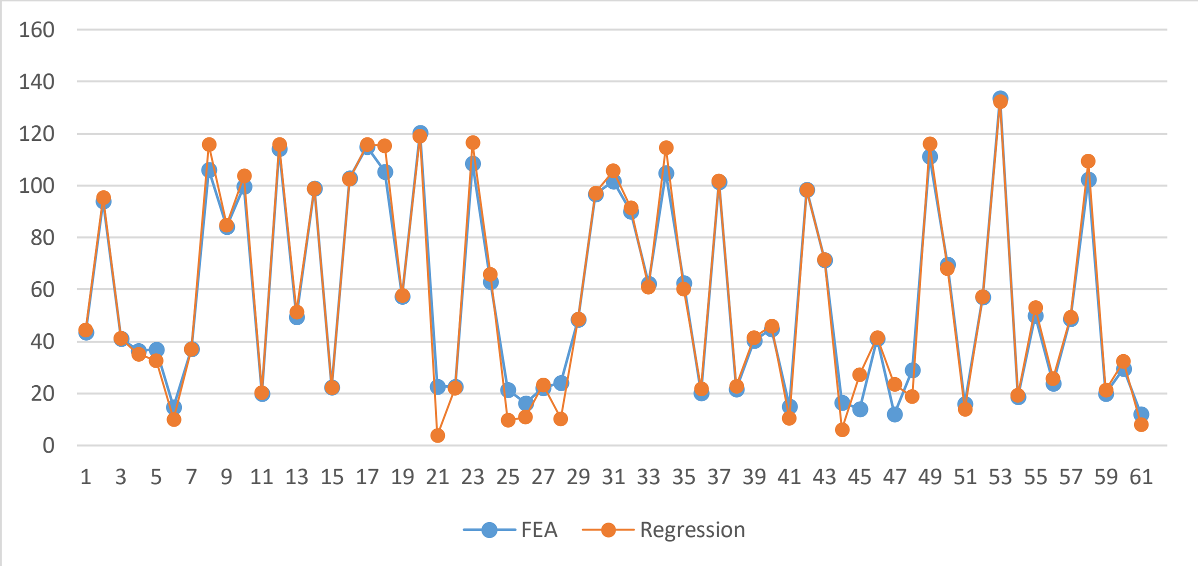

Figure 9: Validating the Kernel Ridge regression model. The model's predictions (in orange) closely match the portion of the data set (in blue) that was not used to create the regression model.

For our current purposes, the most significant data point was the maximum displacement value of the inner ring. We recorded this data point for simulations of 400 different paper-cutting patterns, allowing us to create a regression model in the next phase of the project. Other outputs, such as the maximum stress load, will be addressed in future.

DATA COLLECTION AND REGRESSION ANALYSIS

To quantify the results of changes in the model's variables, we ran simulations on 400 different circular paper-cutting patterns. In all of these simulations the structural-steel material parameters and the applied forces were kept the same. The independent variables that were changed during the simulations included the outside circle radius, the inside circle radius, the number of rings, the number of nodes per ring, and the node length (Table 1). These independent variables were manipulated in various combinations to measure their effect on the dependent variable of maximum structural displacement.

Table 1:. Independent Variables That Were Adjusted in the Structural Simulations

Variables	Range
Outside Circle Radius	500mm to 800mm
Inside Circle Radius	100mm to 200mm
Number of Rings	2 to 25
Node Length	1mm to 10mm
Number of Nodes Per Ring	6 to 15

To create a more rigorous predictive model based on this data, we applied a formal statistical regression analysis. This technique can be thought of as a machine-learning approach in which the outcome of a multi-variable system is estimated based on an extensive data set. Several regression algorithms were explored in order to find the algorithm that had the lowest predictive error for our data. Ultimately the "Kernel Ridge" algorithm was selected as the best suited to accurately identify patterns in the circular

paper-cutting dataset. This approach combines "ridge regression" (the method of linear least squares with l2-norm regularization) with the "kernel trick" (a pattern analysis based on computing the inner relationships between paired data points) (Theodoridis & Konstantinos, 2008).

The prediction performance of the Kernel Ridge regression algorithm was assessed by validating its results against a portion of the data (15%) that had not been used in the development of the regression model (Ling, 2014). Using Kernel Ridge, we were able to obtain a Root Mean Square Error (RMSE) of 5.4860, with a polynomial kernel of degree 8 and an alpha of 0.5. The low RMSE indicates that the regression model closely fits this external data set and thus has a strong predictive power (Figure 9). The nonlinear kernel of 8 confirms the complexity of the model's behavior and indicates that there is a highly nonlinear relationship among the independent variables.

PHYSICAL PROTOTYPE ASSESSMENT

After analyzing the relationships among the relevant variables and creating a mathematical model to predict the transformation volume of a circular paper-cut structure, we needed to validate the accuracy and applicability of the predictions. To accomplish this, we designed several different test structures with diverse cutting patterns. These designs were sent to a CNC fabrication lab to be cut into a steel sheet. Fabricating the designs allowed us to physically measure their structural properties and behavior. Then, independently, the relevant features of the designs were entered into our predictive model to estimate the expansion. The predicted results could thus be directly compared against the actual results in the fabricated structures.

PROTOTYPING METHOD

Three experiments were carried out to test the predictive model. Following the simulation conditions, a sheet of 1 mm thick steel was used for each of the prototypes. This presented an appropriate material strength for the scale that was being studied. Although steel-sheet material is often viewed as one of the more traditional and static architectural components, it has been re-conceptualized as an adaptive and flexible material in this study. Our work shows that even conventional steel surfaces have the potential to be transformed into a variety of expandable shapes through the application of the appropriate CNC cutting techniques.

Three circular paper-cutting patterns were generated via Python script in "Ansys Space Claim." The parameters of these structures were intended to produce diverse geometries that would cover the full range of possibilities that our prediction model supports, in regard to the inner circle radius, the number of rings, the number of nodes per ring, etc. (Table 2).

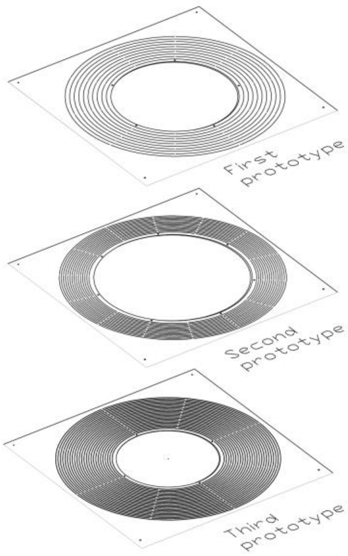

Figure 10: Three circular paper-cutting patterns were generated via Python script in "Ansys Space Claim."

Table 2. The Parameters of Three Prototypes

	First Prototype	Second Prototype	Third Prototype
Number of nodes per ring	8	10	12
Number of rings	25	10	15
Node length	0.8 cm	0.6 cm	0.4 cm
Ring width	1.3 cm	2.0 cm	1.0 cm
Inside circle radius	30 cm	40 cm	60

Using a CNC machine, we applied each circular cutting pattern to a sheet of 125x125x0.1 cm steel. Cables were then passed through the parts of the innermost ring where material had been removed. These cables were used to apply an upward force and expand the structures, while the outer ring remained fixed into place. All three experiments were performed with identical materials and processes and in the same environmental conditions.

To avoid any material damages, a steadily increasing and controlled force of up to 300N was applied to each model for 5 seconds (Figure 10 and 12). The force was then gradually removed over another 5 seconds, allowing the structures to return to a flat position. The maximum deformation achieved was recorded for each structure.

RESULTS AND DISCUSSION

To help disseminate the results of our work, we created a simple, console-based software tool that accepts circular-cutting-pattern design variables as inputs and calculates the predicted amount of maximum structural deformation based on our regression model (Figure 11). The goal of this tool is to provide a good estimate of a circular paper-cut structure's behavior for use during early design stages. The completed software tool was used during the evaluation of our mathematical model's accuracy. Table 3 shows the comparison between the software tool's predictions and the actual measured results for our three prototype structures. While the first two prototypes closely matched the predicted results (within 12% error), the third prototype diverged from the prediction by almost 61%.

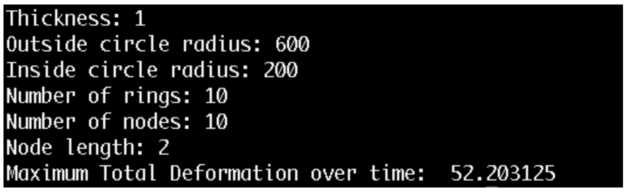

Figure 11: We created a simple software tool with a console-based user interface that predicts the maximum amount of deformation for various circular paper-cutting patterns.

Our results indicate that the complex, multifactorial behavior of circular paper-cut structural transformations can be successfully modelled with regression analysis. It is important to note that this work is only a first step in developing a more sophisticated paper-cutting design software tool. There are a remarkable number of factors that we did not include in our simulation model, such as different potential node properties, different types of structural materials, different force amounts and force dynamics during transformation, overall scale, stress calculations, and error allowances. The complexity of these issues make the simulation process extremely complicated and time-consuming. In addition, there are a variety of other paper-cut techniques beyond the circular pattern that may have valuable uses in architectural design.

Nonetheless, our initial regression model demonstrates that these obstacles can be overcome, and that quick, rigorous methods to identify optimal cutting patterns for various design tasks can be created. For designers who are seeking to create transformable structures based on a circular paper-cutting approach, our model will quickly identify the amount of deformation that is likely to be achieved by specific cutting patterns. This in itself can help to eliminate a lot of the ad-hoc experimentation and guesswork that is currently required in developing such designs.

Figure 12: The model created by applying changing ring width over surface. Source: authors.

Table 3. Comparison Between the Software Tool's Predictions and the Actual Measured Results for Our Three Prototype Structures.

	Max. Force	Predicted Deformation	Measured Prototype Deformation
First prototype	300 N	76 cm	80 cm
Second prototype	300 N	53 cm	60 cm
Third prototype	300 N	90 cm	230 cm

FUTURE WORK

We are currently working to increase the accuracy of prediction in our model by ameliorating the finite element analysis boundary conditions, the size of data set, and the regression algorithm. This will lead to a reduction in prediction error as well as the ability to incorporate a larger number of variables into the analysis, thereby making the model more appropriate for real-world conditions. There are some factors we did not include in our simulation model, such as different potential node properties, different types of structural materials, different force dynamics during transformation, stress calculations, and error allowances which we are working to be covered in future works. Additionally, Paper-cut models are able to adopt an endless number of shapes and positions to adapt to environmental conditions and user's needs. So we plan to continue the study on more complicated geometries and cut patterns that will encourage alternative expansion forms and degrees of flexibility. For example, by applying changing ring width or node number over a single surface the expanded configuration changes dramatically and a 3D model with varying pattern will be created figure (13) and figure (14). Eventually, we hope to expand our paper-cutting design tool to provide a large array of structural predictions for a variety of cutting patterns.

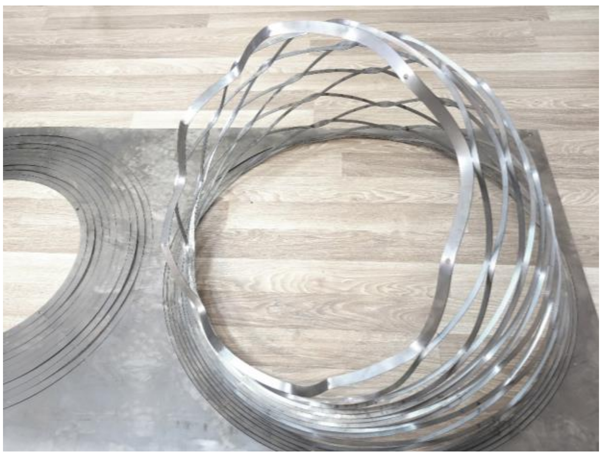

Figure 13: The model created by applying changing node number in left and right side. The side with higher node density expanded less than the other side. Source: authors.

Figure 14: model created by applying changing ring width over surface. Parts with more narrow rings expand more. Source: authors.

Figure 15: The model created by applying changing ring width over surface. Source: authors.

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